National Conference on Synergetic Trends in engineering and Technology (STET-2014)
International Journal of Engineering and Technical Research ISSN: 2321-0869, Special Issue

A Technical Review on Discrete-Time Sliding Mode Controller for Linear Time-Varying Systems

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***Abstract*—This paper contains the linear time varying systems with discrete time sliding mode control with modified switching function. The changed sliding surface improves control input in the initial phase and reduces the chattering with improved robustness. Also the system state is uniformly eventually bounded with better asymptotic convergence under the existence of time varying disturbances.**

***Index Terms*— Discrete-Time Systems, Linear Time-Varying System, Sliding Mode Control**

I. INTRODUCTION

The Sliding Mode Control (SMC) is governed by discontinuous feedback control law which is used for switching the structure of the system by changing the states for getting the desired response[1-2].While changing the states it moves along a sliding manifold for obtaining a closed loop system behavior[3-4].Parametric deviations and external disturbances are minimized by the introduction of sliding modes obeying the stability of the system [5].Although Continuous-Time SMC are robust to uncertainties but real time systems are built on digital device hence the discrete time SMC are subject to chattering and also a long sampling interval may lead to instability. A stable discrete time control which is independent of the sampling interval yields less chattering. The stability of the control system is designed by Lyapunov Function with the sufficient condition of the control gain was given [6].Also the phenomena of Switching, Reaching and Quasi Sliding Mode was examined thoroughly and for robustness the reaching law approach was used to develop the control law. In this work the sliding mode and reaching condition of the Discrete Variable Structure Control Systems are different from continuous time system. This reaching condition reflects all the specific properties of the process occurring in Discrete VSC Systems [8]. But crossing the sliding surface may lead to more chattering so the quasi sliding mode motion was developed which will not cross the sliding hyper plane in each step, but exists in a certain band around it. Then new reaching law technique control strategy is applied. Since the state is not crossing the sliding hyper plane consequently undesirable chattering is avoided. He guarantees convergence of the system state to a vicinity of predetermined fixed plane in finite time. In this way they achieve better regulation quality by reducing control effort [10]. Later on a discrete time sliding mode control was developed by Kang-Bak Park which works for linear time-varying systems. According to him the system state is globally uniformly ultimately bounded under the existence of the Time Varying Disturbance and uncertainties and asymptotically stable for the closed loop system. He set the magnitude of the ultimate bound to be very small simply by increasing the sampling frequency [12]. The above discrete time sliding mode control was also used practically in steering the Highway vehicles and in changing the lanes even in presence of the disturbance it found to be stable [13]. Since the presence of uncertainties and disturbances in the discrete-time sliding mode control does not guarantee the invariant property and does not assure the asymptotic convergence of the system state. And at the initial level of reaching there is a lot of chattering occurs due to uncertain disturbances.

Thus in this paper a discrete time sliding mode controller with modified function for Linear Time-Varying Systems with different disturbances is proposed and verified by simulations that the new control is able to converge better than the previous control law. It guarantees that the system state is globally uniformly ultimately bounded under the existence of Time Varying disturbance and uncertainty and yields less chattering in the initial phase of the iterations.

This paper can be organized as follows. Section II gives the problem formulation for the Linear Time Varying Systems. The deduction of the Reaching Law and Stability Analysis is carried out in Section III. At last conclusion of comparison is discussed in Section V.

II. PROBLEM FORMULATION

Let us consider a linear discrete time varying plant of the following form [12]:

$$x(k+1) = A(k)x(k) + Bu(k) + D(\qquad (1)$$

where k=1,2,3...., $x(\cdot) \in R^n$ is the state vector, $u(\cdot) \in R$ is the scalar input. $D(\cdot) \in R^n$ is the vector for external disturbance. $A(\cdot) \in R^{n*n}$ is the linear time varying system matrix. $B \in R^{n*1}$ is the input matrix.

Let the sliding surface as

$$S(k) = \tau x(\qquad (2)$$

Where $^{T} = R^n$ is assumed to be designed such that *B is non-singular.

For the system to be uniform asymptotically stable, the sliding dynamics can be represented as:

$$S(k) \equiv 0 \qquad (3)$$

For the system (1) the sliding dynamics (3) can be written as

$$\begin{aligned} S(k+1) &= \tau x(k+1) \\ &= \tau(A(k)x(k) + Bu(k) + D(k)) \\ &= S(k) = 0 \end{aligned} \qquad (4)$$

where $k = 0,1,2$

For the above equation, the equivalent control input can be written as:

$$Ueq(k) = -(\tau B) - 1[\tau A(k)x(k) + \tau D(k)] \qquad (5)$$

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Therefore for the above system, the dynamics in the sliding mode can be expressed as

$$x(k+1) = A(k)x(k) + BUeq(k) + D(k)$$
$$= A(k)x(k) + B[-(\tau B)^{-1}[\tau A(k)x(k) + \tau D(k)]] + D(k)$$
$$= A(k)x(k) - B(\tau B)^{-1}\tau A(k)x(k) - B(\tau B)^{-1} CD((+ D($$
$$= (I - B(\tau B)^{-1})A(k)x(k) + (I - B(\tau B)^{-1})D(k)$$
$$= (I - B(\tau B)^{-1})[A(k)x(k) + D(k)] \quad (6)$$

Thus we can say the is assumed to be chosen such that the above sliding dynamics turns to be stable.

A system is in a quasi-sliding mode when the dynamics of $S($ meets the following conditions [6].

1. Starting from any state, the $S($ moves towards the quasi-sliding surface defined by $S(k) =$ and crosses it in a finite period of time.
2. As the sliding surface is crossed by the first time the s(k) value changes around the surface in a zig-zag motion.
3. The sliding zig-zag motion is stable and stays inside a fixed band.
4.

III. Definitions[8]:

1. The motion of a discrete-time system satisfying attributes (2) and (3) is called a quasi-sliding mode. The specified band which contains the quasi-sliding mode is called the quasi-sliding mode band is defined as:

$$\{x| -\Delta < s(k) < + \quad (7)$$

where 2Δ is the width of the band.

2. The quasi- sliding mode becomes an ideal quasi- sliding mode when $\Delta =$.

3. A discrete time system is said to satisfy a reaching condition if the resulting system possesses all three conditions (1),(2) and (3).

The reaching law straightly dictates the dynamics of the switching function and for a discrete time system it can be stated as [8]:

$$S(k+1) - S(k) = -qTs(k) - Tsgn(s(k)) \quad (8)$$

where $\in > 0, q > 0, 1 - qT >$

$T >$ is the sampling period.

IV. Reaching Law and Stability Analysis

Let's begin with the incremental change of the switching mode $s($ i.e.

$$s(k+1) - s(k) = \tau x(k+1) - \tau x(\quad (9)$$

Substituting (1) into (9) gives:

$$s(k+1) - s(k) = \tau A(k)x(k) + \tau Bu(k) + \tau D(k) - \tau x(k) \quad (10)$$

Now comparing it to reaching law (8):

$$s(k+1) - s(k) = -qTs(k) - \in Tsgn(s(= \tau A(k)x(k) + \tau Bu(k) + \tau D(k) - \tau x(k) \quad (11)$$

Solving for $u($ gives the control law:

$$u(k) = -(\tau B)^{-1}[\tau A(k)x(k) + \tau D(k) - \tau x(k) + qT\tau x(k) + \in Tsgn(\tau x(k))]$$

Substituting (12) into (1) gives the response of the linear discrete time linear Time varying plant.

$$x(k+1) = A(k)x(k) - B(\tau B)^{-1}[\tau A(k)x(k) + \tau D(k) - \tau x(k) + qT\tau x(k) + \in Tsgn(\tau x(k)] + D(k) \quad (13)$$

The sliding mode band is given by (7).The objective is to determine the bandwidth (2Δ) so that when the trajectory moves into the band it remains stay within the band.

Multiply '$\boldsymbol{\tau}$' on both sides of (13) yield.

$$S(k+1) = (1 - qT)S(k) - \in Tsgn(S(k)) \quad (14)$$

where $1 - \quad > 0$

In the above equation the sign of $S(k +$ must be opposite to that of $S($ & this reason is given by [3]

$$\{x|s(x)| < \frac{\in T}{1 - qT}\}$$

Therefore, the bandwidth is given by

$$= 2\frac{\in T}{1 - qT} \quad (15)$$

In (15) the trajectory will more with a smaller band & with decreasing sampling period, the width of both the bands decreases.

In order to proof the system is in quasi-sliding mode, the reaching law can be established by mathematically proving:

$$S(k)[S(k+1) - S(k)] < 0$$
$$S(k).S(k+1) - S^2(k) < \quad (16)$$

Since $S(k) =$, hence $S^2)$ is negligible and assumed to be zero.

Therefore,

$$S(k+1).S(k) < 0 \quad (17)$$

Now from (2) we can write (17) as

$$\tau x(k+1).\tau x(k) < 0$$
$$\tau^2[A(k)x(k) + Bu(k) + D(k)]x(k) < 0$$
$$\tau^2[A(k)x(k) - B(\tau B)^{-1}[\tau A(k)x(k) + \tau D(k)] + D(k)]x(k) < 0$$
$$x(k)[A(k)x(k) - B(\tau B)^{-1}\tau A(k)x(k) - B(\tau B)^{-1}\tau D(k) + D(x)] < 0$$
$$\tau^2 x(k)\left[(A(k)x(k) + D(k))(I - B(\tau B)^{-1}\tau)\right] < 0 \quad (18)$$

And for fulfilling the reaching law the values for matrices B and C should be chosen such that the product $B(\tau B)^{-1}\tau$ must be greater than identity matrix I. And in our simulation it is shown that $A(k)B(\tau B)^{-1}\tau$ is greater than $A(k)$ hence obeying the condition for reaching law.

V. Conclusion

In this paper, the discrete time varying system with modified sliding surface is proposed which improves the initial phase of chattering and smooth trajectory is established which is uniformly eventually bounded.

References

[1] V. I. Utkin, "Variable structure systems with sliding modes,"IEEE Trans. Automat. Contr., vol. AC-22, no. 2. pp. 212-222, 1977.

[2] V. I. Utkin, "Sliding Modes and Their Application to Variable Structure Systems," Moscow: Mir. 1977.

[3] J. Y. Hung, W. Gao and J. C. Hung, "Variable structure control: A survey," IEEE Trans. Ind. Electron., vol. 40, no. 1, pp. 2–21, Feb. 1993.

[4] K. D. Young, V. I. Utkin and U. Ozguner, “A control engineer’s guide to sliding mode control,” IEEE Trans. Control Syst., vol. 7, no. 3,pp. 328–342, May 1999.
[5] B. Drazenovic,“The invariance conditions in variable structure systems,”Automatica, vol. 5, pp. 287-295. 1969.
[6] K. Furuta, “Sliding mode control of a discrete system,” Syst. Contr. Lett., vol. 14, no. 2, pp. 145–152, Feb. 1990.
[7] A. Bartolini, A. Ferrara, and V. Utkin, “Adaptive mode control in discrete-time systems,” Automatica, vol. 31, no. 5, pp. 769–773, 1995.
[8] W. Gao, Y. Wang and A. Homaifa, “Discrete time variable structure control systems,” IEEE Trans. Ind. Electron., vol. 42, pp. 117–122, Apr.1995.
[9] C.Y.Chan, “Discrete Adaptive Sliding-mode Tracking Controller,” Automatica, vol. 33, no. 5, pp. 999–1002, 1997.
[10] Andrzej Bartoszewicz, “Discrete-Time Quasi-Sliding-Mode Control Strategies,” IEEE Trans. Industrial. Electronics, vol. 45,No.4 Aug. 1998.
[11] David Munoz et.al, "An Adaptive Sliding-Mode Controller for Discrete Nonlinear Systems," IEEE Trans. Industrial. Electronics, vol. 47,No.3 Jun. 2000.
[12] Kang-Bak Park,”Discrete-Time Sliding Mode Controller for Linear Time-Varying Systems with Disturbances,” ICASE, Korea, Vol 2,No.4,Dec.2000.
[13] J.Kharaajoo, "Discrete-time sliding mode control design for linear time-varying systems: application to steering control of highway vehicles," Mechatronics, ICM-04. Proceedings of the IEEE International Conference on, On page(s): 113 - 116, 2004.
[14] O. Kaynak and A. Denker, “Discrete-time sliding mode control in presence of system uncertainty,” Int. J. Contr., vol. 57, no. 5, pp. 1177–1189, 1993.
[15] D. Muñoz, “Discrete sliding mode controller for nonlinear systems,” M.S. thesis (in Spanish), Dep. Elect. Eng., Faculty Eng., Univ. Concepción, Concepción, Chile, 1997.
[16] R. Johansson, “Lyapunov design for adaptive control of robots,” in Proceedings IFAC Nonlinear Control Systems Design, Capri, Italy, 1989, pp.391–396.
[17] C.Edwards and S.K. Spurgeon,”Sliding Mode Control:Theory and Applications,”Taylor & Francis,1798-1998.
[18] J. Y. Hung, W. Gao, and J. C. Hung,"Variable structure control: A survey," IEEE Trans. Ind. Electron.,vol. 40, no. 1, pp. 2-22, 1993.
[19] S. Z. Sarpturk, Y. Istefanopulos, and O. Kaynak,"On the stability of discrete-time sliding mode control systems," IEEE Trans. Automat. Contr., Vol. 32, No. 10, pp. 930-932, 1987.
[20] W. Gao, Y. Wang, and A. Homaifa,"Discrete-time variable structure control systems," IEEE Trans. Ind.Electron., vol. 42, no. 2, pp. 117-122, 1995.
[21] Y. Pan and K. Furuta,"Discrete-time VSS controller design," International Journal of Robust and Nonlinear Control, vol. 7,pp. 373-386, 1997.
[22] P. Myszkorowski, "Robust controller for the linear discrete-time systems," Proc. 30th IEEE Conf. on Decision and Control, England, pp. 2972-2973, 1991