Effect of Prony Series Parameters on Fracture Behavior of Mixed-Mode of Viscoelastic Materials

Varun Chhabra, Sanjeev Gaur, Ramendra Singh

***Abstract— In* this paper Deformations such as crack opening and a crack sliding displacement in a viscoelastic medium is studied using finite element analysis (FEM). The solution is carried out directly in time domain with a mesh not conforming to the crack geometry. The planar fracture problem of linear viscoelastic materials is established with the help of a finite element model. This finite element model describes time-dependent deformation behavior of viscoelastic materials with changing microstructures. Finite element software Abaqus has been used with time- dependent Prony series parameters. The numerical example shows the good agreement between the present results and those from analytical.**

***Index Terms*— Linear viscoelasticity, Finite element method (FEM), Prony series, Crack opening & crack sliding displacement, Abaqus.**

I. Introduction

In the present era the use of polymeric materials for structural applications in engineering is increasing day by day which demands a better understanding of their deformation behaviors. A Viscoelastic material shows the time-dependent behavior so that the fracture behavior of viscoelastic problem is dependent on both current state and the whole history [1]. No long, so the mechanical properties of many viscoelastic materials are modeled as linear viscoelastic materials.

Numerical method, especially the representative finite element method (FEM), is currently used to model viscoelastic behavior [2]. Generally, the stress level is relatively low and loading time is high. It is one of the most common computational methods.

The crack growth process is generally concentrated in the bitumen, with crack bifurcations being influenced by aggregates and their thin film interfaces. A study of crack initiation and its propagation promotes integrating a time-dependent behavior associated with an energy-based approach focused on the crack tip behavior and employing

an energy release rate concept. The primary objective of this paper is to develop a FEM model for analyzing the effect of Prony series parameters on crack opening and crack sliding displacement in a viscoelastic medium. As an assumption Poisson's ratio is a constant, asymptotic displacement field in the vicinity of a crack in linear viscoelastic isotropic medium is obtained through the classical viscoelastic- elastic principle [3].

Abaqus is one of the most advanced large-scaled finite element software possesses robust computing function and extensive simulated performance and it has a large number of different kinds of element models, material models and analytic processes, especially its good nonlinearity mechanical analysis function is at the world leading level.

The integral constitutive equations of relaxation shear modulus is adopted in Abaqus viscoelastic models and expanded by Prony series [4]. Time domain viscoelasticity is available in Abaqus for small-strain applications where the rate independent elastic response can be defined with a linear elastic material model and for large strain applications where the rate-independent elastic response must be defined with a hyperelastic or hyperfoam material model [5]. To check the validity of the formulae, the numerical results of the representative example of edge cracked viscoelastic strip are presented and compared with analytical solutions.

II. CONSTITUTIVE RELATION

In this section, we briefly present the constitutive relation. For a two-dimensional case, in integral form, the linear viscoelastic constitutive relation is [3]

$$\sigma(t) = E(t)\boldsymbol{D}\varepsilon(0) + \int_0^t E(t-\xi)D\frac{d\varepsilon(\xi)}{d\xi}d\xi \qquad Eq.(1)$$

where, σ and ε are stresses and strains in array form, respectively and t denotes time. In general, as a linear viscoelastic material, the relaxation modulus E (t) is expressed in Prony series as [6]

$$E(t) = E_\infty + \sum_{m=1}^{M} E_m \exp\left(-{}^{t}/_{\tau_m}\right) \qquad Eq.(2)$$

Matrix D depends only on the Poisson's ratio ν, taking the form of [7]

$$D = \frac{1-\nu}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & {(1-2\nu)}/{2(1-\nu)} \end{bmatrix} \qquad Eq.(3)$$

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III. NUMERICAL PROBLEM

A fracture example is studied to validate the effectiveness and accuracy of the current method for a cracked viscoelastic body and the results obtained from Abaqus software are compared to the available analytical reference solutions. For the following example the material is assumed to be a three-parameter linear viscoelastic solid as shown in Figure 1. Two groups of assumed material properties are used in the validation case studies shown in Table 1.

A viscoelastic strip with an oblique crack at the centre as shown in is examined. The height of the strip is H = W=2 mm. The crack depth is a =0.5 mm, applied load is 1MPa and the total loading time is 200sec.

The formulation of displacement field at crack tip under remotely applied tension is [8].

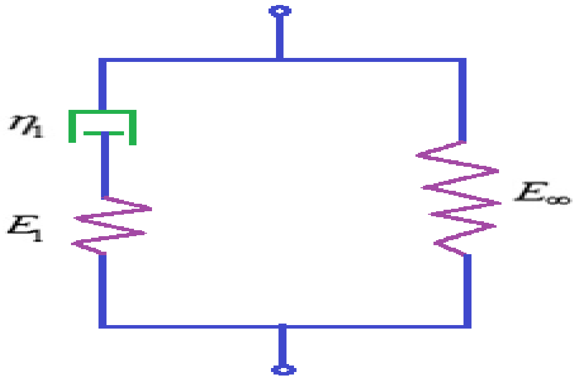

Figure 1: The three-parameter linear viscoelastic solid.

For Mode I Crack

$$\begin{cases} u_1(r,\theta,t) = (1+\nu)J(t)K_I(t)\sqrt{\dfrac{r}{2\pi}}\left(k-1+2\sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} \\ u_2(r,\theta,t) = (1+\nu)J(t)K_I(t)\sqrt{\dfrac{r}{2\pi}}\left(k+1-2\cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} \end{cases}$$

For Mode II Crack

$$\begin{cases} u_1(r,\theta,t) = (1+\nu)J(t)K_{II}(t)\sqrt{\dfrac{r}{2\pi}}\left(k+1+2\cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} \\ u_2(r,\theta,t) = (1+\nu)J(t)K_{II}(t)\sqrt{\dfrac{r}{2\pi}}\left(k-1-2\sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} \end{cases}$$

where, u_2 is the y-direction displacement component in Cartesian coordinate system located at crack tip. r and θ are the polar coordinates defined around the crack tip. F is a non-dimensional coefficient for stress intensity factor with the value 1.6599, J (t) is the creep compliance, and k is the Kolosov constant takes the value of k= (3- 4ν) for plane strain state and σ_0 is assumed to be 1 [8].

TABLE I
MATERIAL PARAMETERS OF THE LINEAR VISCOELASTIC SOLID.

Group	E_∞	E_1	η_1
I	1.0	1.0	10.0
II	1.0	2.0	10.0

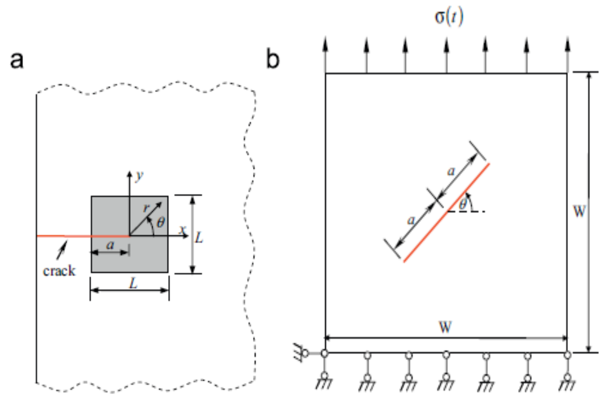

Figure 2a&2b: Viscoelastic strip with an oblique crack

IV. MODELLING

The viscoelastic material shows the time-dependent behavior. The time domain viscoelastic material model:

- describes isotropic rate-dependent material behavior for materials in which dissipative losses primarily caused by "viscous" (internal damping) effects must be modeled in the time domain.
- can be used only in conjunction with "Linear elastic behavior,"
- is active only during a transient static analysis ("Quasi-static analysis,") [5]

In this present work crack opening displacement is computed to analyze the fracture behavior of viscoelastic materials. Mathematical crack opening displacement (COD) and crack sliding displacement (CSD) represents as

$$COD = u_2(r,\pi,t) - u_2(r,-\pi,t) \qquad Eq.(4)$$

$$CSD = u_1(r,\pi,t) - u_1(r,-\pi,t) \qquad Eq.(5)$$

where, u_1 is the x-direction and u_2 is the y-direction displacement component in Cartesian coordinate system located at the crack tip. r and θ are the polar coordinates defined around the crack tip.

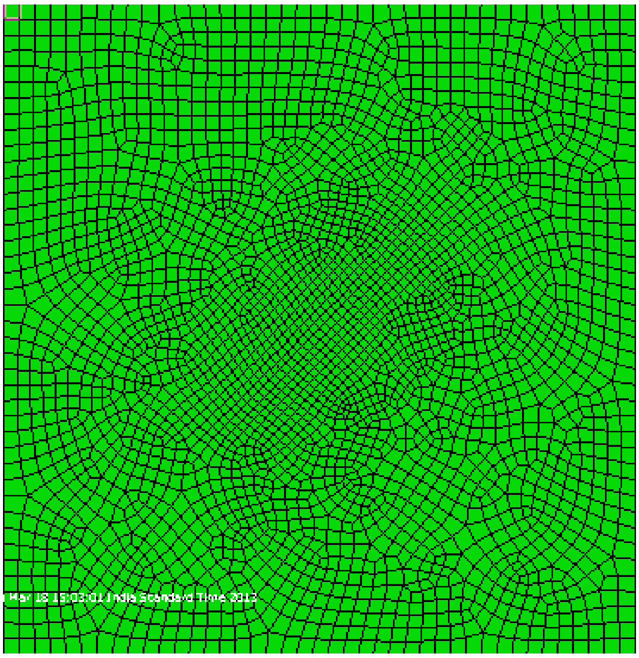

Figure 3a: Finite element mesh mixed mode model of crack,

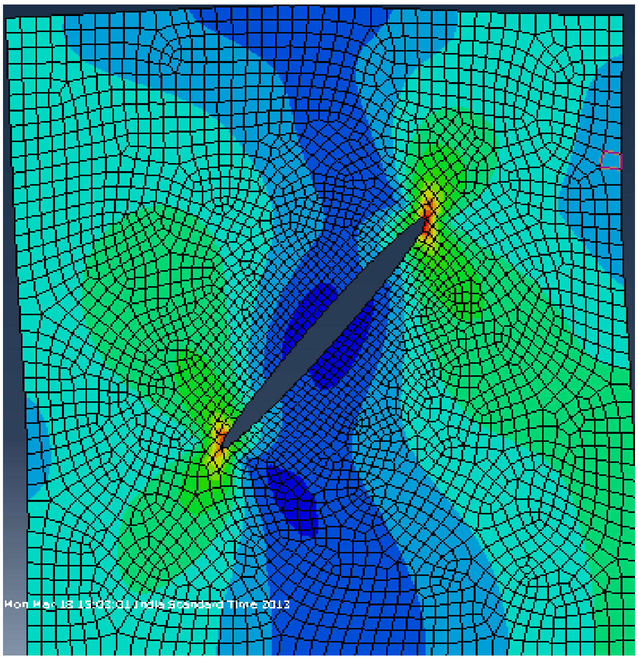

Figure 3b: Deformed shape of the mixed mode model

For assigning the viscoelastic material property in Abaqus shear test data has been used. The following mathematical relations between Prony series parameters has been used to calculate the shear test data:

In this case, the relaxation modulus in Eq. 2 becomes

$$E(t) = E_{\infty} + E_1 \exp\left({}^{-t}/_{\tau_1'}\right) \qquad Eq.(6)$$

and the corresponding creep compliance J (t) in Eq. 5 is [7]

$$J(t) = \frac{1}{E_{\infty}} - \frac{E_1}{E_{\infty}(E_{\infty} + E_1)} \exp\left({}^{-t}/_{\tau_1'}\right) \qquad Eq.(7)$$

with

$$\tau_1' = \left({}^{1}/_{E_{\infty}} + {}^{1}/_{E_1}\right)\eta_1 \qquad Eq.(8)$$

where, η_1 is the dashpot constant.

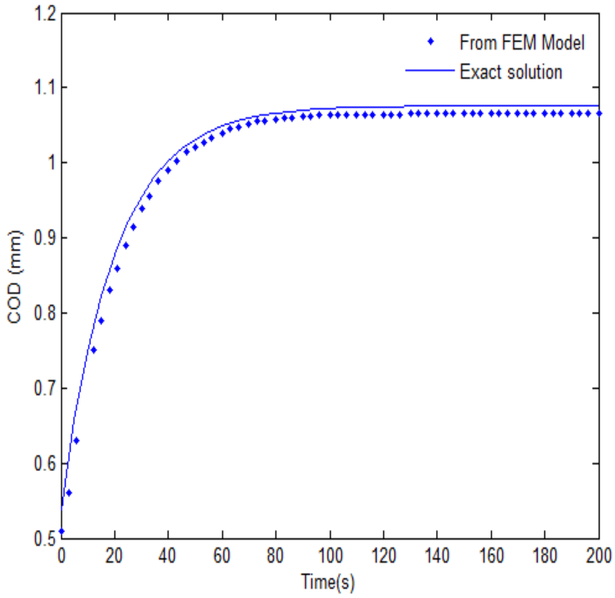

Figure 4a: Comparison of the COD for the strip subjected to tension load with exact solution of Group I Prony series parameters.

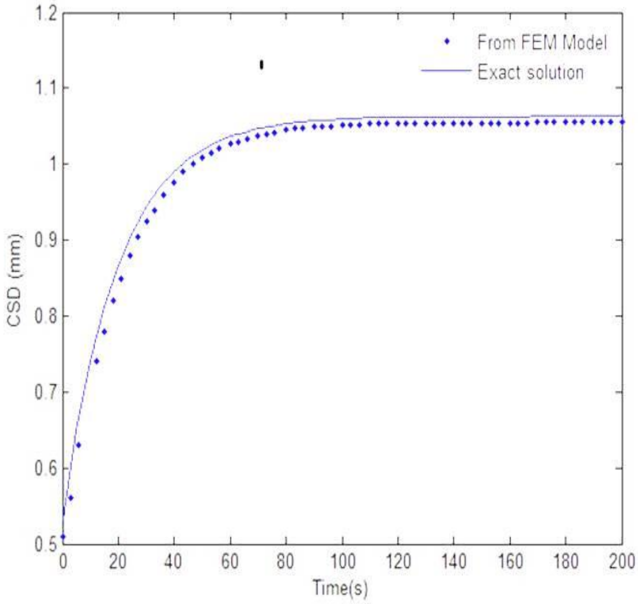

Figure 4b: Comparison of the CSD for the strip subjected to tension load with exact solution of Group I Prony series parameters.

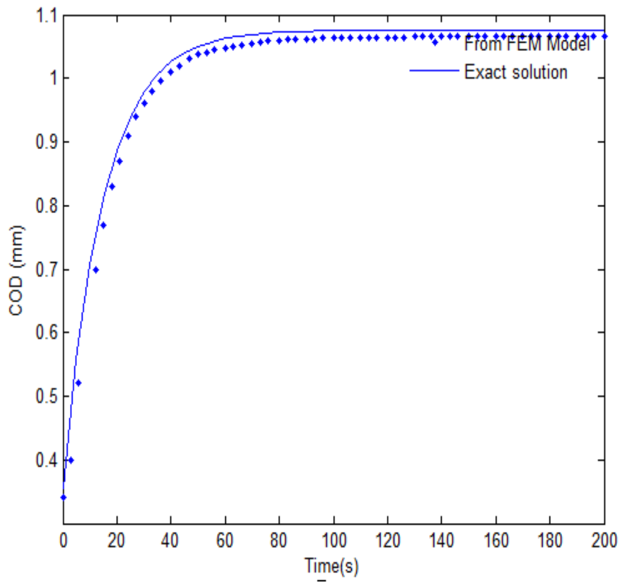

Figure 5a: Comparison of the COD for the strip subjected to tension load with exact solution of Group II Prony series parameters

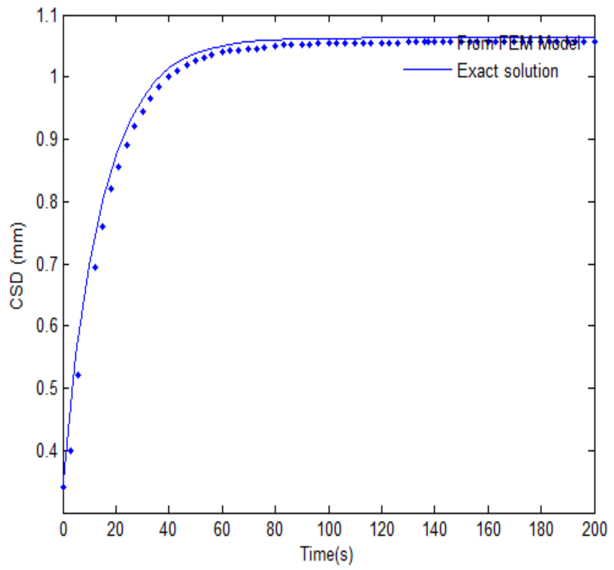

Figure 5b: Comparison of the CSD for the strip subjected to tension load with exact solution of Group I Prony series parameters

The boundary condition has been assigned to the strip, in which the displacement at the lower part of the section in y-direction is zero (Uy=0) and 1MPa load has been applied to upper part of the section (σ_0=1MPa) as shown in Figure 4(a). Four noded quadratic elements have been selected to mesh the model of a crack problem as shown in Figure.The full integration scheme has been selected to solve this problem.

V. RESULTS

For Group I Prony series parameters the variation of COD and CSD with time are shown in Figure 4a & 4b. The value of COD at initial time in present solution is 0.51 which is much closer to the exact solution value 0.53. The value of CSD at initial time in present solution is 0.52 which is much closer to the exact solution value 0.50. The COD increased with time and at a certain instant, it became constant upto the loading time 200 sec. Its value during this period in present solution is 1.065 for COD which is much closer to the exact solution value 1.07. Its value during this period in present solution is 1.050 for CSD which is much closer to the exact solution value 1.055. It is clearly seen that the present results of Group I Prony series parameters are in good agreement with the exact solutions.

For Group II Prony series parameters the variation of COD and CSD with time are shown in Figure 5a & 5b. The value of COD at initial time in present solution is 0.38 which is much closer to the exact solution value 0.40. The value of CSD at initial time in present solution is 0.37 which is much closer to the exact solution value 0.39. The COD increased with time and at a certain instant, it became constant upto the loading time 200 sec. Its value during this period in present solution is 1.06 for COD which is much closer to the exact solution value 1.065. Its value during this period in present solution is 1.050 for CSD which is much closer to the exact solution value 1.055. It is clearly seen that the present results of Group I Prony series parameters are in good agreement with the exact solutions.

VI. CONCLUSIONS AND DISCUSSION

The FEM was applied to solve crack problems in linear viscoelastic materials using Prony series parameters. One problem of mode I and mode II with two groups of Prony series parameters was examined to check the validity of the current method. Through the comparison with analytical solutions from the literature, the present method proved to be accurate and efficient. In the future this viscoelastic model can also be used to analyze the effect of inclusions voids and minor cracks on the fracture behavior of viscoelastic materials.

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