Melting Heat Transfer in Boundary Layer Stagnation Point Flow of Micropolar Fluid towards a Porous Stretching/Shrinking Surface

Khilap Singh, Padam Singh, Manoj Kumar

Abstract— Abstract— **The present investigation deals with the study of fluid flow and heat transfer characteristics in the melting process towards a porous stretching/shrinking surface. The governing equations representing fluid flow have been transformed into nonlinear ordinary differential equations using similarity transformation. The equations thus obtained have been solved numerically using Runge–Kutta Fehlberg method with shooting technique. The effects of the permeability parameters on the fluid flow and heat transfer characteristics have been illustrated graphically and discussed in detail. Significant changes were observed in the fluid flow and heat transfer with respect to permeability parameter.**

Index Terms— **Boundary layer, Stretching sheet, Melting, Micropolar fluid, Stagnation-point, Porosity.**

I. Introduction

Eringen [1] has introduced the theory of micropolar fluids that is capable to describe those fluids by taking into account the effect arising from local structure and micromotions of the fluid elements. Tamayol and Bahrami [2] studied that porous materials can be used to enhance the heat transfer rate from stretching surfaces to improve processes such as hot rolling and composite fabrication. Sharma and Gupta [3] studied the effect of medium permeability on thermal convection in micropolar fluids and found that the presence of coupling between thermal and micropolar effects may introduce oscillatory motions in the system. Raptis [4] investigated the boundary layer flow of a microplar fluid through a porous medium. The unsteady MHD boundary layer flow of a micropolar fluid near the stagnation point of a two-dimensional plane surface through a porous medium has been studied by Nadeem et al. [5]. Ishak et al. [6] investigated heat transfer over a stretching surface with variable heat flux in micropolar fluid. Wang [7] investigated the shrinking flow where velocity of boundary layer moves toward a fixed point and they found an exact solution of Navier-Stokes equations. A good list of references for micropolar fluids is available in Łukaszewicz [8]. Tien and Yen [9] investigated the effect of melting on forced convection heat transfer between a melting body and surrounding fluid. Epstein and Cho [10] analyzed the melting heat transfer of the steady laminar flows over a flat plate. The steady laminar boundary layer flow and heat transfer from a warm, laminar liquid flow to a melting surface moving parallel to a constant free stream has been studied by Ishak et al. [11]. Rosali et al. [12] studied micropolar fluid flow towards a permeable stretching /shrinking sheet in a porous medium numerically. Yacob et al. [13] investigated a model to study the heat transfer characteristics occurring during the melting process due to a stretching / shrinking sheet and they have studied the effects of the material parameter, melting parameter and the stretching /shrinking parameter on the skin friction coefficient and the local Nusselt number. Cheng and Lin [14] analyzed the melting effect on transient mixed convective heat transfer from a vertical plate in a liquid saturated porous medium.

In the present work, we consider the stagnation point flow and melting heat transfer of a micropolar fluid towards a porous stretching / shrinking surface have been considered.

II. Mathematical Formulation

The graphical model of the problem has been given along with flow configuration and coordinate system. The system deals with two dimensional stagnation point steady

values of $f''(0)$ are positive when ε < 1, and become negative when ε > 1. Physically, positive value of $f''(0)$flow of micropolar fluids towards a porous stretching/ shrinking surface. The velocity of the external flow is $u_e(x) = ax$

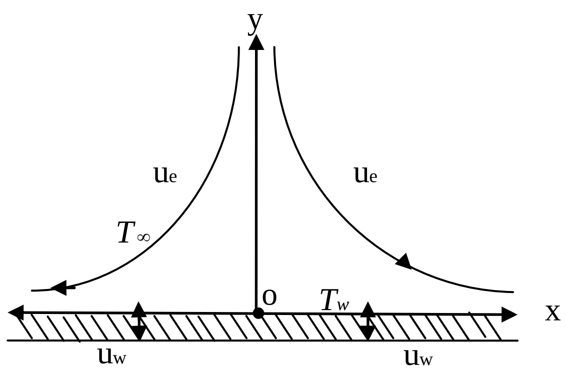

Figure 1: Flow configuration and Coordinate system

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values of $f''(0)$ are positive when ε < 1, and become negative when ε > 1. Physically, positive value of $f''(0)$flow of micropolar fluids towards a porous stretching/ shrinking surface. The velocity of the external flow is $u_e(x) = ax$ and the velocity of the stretching surface is $u_w(x) = cx$, where a is a positive constant, while c is a positive (stretching surface) or a negative (shrinking surface) constant and x is the coordinate measured along the surface. It is also assumed that the temperature of the melting surface and free stream condition is T_m and T_∞, where $T_\infty > T_m$. The viscous dissipation and the heat generation or absorption has assumed to be negligible. Under these assumptions, the governing equations representing flow are as follow:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{\partial u_e}{\partial x} + \left(\frac{\mu+k}{\rho}\right)\frac{\partial^2 u}{\partial y^2} + \frac{k}{\rho}\frac{\partial N}{\partial y} + \frac{\epsilon\mu_{eff}}{\rho K_1}(u_e - u) \quad (2)$$

$$u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = \frac{\gamma}{\rho j}\frac{\partial^2 N}{\partial y^2} - \frac{k}{\rho j}\left(2N + \frac{\partial u}{\partial y}\right) \quad (3)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \quad (4)$$

with the following boundary conditions:

$$u = u_w(x) = cx,\ N = -n\frac{\partial u}{\partial y},\ T = T_m \text{ at } y = 0$$

$$u = u_e(x) = ax,\ N = 0, T = T_\infty, \text{ at } y = \infty$$

And $$\kappa\left(\frac{\partial x}{\partial y}\right)_{y=0} = \rho[\lambda + c_s(T_m - T_0)]v(x,0) \quad (5)$$

Here u and v are the velocity component along the x and y axis respectively. Further, μ is dynamic viscosity, k is vortex viscosity, ρ is fluid density, T is fluid temperature, j is micro inertia density, N is microrotation, γ is spin gradient viscosity, K_1 is permeability of porous media, ϵ is porosity, μ_{eff} is effective dynamic viscosity, α is thermal diffusivity, κ is the thermal conductivity, λ is the latent heat of the fluid and c_s is the heat capacity of the solid surface. The case when $n = 0$, is called strong concentration which indicates that no microtation near the wall. In case $n = 1/2$ it indicates that the vanishing of anti-symmetric part of the stress tensor and denote weak concentration and the case $n = 1$ is used for the modeling of turbulent boundary layer flows by Yacob et al.[13] $\gamma = \left(\mu + \frac{k}{2}\right)l = \mu(1 + K/2)l$, where $K = k/\mu$ is the micropolar or material parameter and $l = \upsilon/a$ as reference length. The microstructure effects become negligible and the total spin N reduce to the angular velocity.

III. Problem Solution

Equations (2) – (4) can be transformed into a set of non linear ordinary differential equations by using the following similarity variables:

$$\psi = (a\upsilon)^{1/2}xf(\eta), \qquad N = xa\left(\frac{a}{\upsilon}\right)^{1/2}g(\eta)$$

$$\theta(\eta) = \frac{T - T_m}{T_\infty - T_m}, \qquad \eta = \left(\frac{a}{\upsilon}\right)^{1/2}y \quad (6)$$

The transformed ordinary differential equations are:

$$(1 + K)f''' + ff'' + 1 - f'^2 + Kg' + \Lambda(1 - f') = 0 \quad (7)$$

$$(1 + K/2)g'' + fg' - f'g - K(2g + f'') = 0 \quad (8)$$

$$\theta'' + \Pr f\theta' = 0 \quad (9)$$

where primes denote differentiation with respect to η and Pr = υ/α is Prandtl number. The boundary conditions (5) become

$$f'(0) = \varepsilon, g(0) = -nf''(0),$$

$$\Pr f(0) + M\theta'(0) = 0, \quad \theta(0) = 0,$$

$$f'(\infty) = 1, \quad g(\infty) = 0, \quad \theta(\infty) = 1, \quad (10)$$

where ε = c/a is the stretching(ε > 0) or shrinking(ε < 0) parameter, M is the dimensionless melting parameter and Λ is permeability parameter which are defined as

$$M = \frac{c_f(T_\infty - T_m)}{\lambda + c_s(T_m - T_0)}, \qquad \Lambda = \frac{\epsilon\mu_{eff}}{aK_1\rho} \quad (11)$$

The physical parameters of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$C_f = \frac{\tau_w}{\rho u_e^2}, \qquad Nu_x = \frac{xq_w}{\kappa(T_\infty - T_m)} \quad (12)$$

where τ_w and q_w are the surface shear stress and the surface heat flux, which are given by

$$\tau_w = \left[(\mu + k)\left(\frac{\partial u}{\partial y}\right)\right]_{y=0}, \quad q_w = -\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (13)$$

hence using (6), we get

$$Re_x^{1/2}C_f = [1 + (1 - n)K]f''(0),$$

$$Re_x^{1/2}Nu_x = -\theta'(0) \quad (14)$$

Where $Re_x = u_e(x)x/\upsilon$ is the local Reynolds number.

The transformed equations (7)-(9) subject to boundary conditions (10) were solved numerically using the Runge-Kutta-Fehlberg method with shooting technique to obtain the missing values of $f''(0)$ and $-\theta'(0)$, for some value of the permeability parameter Λ and the stretching / shrinking parameter ε, while the Prandtl number Pr is fixed to unity and we takes $n = 0.5$(weak concentration).

IV. Results and Discussion

In order to validate the numerical results obtained, we compare our results with reported by Yacob et al.[13] as shown in Tables 1, and they are found to be in a favorable agreement. Figures 2 and 3 show the variations of the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ with ε for different value of Λ when M=1,K=1. It is worth mentioning that for the shrinking case (ε < 0), the solution exists up to a critical value of ε (say ε_c) beyond which no solution exists. For stretching case (ε > 0), the means the fluid exerts a drag force on the solid surface and negative value means the solid surface exerts a drag force on the fluid. The negative value of $-\theta'(0)$ presented in Figure 3 shows that the heat is transferred from the warm fluid to cool solid surface. It is evident from Figure 2 that the increasing of permeability parameter Λ is to increase the value of $f''(0)$, in absolute sense. This result in increasing manner of the heat transfer rate at the fluid-solid interface $|-\theta'(0)|$ for ε < 1, but opposite behaviours are observed for ε > 1. All velocity and temperature profiles presented in Figures 4 and 5 which satisfy the far field boundary conditions (10) asymptotically thus support the validity of the numerical result obtained.

Table 1: Values of $f''(0)$ and $-\theta'(0)$ for various values of ∧ and ε where M=1, K=1

		Yacob et al.[13]		Present Result	
ε	Λ	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
0	0	0.879324	-0.347892	0.879324	-0.347892
	0.3			0.967891	-0.35500
	0.5			1.022730	-0.35870
0.5	0	0.506342	-0.432443	0.506325	-0.432443
	0.3			0.545101	-0.435080
	0.5			0.569500	-0.436500

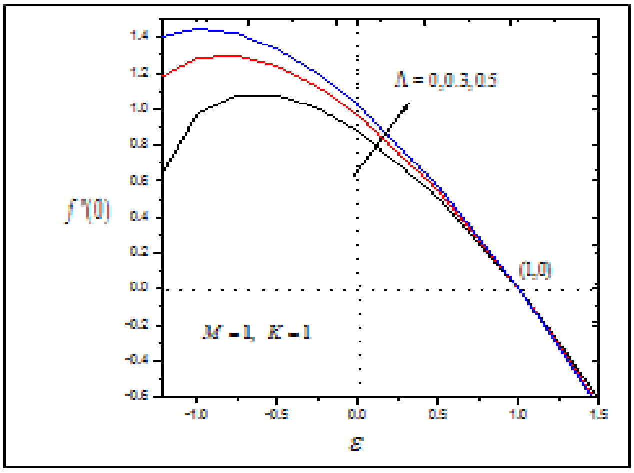

Fig.2: Skin friction vs stretching / shrinking parameter.

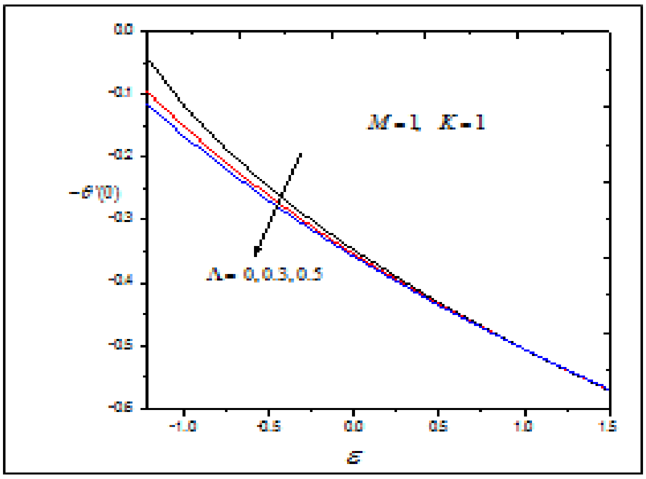

Fig.3: Local Nusselt number vs stretching / shrinking parameter

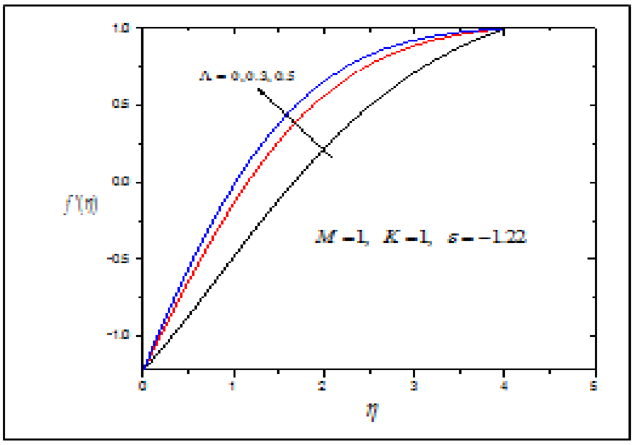

Fig.4: Effect of permeability on velocity

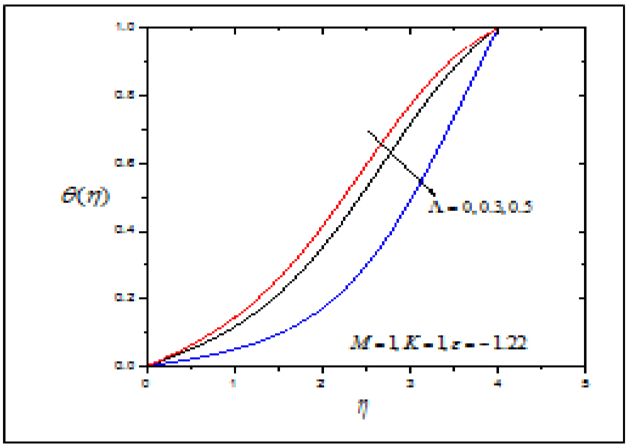

Fig.5: Effect of permeability on temperature.

V. Conclusion

We have studied the effects of the permeability parameter on the skin friction coefficient and the local Nusselt number (which represents the heat transfer rate at the surface) for the steady laminar boundary layer flow and heat transfer from a warm micropolar fluid to a melting solid surface of the same material. It has been found that permeability parameter increase the friction and the heat transfer rate at the fluid-solid interface.

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