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MHD Slip Flow of Alumina Water Nanofluid over a Flat Plate

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Abstract— Present paper deals with analysis of magnetohydrodynamic (MHD) flow of an incompressible, viscous, electrically conducting and steady alumina-water nanofluid in the presence of transverse magnetic field over a flat plate with slip conditions at the boundary. The governing equations are transformed into a set of simultaneous ordinary differential equations by using appropriate similarity transformation. The set of equations thus obtained has been solved numerically by using Adaptive Runge-Kutta method with shooting technique. The effects of various physical quantities, viz. magnetic parameter, velocity slip parameter, thermal slip parameter and Prandtal number on velocity distribution, temperature distribution, shear stress, temperature gradient were depicted graphically and analyzed. Significant change was observed in the heat transfer rate due to magnetic field under boundary layer slip.

***Index Terms*— Magnetohydrodynamic, Nanofluid, Boundary Layer Slip Flow, Volume Fraction.**

I. INTRODUCTION

Alumina water nanofluid has wide range of applications in real life situations. Conventional fluids, such as water, lubricants, and other coolant are normally used as heat transfer fluids. Although, various techniques are applied to enhance the heat transfer, the low heat transfer performance of these conventional fluids obstructs the performance enhancement and the compactness of heat exchangers. The use of solid particles as an additive suspended into the base fluid is technique for the heat transfer enhancement. Improving the thermal conductivity is the key idea to improve the heat transfer characteristics of conventional fluids. Since a solid metal has a larger thermal conductivity than a base fluid, suspending metallic solid fine particles into the base fluid improves the thermal conductivity of that fluid. The enhancement of thermal conductivity of conventional fluids by the suspension of solid particles of millimeter/micrometer-sized particles has been well-known for many years. Specific heat measurements of aluminum oxide nanofluid with various particle concentrations were studied and showed that applying nanofluids resulted in a reduction of cogeneration efficiency. This is due to the decrease in specific heat capacity, which influences the waste heat recovery from the engine. However, it was found that the efficiency of waste heat recovery heat exchanger increased for nanofluid due to its superior convective heat transfer coefficient. Addition of aluminum oxide nanoparticles remarkably decreases the super cooling degree of water, advances the beginning freezing time, and reduces the total freezing time. Xuan and Roetzel [1] investigated the mechanism of heat transfer enhancement of the nanofluid. The effects of transport properties of the nanofluid and thermal dispersion were included in their study. Seddek [2] analyzed the effect of magnetic field on the flow of a micropolar fluid past a continuously moving plate. The effect of the magnetic parameter on the microrotation was discussed through graph. Yang et al.[3] measured the convective heat transfer coefficients of several nanoparticles-in-liquid dispersions (nanofluids) under laminar flow in a horizontal tube heat exchanger. The authors found that the graphite nanoparticles increased the static thermal conductivities of the fluid significantly at low weight fraction loadings. Wen and Ding [4] presented formulation of aqueous based nanofluids and its application under natural convective heat transfer conditions. Transient and steady heat transfer coefficients were measured and the results shown a systematic decrease in the natural convective heat transfer coefficient with increasing particle concentration. Patel et al. [5] presented the model to determine effective thermal conductivity of nanofluid. A new way of modeling thermal conductivity of nanofluids has been explored which is found to agree excellently with a wide range of experimental data obtained by the present authors as well as the data published in literature. Jou and Tzeng [6] investigated heat transfer enhancement utilizing nanofluids in a two-dimensional enclosure for various pertinent parameters. They found that increasing the buoyancy parameter and volume fraction of nanofluids cause an increase in the average heat transfer coefficient. Ding et al.[7] studied the mixed convective flow of nanofluid in the nucleate regime. The results were shown that the boiling heat transfer enhanced in the nucleate regime for both alumina and titanium nanofluids, and the enhancement is more sensitive to the concentration change for TiO_2 nanofluids. Cheng [8] presents an overall review of a number of patents on nanofluid heat transfer technologies and their applications for the energy efficiency improvement in various thermal systems in recent years. According to that review, the future developments of those technologies were discussed. Santra et al.[9] studied the heat transfer in copper water nanofluid through parallel plates. It has been observed that the heat transfer augmentation is possible using nanofluid in comparison to conventional fluids. The rate of heat transfer increases with the increase in flow as well as increase in solid volume fraction of the nanofluid. Aziz [10] analyzed the hydrodynamic and thermal slip flow with constant heat flux over a flat plate. The author observed that an increment in Prandtl number and the slip parameter reduces the dimensionless surface temperature. Timofeeva et. al [11]

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presented some studies that address the complexity of nanofluid systems and advance the understanding of nano scale contributions to viscosity, thermal conductivity, and cooling efficiency of nanofluids. In this article the relative importance of nanofluid parameters for heat transfer were evaluated which allowed engineering nanofluids with desired set of properties. Bouihi and Sehaqui [14] studied the natural convection in a two-dimensional enclosure with a sinusoidal boundary thermal condition utilizing nanofluid numerically. The authors proposed different correlations for predicting heat transfer for uniform and sinusoidal boundary thermal conditions. Uddin et al. [15] studied steady two dimensional MHD laminar free convective boundary layer flows of an electrically conducting Newtonian nanofluid over a solid stationary vertical plate in a quiescent fluid taking into account the Newtonian heating boundary condition. It was found that the rate of heat and mass transfer increase as Newtonian heating parameter increases. The dimensionless velocity and temperature distributions increase with the increase of Newtonian heating parameter. The results of the reduced heat transfer rate were compared for convective heating boundary condition and found an excellent agreement. Colla et al. [16] presented an experimental investigation on water-based nanofluids containing iron oxide (Fe_2O_3) in concentrations ranging between 5 and 20% in mass. The temperature and nanoparticles concentration effects on viscosity were analyzed, obtaining a significant increase with respect to water. All the fluids exhibited a Newtonian behavior. The experimental values were compared with some theoretical models for both thermal conductivity and dynamic viscosity. Aladag et. al. [17] presented experimental investigations of the viscosity of nanofluids at low temperatures. Experiments also showed that CNT water based nanofluid behaves as Newtonian fluid at high shear rate whereas Al_2O_3 water based nanofluid is non-Newtonian within the range of low temperatures investigated.

The present investigation deals with two dimensional MHD flow with slip of an incompressible, viscous, forced convective, electrically conducting and steady alumina water nanofluid. This nanofluid is a colloidal suspension of spherical alumina particles with diameter less than 100nm fluid water at room temperature.

II. MATHEMATICAL DESCRIPTION AND SOLUTION OF THE PROBLEM

The figure of the problem considered has been given below along with flow configuration and coordinate system. The system deals with two dimensional MHD flow with slip of an incompressible, viscous, forced convective, electrically conducting and steady water based nanofluid with colloidal suspension of spherical alumina particles. The magnetic field B_0 is imposed in transverse direction. The plate length is considered infinite; The uniform velocity at infinity is u_∞. The temperature on the wall is T_w, and far from the wall is T_∞. The continuity, momentum and energy equations representing flow are following:

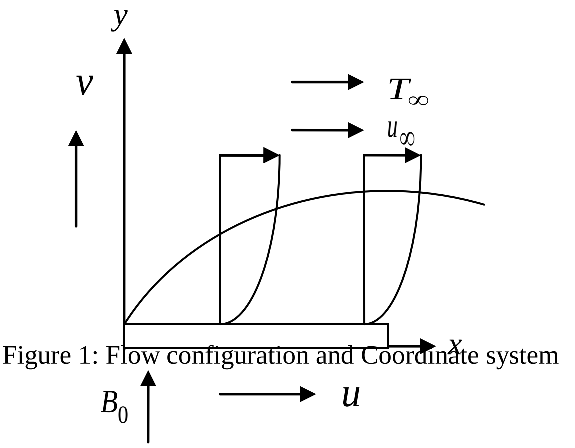

Figure 1: Flow configuration and Coordinate system

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\vartheta_{nf}\frac{\partial^2 u}{\partial y^2}+\frac{\sigma_{nf}B_0}{\rho_{nf}}(u_\infty-u) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{nf}}{\rho_{nf}C_{pnf}}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

With the boundary conditions:

$u=V_{sf}\frac{\partial u}{\partial y}$, v = 0 and $T=T_w+T_{sf}\frac{\partial T}{\partial y}$ at y = 0 and

$u\rightarrow u_\infty$, $T\rightarrow T_\infty$ when $y\rightarrow\infty$ (at free stream) (4)

Where u and v represent the velocity components in the x and y directions respectively, and $V_{sf}=V_{sf}^0\sqrt{\frac{u_\infty x}{\vartheta_{nf}}}$, $T_{sf}=T_{sf}^0\sqrt{\frac{u_\infty x}{\vartheta_{nf}}}$ are velocity slip factor and thermal slip factor with initial values V_{sf}^0, T_{sf}^0 respectively. ϑ_{nf} is the kinematic viscosity, ρ_{nf} is the density, k_{nf} is the thermal conductivity, C_{pnf} is the specific heat capacity and σ_{nf} is the electrical conductivity of the nanofluid.

The effective thermal conductivity of the nanofluid is given by Patel et al.[5] as follows:

$$k_{nf}=k_f\left\{\frac{(k_s/k_f)+(n-1)-(n-1)\varnothing(1-(k_s/k_f))}{(k_s/k_f)+(n-1)-\varnothing(1-(k_s/k_f))}\right\} \tag{5}$$

Density of nanofluid is given by Santra et al.[9] as follows:

$$\rho_{nf}=(1-\varnothing)\rho_f+\varnothing\rho_s \tag{6}$$

and dynamic viscosity of nanofluid is given as:

$$\mu_{nf}=\frac{\mu_f}{(1-\varnothing)^{2.5}} \tag{7}$$

Where $\varnothing$ is the volume fraction of the Alumina solid particles and $\mu_f=0.001002\ Ns/m^2$ is the dynamic viscosity of water and $n=\frac{3}{\psi}$, $\psi=1$ for spherical particles, ψ is the spherecity and n is the empirical shape factor.

The kinematic viscosity of nanofluid is given as:

$$\vartheta_{nf}=\frac{\mu_{nf}}{\rho_{nf}} \tag{8}$$

Table 1: The physical properties of Al_2O_3 and base fluid water at 20^oC have been suggested by Anjali et. al. [12] given below:

	Density (Kg/m^3)	Specific Heat (J/Kg.K)	Thermal Conductivity (W/m.K)
Water	1000.52	4181.8	0.597
Alumina	3970.0	769.0	36.0

To solve the governing equations (1), (2) and (3) following similarity transformation has been introduced as given Bhattacharyya et al.[13]:

$\psi = \sqrt{u_\infty v x}\, f(\eta)$, $\eta = \frac{y}{x}\sqrt{\frac{x u_\infty}{v}}$ and $\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$ where ψ is the stream function which satisfies equation (1) with $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$.

After using above transformation the equations (2) and (3) reduced to the nonlinear differential equations as follows:

$$2f'' + ff' - 2M(f' - 1) = 0 \qquad (9)$$

$$2\theta'' + Prf\theta' = 0 \qquad (10)$$

and the boundary conditions (4) reduced as follows:

$$f(0) = 0, \quad f'(0) = \gamma f''(0), \quad \theta(0) = 1 + \epsilon\theta'(0) \text{ and } f'(\eta) \to 1,\ \theta(\eta) \to 0 \quad \text{as } \eta \to \infty \qquad (11)$$

Where $Pr = \frac{\mu_{nf} C_{pnf}}{k_{nf}}$ is the Prandtal number, $M = \frac{\sigma_{nf} B_0^2}{\rho_{nf} u_\infty}$ is the magnetic parameter, $\gamma = \frac{V_{sf}^0\, u_\infty}{v}$ is he velocity slip parameter and $\epsilon = \frac{T_{sf}^0\, u_\infty}{v}$ is the thermal slip parameter.

The set of non-linear differential equations (9) and (10) subject to the boundary conditions (11) form a two point boundary value problem. To solve this system of non-linear differential Adaptive Runge-Kutta method with shooting technique have been applied.

III. RESULTS AND DISCUSSION

The numerical computations have been made for velocity profile, temperature profile, Shear stress and other parameters involved in the flow. The results were depicted graphically and their physical explanation also given corresponding to different values of velocity slip parameter, magnetic parameter, Prandtal number, thermal slip parameter and volume fraction of alumina nanoparticles. The physical and thermal properties of alumina water nanofluid corresponding to different volume fractions have also been tabulated in table 2.

The effect of magnetic parameter M on velocity profile $f'(\eta)$ and temperature profile $\theta(\eta)$ under slip and no slip conditions have been present graphically through figures 2 – 3 and figures 6 - 7. It is noticed that in both the cases, velocity increases with an increase in magnetic parameter consequently boundary layer thickness decreases. It was observed that temperature of alumina water nanofluid decreases with an increase in magnetic parameter. Increment in the magnetic parameter value causes decay in thermal boundary layer thickness of nanofluid. Figures 4 - 5 show the effect of magnetic parameter on shear stress profile $f''(\eta)$ under no slip and slip conditions respectively. It was observed that the shear stress decreases with an increase in magnetic parameter. The effect of magnetic parameter on temperature gradient $-\theta'(\eta)$ has been studied through figures 8 and 9 under no slip and slip conditions. It is observed that enhanced velocity of the nanofluid due to boundary slip near the wall of the plate increasing the heat transfer rate. The effect of thermal slip parameter on temperature profile shown in figure 10. The effect of velocity slip parameter on temperature gradient shown in figure 11. It is observed that the effect of nanoparticles volume concentration is very small on temperature gradient. The effect of magnetic interaction parameter on temperature gradient $-\theta'(\eta)$ at the wall of the plate against thermal slip parameter ε has also been shown in figure 12 and analyzed. Figure13 shows the effect of volume fraction ϕ of alumina nano particles on temperature gradient $-\theta'(\eta)$ under boundary slip conditions. Figure 14 display the effect of Prandtal number of alumina water nanofluid on shear stress profile. Figure 15 shows the skin friction coefficient $f''(0)$ against the velocity slip parameter γ for different values of the magnetic parameter. It is observed that the skin friction coefficient decreases as the velocity slip parameter increases and the magnetic field affects conversely.

Table 2: Physical parameters for Al_2O_3 Nanofluid at 20^0C

Vol. Fraction ϕ	Thermal Conductivity K_{nf}	Dynamic Viscosity μ_{nf}	Prandtal Number Pr	Density ρ_{nf}
0.00	0.597000000	1.002000000	7.018699497	1000.5200
0.01	0.614211407	1.027495042	6.938510692	1030.2148
0.02	0.631756826	1.053907590	6.862284771	1059.9096
0.03	0.649646074	1.081280537	6.789840351	1089.6044
0.04	0.667889357	1.109659253	6.721010316	1119.2992
0.05	0.686497292	1.139091755	6.655640650	1148.9940
0.06	0.705480921	1.169628892	6.593589409	1178.6888
0.07	0.724851739	1.201324544	6.534725789	1208.3836
0.08	0.744621712	1.234235839	6.478929299	1238.0784
0.09	0.764803307	1.268423387	6.426089022	1267.7732
0.10	0.785409515	1.303951529	6.376102950	1297.4680

Table 3: Values of $f''(0)$ and $-\theta'(0)$ for various values of Magnetic Parameter M, velocity slip parameter γ, thermal slip parameter ε, Prandtal number Pr.

Magnetic Parameter M	Velosity slip parameter γ	Thermal slip parameter ε	Prandtal Number Pr	$f''(0)$	$\theta'(0)$
0	0	0	6.938510692	0.332	0.643
0	0.3	0.2	6.938510692	0.325	0.657
0.2	0.3	0.2	6.938510692	0.4895	0.737
0.4	0.3	0.2	6.938510692	0.5975	0.781
0.5	0.3	0.2	6.721010316	0.6409	0.788
0.5	0.3	0.2	6.478929299	0.6409	0.777
0.5	0.3	0.2	6.37610295	0.6409	0.773
1	0.3	0.2	6.938510692	0.8069	0.8514

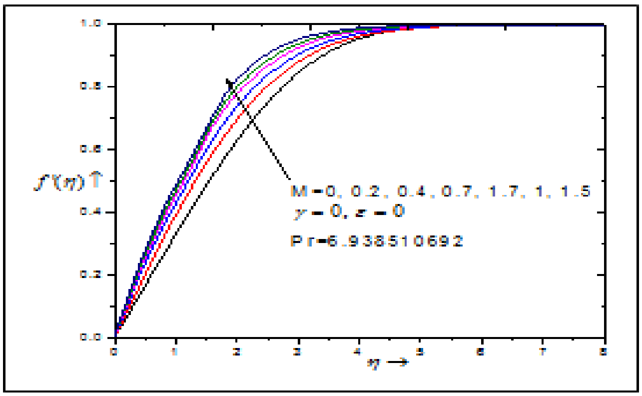

Fig. 2: Variation of magnetic parameter on velocity profile under no slip condition

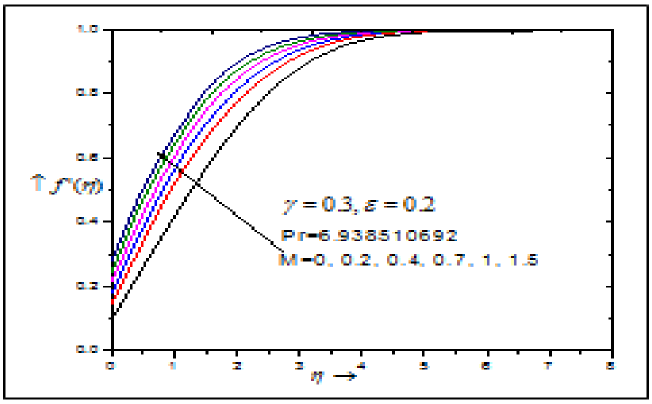

Fig.3: Variation of magnetic parameter on velocity profile under slip condition

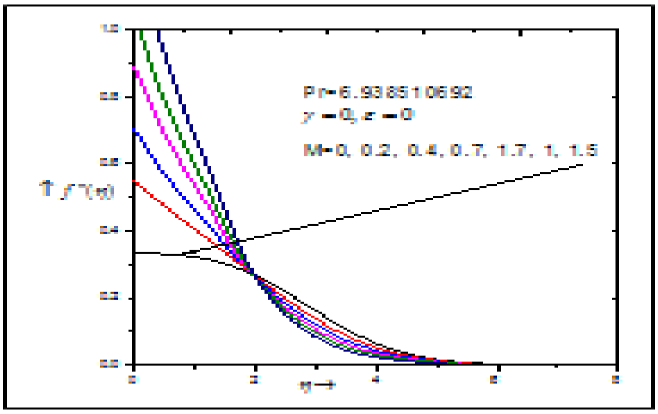

Fig. 4: Shear stress under no slip condition

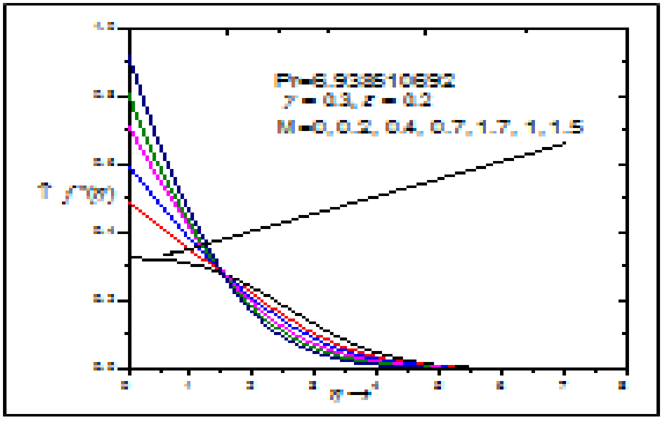

Fig. 5: Shear stress under slip condition

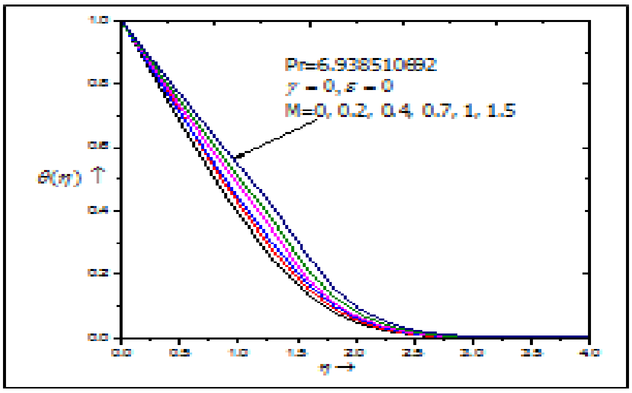

Fig. 6: Temperature profile under no slip conditions

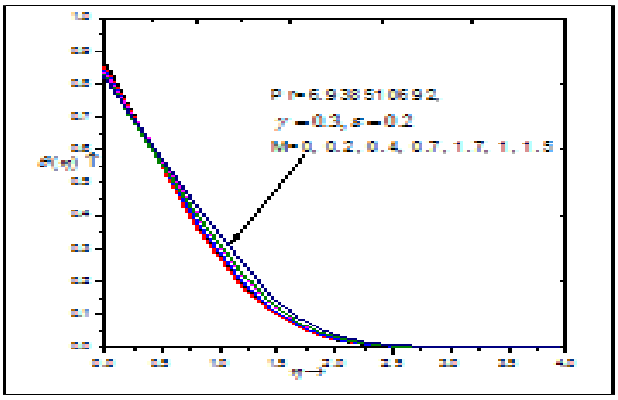

Fig. 7: Temperature profile under slip conditions

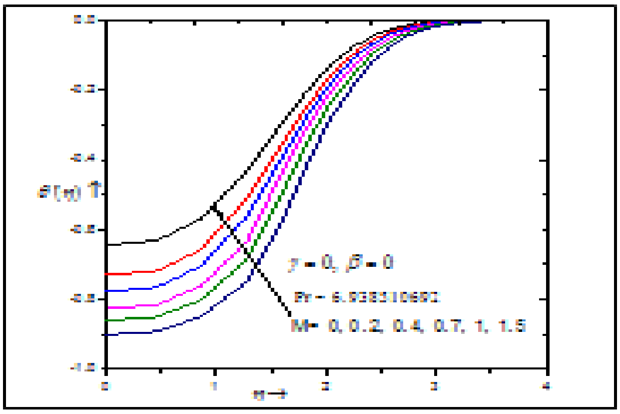

Fig. 8: Temperature Gradient under no slip condition

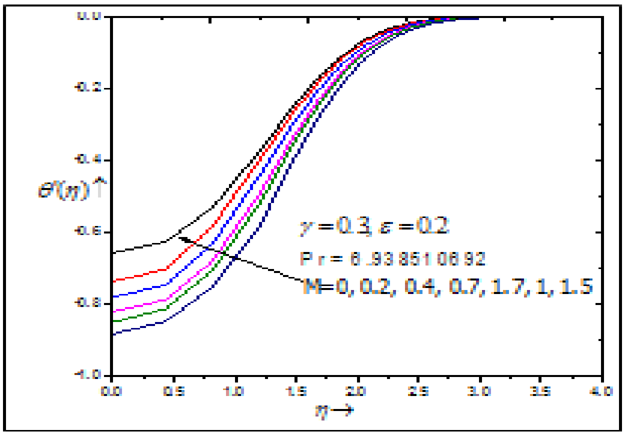

Fig. 9: Temperature Gradient under slip conditions

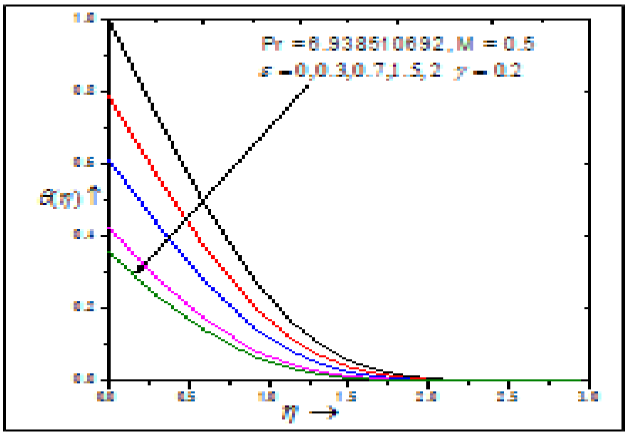

Fig. 10: Effect of thermal slip on temperature

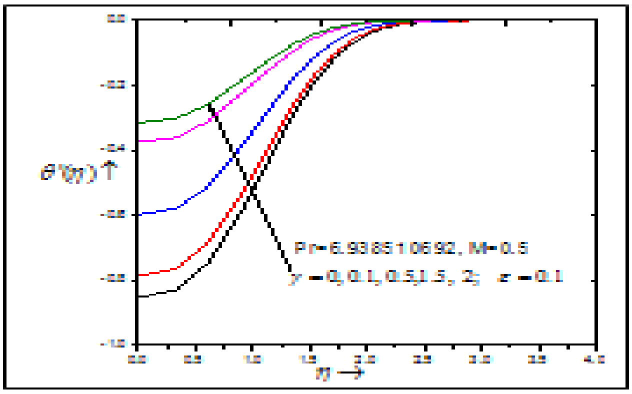

Fig. 11: Effect of velocity slip on temperature gradient

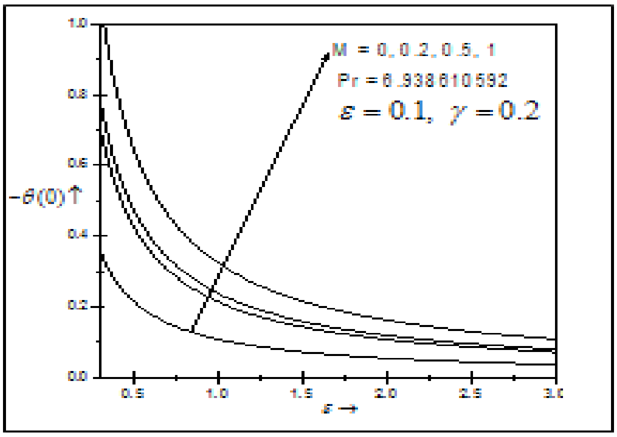

Fig. 12: Temperature gradient at the plate against thermal slip parameter

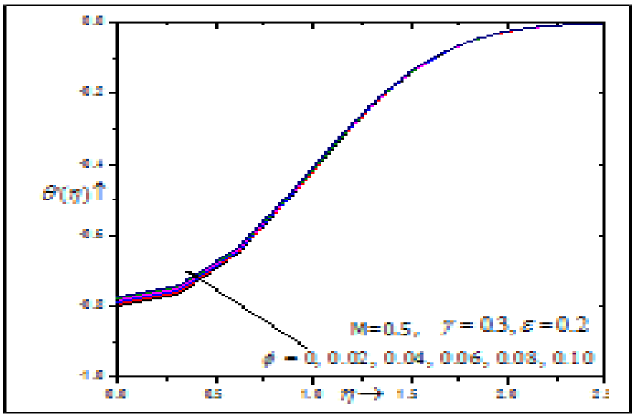

Fig. 13: Temperature gradient against volume fraction

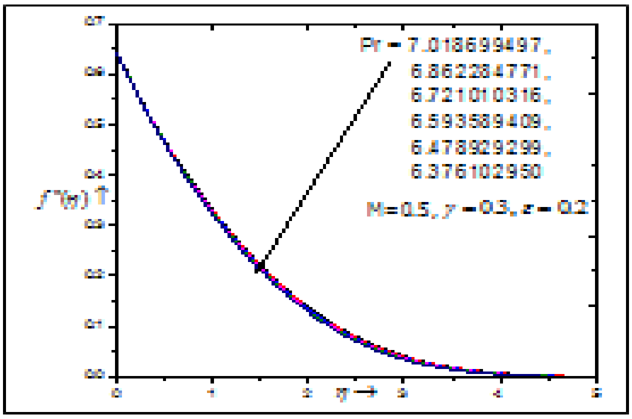

Fig. 14: Shear stress profile for various values of Prandtal number

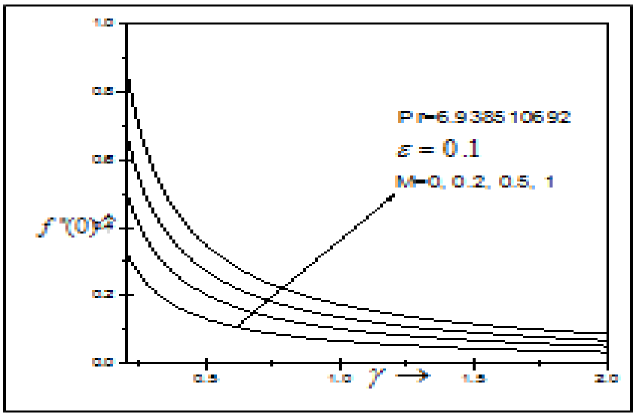

Fig. 15: Skin friction coeff. against velocity slip parameter

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