MHD Free Convection Flow past a Vertical Porous Plate with Heat Source: A Finite Difference Approach

Shailendra Kumar Gautam and AK Singh

***Abstract*— In this chapter we study the Magneto hydrodynamics (MHD) free convective heat and mass transfer flow past a vertical plate with heat source taking viscous and Darcy resistance terms into account and the constant permeability of the medium numerically. The governing nonlinear partial differential equations are solved numerically by finite difference method. The results are obtained for velocity, temperature and concentration profiles. The numerical results are presented graphically for different values of the parameters entering into the problem. Finally, the numerical values of skin friction are presented in tabular form.**

***Index Terms*— Heat and Mass transfer, free convection, MHD, Porous medium, vertical plate, Finite difference method**

I. INTRODUCTION

The convection problem in a porous medium has important applications in geothermal reservoirs and geothermal extractions. The process of heat and mass transfer is encountered in aeronautics, fluid fuel nuclear reactor, chemical process industries and many engineering applications in which the fluid is the working medium. The wide range of technological and industrial applications has stimulated considerable amount of interest in the study of heat and mass transfer in convection flows. Free convective flow past a vertical plate has been studied extensive by **Ostrach (1953)**. **Siegel (1958)** investigated the transient free convection form a vertical flat plate. **Cheng and Lau (1977)** and **Cheng and Teckchandani (1977)** obtained numerical solutions for the convective flow in a porous medium bounded by two isothermal parallel plates in the presence of the withdrawal of the fluid. In all the above mentioned studies, the effect of porosity, permeability and the thermal resistance of the medium is ignored or treated as constant. However, porosity measurements by **Benenati and Broselow (1962)** show that porosity is not constant but varies from the surface of the plate to its interior to which as a result permeability also varies. In case of unsteady free convective flows, Soundalgekar (1972) studied the effects of viscous dissipation on the flow past an infinite vertical porous plate. The combined effect of buoyancy forces from thermal and mass diffusion on forced convection was studied by **Chen et al. (1980)**. The free convection on a horizontal plate in a saturated porous medium with prescribed heat transfer coefficient was studied by **Ramanaiah and Malarvizhi (1991)**. **Bejan and Khair (1985)** have investigated the vertical free convective boundary layer flow embedded in a porous medium resulting from the combined heat and mass transfer. **Lin and Wu (1995)** analyzed the problem of simultaneous heat and mass transfer with the entire range of buoyancy ratio for most practical and chemical specieds in dilute and aqueous solutions. **Rushi kumar and Nagarajan (2007)** studied the mass transfer effect of MHD free convection flow of an in compressible viscous dissipative fluid past an infinite vertical plate. Mass transfer effects on free convection flow of an incompressible viscous dissipative fluid have been studied by **Manohar and Nagarajan (2001)**. Recently, **Sivaiah et al. (2009)** have discussed on heat and mass transfer effects on MHD free convection flow past a vertical porous plate.

Therefore, the aim of the present chapter to study the MHD free convective heat and mass transfer flow of viscous incompressible fluid through a porous medium confined in a porous plate with heat source. The velocity, temperature, concentration profiles and the skin friction have been analyzed for variations in the different parameters involved in the problem.

II. MATHEMATICAL FORMULATION:

Let us consider the two dimensional free convection and mass transfer flow of an in compressible viscous Newtonian fluid past an infinite vertical porous plate with heat source under the following assumption:

1. The plate temperature is oscillating with time, about a constant non-zero mean value.
2. Viscous and Darcy's resistance terms are taken into account with constant permeability of the medium.
3. Boussinesq's approximation is valid
4. The suction velocity normal to the plate is a function of time and can be written as $v = -U_0$

A system of rectangular coordinates $O(x^*, y^*, z^*)$ is taken such that $y^* = 0$ on the plate and z^* axis is along its leading edge. All the fluid properties are considered constant expect that the influence of the density variation with temperature is considered. The influence of the density variation in other terms of the momentum and the energy equation and the variation of the expansion coefficient with temperature is considered negligible. This is the well known Boussinesq approximation.

Shailendra Kumar Gautam, Deptt of Mathematics, Eshan College of Engineering
AK Singh, Deptt of Mathematics, RBS Engineering Technical campus Agra

We regard the porous medium as an assemblage of small identical and spherical particles of fixed space. Under these conditions, the problem is governed by the following system of equations:

$$\frac{\partial v^*}{\partial y^*} = 0 \qquad (2.1)$$

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = g\beta(T^* - T^*_\infty) + g\beta'(C^* - C^*_\infty) + \upsilon \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\upsilon}{K^*}u^* - \frac{\sigma B_0^2}{\rho}u^* \qquad (2.2)$$

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{k}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} + S^*(T^* - T^*_\infty) \qquad (2.3)$$

$$\frac{\partial C^*}{\partial t^*} + v^* \frac{\partial C^*}{\partial y^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} \qquad (2.4)$$

Where (u^*, v^*) are the velocity components, (T^*, C^*) are the temperature and concentration components and ρ is the density, $\alpha = \frac{k}{\rho C_p}$ is the thermal diffusivity and S^* is the heat source parameter.

The boundary conditions for the velocity and temperature and concentration fields are:

$$\begin{cases} u^* = 0, T^* = T_\infty^*, C^* = C^*_\infty \ at\ y^* = 0 \\ u^* = 0, T^* = T_\infty^*, C^* = C^*_\infty \ at\ y^* \to \infty \end{cases} \qquad (2.5)$$

Let us introduce the non-dimensional variables,

$$u = \frac{u^*}{u_0}, t = \frac{t^* U_0^2}{\upsilon}, y = \frac{y^* U_0}{\upsilon}, \theta = \frac{(T^* - T_\infty^*)}{(T_w^* - T_\infty^*)}, \frac{(T^* - T_\infty^*)}{(C_w^* - C_\infty^*)}$$

$$K = \frac{U_0^2 K^*}{\upsilon^*}, M = \frac{\sigma B_0^2 \upsilon}{\rho U_0^2}, \Pr = \frac{\mu C_p}{k}, Sc = \frac{\upsilon}{D}, S = \frac{S'\upsilon}{U_0^2}$$

$$Gr = \frac{\upsilon g\beta(T_w^* - T_\infty^*)}{U_0^2}, N = \frac{\beta'(C_w^* - C_\infty^*)}{(T_w^* - T_\infty^*)}$$

Where $\Pr$ is the Prandtl number, Gr is the Grashof number, N is the buoyancy ratio, Sc is the Schmidt number, M is the magnetic parameter, D is the mass diffusivity, K is the permeability parameter, β is the thermal expansion coefficient, β' is the concentration expansion coefficient and S is the heat source parameter. Other physical variables have their usual meanings.

The governing equations reduce to

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = Gr(\theta + NC) + \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K}\right)u \qquad (2.6)$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + S\theta \qquad (2.7)$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \qquad (2.8)$$

And the corresponding boundary conditions are

$$\begin{cases} u = 0, \theta = 1, C = 1 \quad ; y = 0 \\ u = 0, T = 0, C = 0 \quad ; y \to \infty \end{cases} \qquad (2.9)$$

III. SOLUTION OF THE PROBLEM:

The governing Equations (2.6-2.8) are to be solved under the initial and boundary conditions of equation (2.9). The finite difference technique is applied to solve these equations.

The equivalent finite difference scheme of equations (2.6-2.8) are given by

$$\left[\frac{u_{i,j+1}-u_{i,j}}{\Delta t}\right]=\left[\frac{u_{i+1,j}-u_{i,j}}{\Delta y}\right]+Gr(\theta_{i,j}+NC_{i,j})+\left[\frac{u_{i+1,j}-2u_{i,j}-u_{i-1,j}}{(\Delta y)^2}\right]-\left(M+\frac{1}{K}\right)u_{i,j}$$
(3.1)

$$\left[\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta t}\right]=\left[\frac{\theta_{i+1,j}-\theta_{i,j}}{\Delta y}\right]+\frac{1}{\Pr}\left[\frac{\theta_{i+1,j}-2\theta_{i,j}-\theta_{i-1,j}}{(\Delta y)^2}\right]+S\theta_{i,j} \quad (3.2)$$

$$\left[\frac{C_{i,j+1}-C_{i,j}}{\Delta t}\right]=\left[\frac{C_{i+1,j}-C_{i,j}}{\Delta y}\right]+\frac{1}{Sc}\left[\frac{C_{i+1,j}-2C_{i,j}-C_{i-1,j}}{(\Delta y)^2}\right] \quad (3.3)$$

Here index i refers to y and j to time t. The mesh is divided by taking $\Delta y = 0.1$
From the initial conditions in Equation (2.9), we have following equivalent

(3.4)

$$\left.\begin{array}{c} u(0,0)=0,\theta(0,0)=1, C\theta(0,0)=1 \\ u(i,0)=0,\theta(i,0)=0, C(i,0)=0,\ for\ all\ i\ except\ i=0 \end{array}\right\}$$

The boundary conditions from equation (3.4) are expressed in finite difference form are as follows

$$\left.\begin{array}{c} u(0,j)=0,\theta(0,j),C(0,j)=1\ \ for\ all\ j \\ u(1,j)=0,\theta(1,j)=1,C(1,j)=1\ for\ all\ j \end{array}\right\}$$

Here, infinity is taken as $y=4.1$. First, the velocity at the end of time step namely $u(i,j+1), i=1\ to\ 10$ is computed form equation (2.1) and temperature $\theta(i,j+1), i=1\ to\ 10$ from equation (2.2) and concentration $C(i,j+1), i=1\ to\ 10$ from equation (2.3). The procedure is repeated until $t=1(i.e.\ j=100)$. During computation, Δt was chosen to be 0.00125. These computations are carried out for different values of parameters $Gr, \Pr, Sc, M, N, S, t\ and\ K$. To judge the accuracy of the convergence of finite difference scheme, the same program was run with smaller values of Δt i.e. $\Delta t = 0.0009, 0.001$ and no significant change was observed. Hence, we conclude that the difference scheme is stable and convergent.

IV. Results and Discussion:

Numerical calculations have been carried out for dimensionless velocity, temperature and concentration distributions for different values of parameters and are displayed in figures (1) to (12).
Figures (1) to (8) represent the velocity profiles for different parameters. Figure (1) shows the variation of velocity u with magnetic parameter M. It is observed that the velocity decreases as M increases. From Figure (2), it is observed that the velocity increases as the Grashoff number Gr increase. The variation of u with N is shown in Figure (3). It is noticed that increase in N leads to increase in velocity. The velocity profile for time variable t is shown in figure (4). It is clear that an increase in t leads to an increase in u. Figure (5) shows that an increase in permeability parameter K causes an increase in velocity. From Figure (6) shows the variation of velocity u with Prandtl number $\Pr$. It is observed that the velocity decreases as $\Pr$ increases. The velocity profile for Schmdit number Sc is shown in figure (7). It is clear that velocity u decreases with increasing in Sc. In figure (8) the velocity profile increases due to increasing heat source parameter S. In the figure (9), it is observed that increase in Prandtl number $\Pr$ causes decrease in temperature. Figures (10) show that an increase in heat source parameter S causes an increase in temperature profile. From figure (11) it is obvious that an increase in Schmidt number Sc leads to decrease in concentration.

Figure (12) shows that the skin friction. Knowing the velocity field, the skin friction is evaluated in non-dimensional form using $\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0}$. The numerical values of τ are calculated by applying Newton's interpolation formula for 11 points and are presented. From figure (12) it is observed that an increase in Grashoff number Gr and heat source parameter S causes an increase in skin friction and an increase in magnetic parameter M leads to decrease in skin friction.

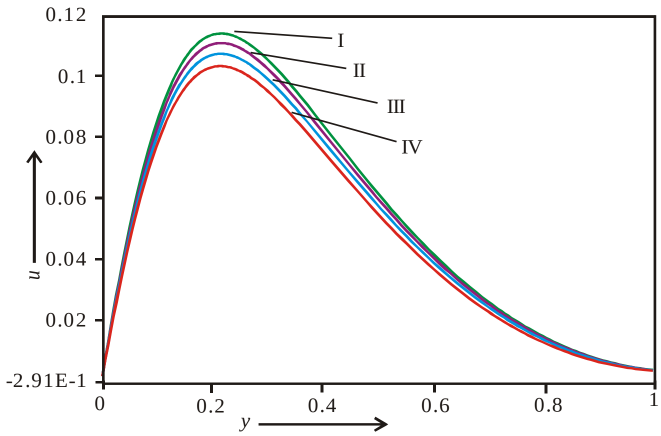

Fig. 1: The velocity profile for different values of M

	M
I	**1.0**
II	**3.0**
III	**5.0**
IV	**8.0**

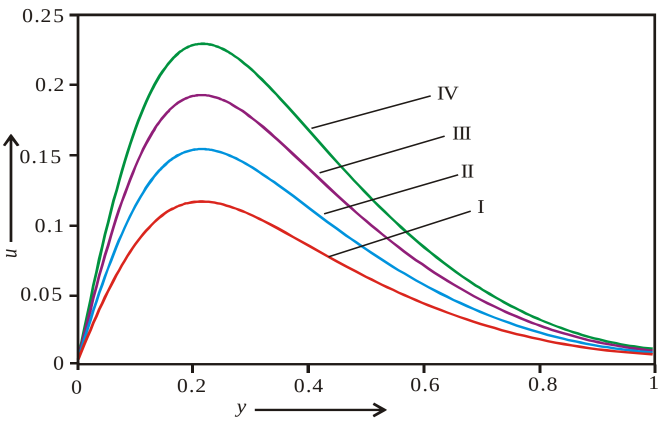

Fig. 2: The velocity profile for different values of Gr

	Gr
I	**3**
II	**4**
III	**5**
IV	**6**

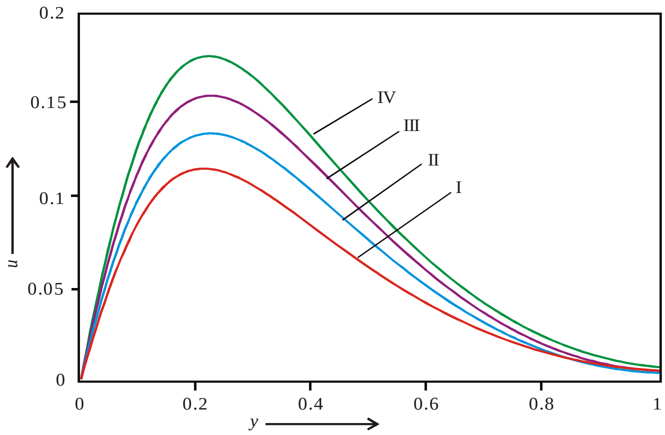

Fig. 3: The velocity profile for different values of N

	N
I	**2.0**
II	**2.5**
III	**3.0**
IV	**3.5**

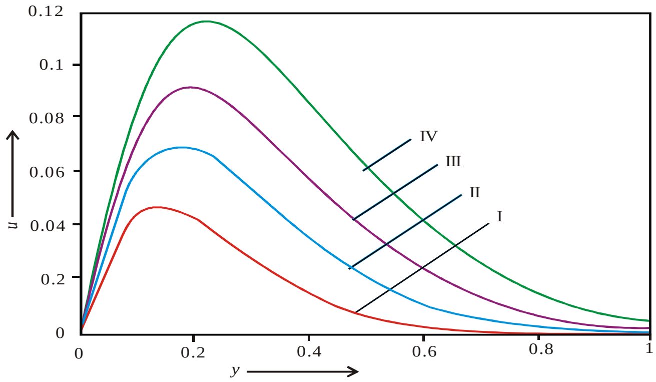

Fig. 4: The velocity profile for different values of t

	t
I	**0.02**
II	**0.03**
III	**0.04**
IV	**0.05**

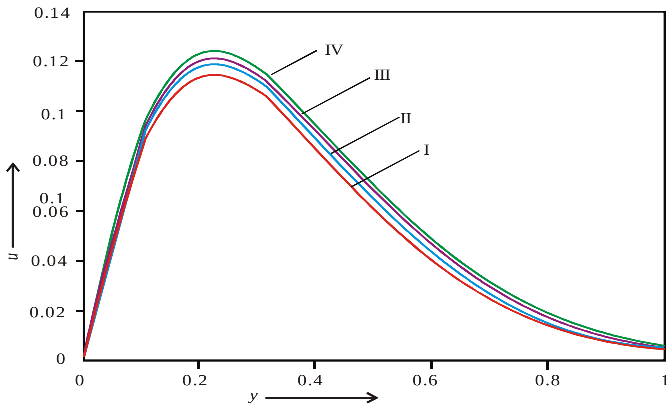

Fig. 5: The velocity profile for different values of K

	K
I	**2**
II	**5**
III	**10**
IV	**15**

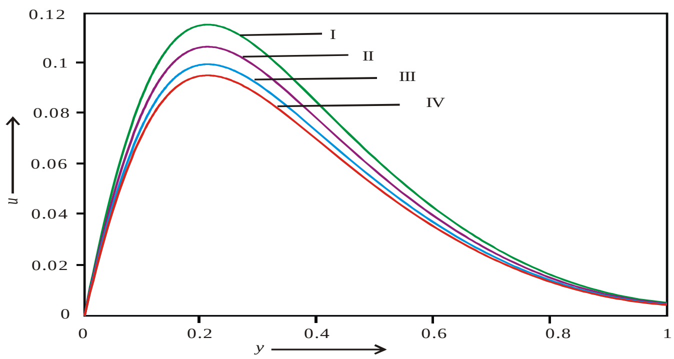

Fig. 6: The velocity profile for different values of $\Pr$

	Pr
I	**0.71**
II	**1.2**
III	**2.0**
IV	**3.0**

	S
I	**0.01**
II	**0.5**
III	**1.0**
IV	**1.5**

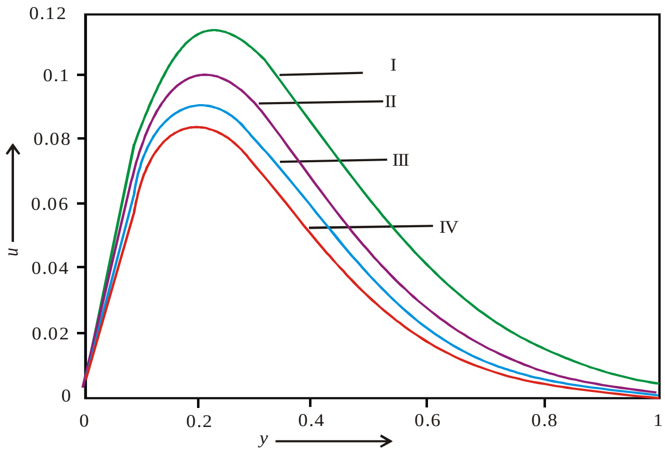

Fig. 7: The velocity profile for different values of Sc

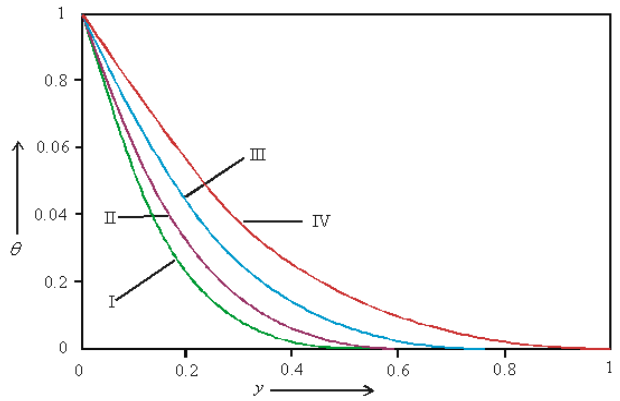

Fig. 9: The temperature profile for different values of Pr

	Sc
I	**0.4**
II	**0.6**
III	**0.8**
IV	**1.0**

	Pr
I	**0.71**
II	**1.2**
III	**2.0**
IV	**3.0**

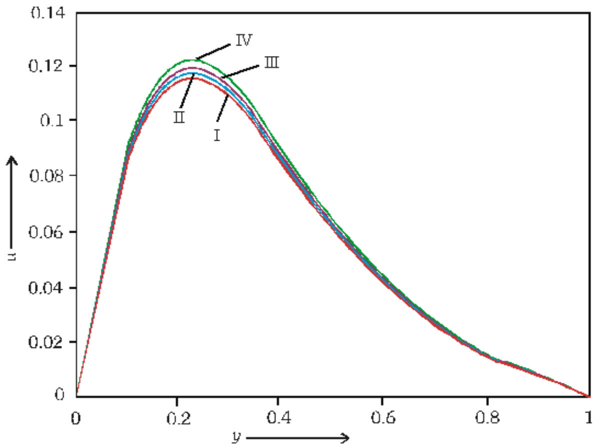

Fig. 8: The velocity profile for different values of S

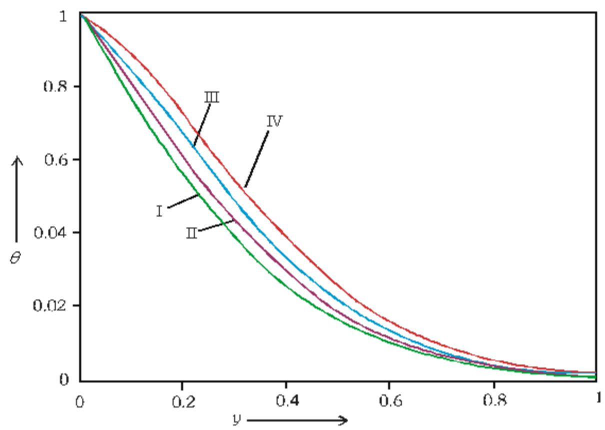

Fig. 10: The temperature profile for different values of S

	S
I	**0.01**
II	**0.5**
III	**1.0**
IV	**1.5**

	M	Gr	S
I	**1**	**3**	**0.01**
II	**3**	**3**	**0.01**
III	**1**	**4**	**0.01**
IV	**1**	**3**	**1.0**

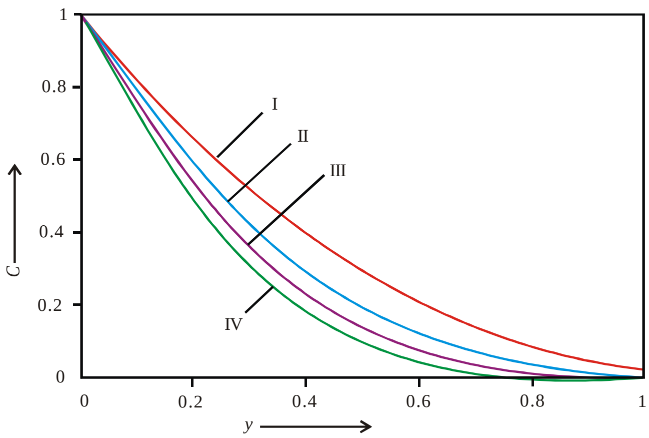

Fig. 11: The Concentration profile for different values of Sc

	Sc
I	**0.4**
II	**0.6**
III	**0.8**
IV	**1.0**

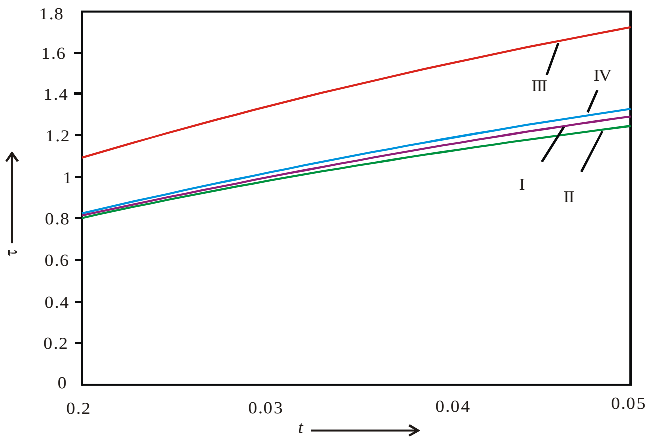

Fig. 12: The skin friction profile for the different value of M , Gr and S

V. References:

[1] Ostrich, S. (1953): "New aspects of natural convection heat transfer", Trans. Am. Soc. Mech. Eng., Vol. 75, pp.1287-1290

[2] Siegel, R. (1958): "Transient free convection from a vertical flat plate", Transactions of ASME, vol. 30,pp. 347-359

[3] Cheng, P. and Lau, K. (1977): "In Proc., 2nd Nation's symposium development", Geothermal Resources, pp.1591-1598

[4] Cheng, P. and Teckchandani, L. (1977) : "Numerical solutions for transient heating and fluid withdrawal in a liquid-dominated Geothermal reservoir", in the Earth's Crust: Its nature and physical properties, Vol. 20, pp.705-721, AGU Monograph, AGU, Washington Dc

[5] Benenati, R.F. and Broselow, C.B. (1962):AI Ch. E.J., Vol. 81, pp.359-361"

[6] Chen, T.S., Yuth, C. and Moutsoglou, A. (1980): "Combined heat and mass transfer in mixed convection along a vertical and inclined plates", Int. J. Heat Mass Transfer, Vol. 23, pp.527-537

[7] Ramanaiah, G. and Malarvizhi, G. (1991): "Free convection on a horizontal plate in a saturated porous medium with prescribed heat transfer coefficient", Acta Mech., Vol. 87, pp. 73-80

[8] Bejan, A. and Khair, K.R. (1985): " Mass transfer to natural convection boundary layer flow driven by heat transfer", ASME J. of Heat Transfer, Vol. 107, pp. 1979-1981

[9] Lin, H.T. and Wu, C.M. (1995): "Combined heat and mass transfer by laminar natural convection form a vertical plate", Int. J. Heat and Mass Transfer, Vol. 30, pp.369-376

[10] Rushi Kumar, B. and Nagarajan, A.S. (2007): "Mass transfer effects of MHD free convection flow of an incompressible viscous dissipative fluid", IRPAM, Vol. 3, No. 1, pp. 145-157

[11] Manohar, D. and Nagarajan, A.S. (2001): "Mass transfer effects on free convection flow of an incompressible viscous dissipative fluid", Journal of Energy, Heat and Mass transfer, vol. 23, pp. 445-454

[12] Siviah, M.; Nagarajan, A.S. and Reddy, P.S. (2009): "Heat and mass transfer effects on MHD free convection flow past a vertical porous plate", The Icfai university journal of computational mathematics, June Vol. II, No. 2, pp. 14-21