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An Optimal Inventory Policy for Deteriorating Items with Additive Exponential Life Time having Linear Trended Demand and Shortages

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Abstract : The present paper deals with an inventory policy of deteriorating items with linear trended demand and shortages. Here we assume that the lifetime of the commodity is random and follows additive exponential distribution. This model is very useful in situation arising at places like fruit and vegetable markets and food processing industries etc. in which the lifetime of the commodity is the sum of two components namely natural lifetime and extended lifetime due to cold storage or chemical treatment. Shortages are allowed and fully backlogged. Lastly the model is illustrated with the help of a numerical example and the sensitivity of the optimal solution towards the changes in the values of different parameters is also studied.

Key Words: Inventory, deterioration, linear tended demand, additive exponential and shortages.

Introduction:

The classical inventory model of Haris F[1] considers the idea care in which depletion of inventory is caused by a constant demand rate alone. But subsequently, it was noticed that the depletion of inventory may take place due to deterioration also. Deterioration is defined as decay or damage such that the items cannot be used for its original purpose. Like hardware, glassware etc, the rate of deterioration is so small so that there is hardly any need to consider its effect. Many researchers like Ghare and Schrader [2], Goel et al [3], Shah [4], Silver E.A [5], Datta and Pal [6] etc names only a few. Several researchers like Covert and Philip [7], Biswaranjan Mandal [8] have approximated the lifetime of a commodity as exponential or as two parameter Weibull distributed. In this paper my development based on the assumption where the lifetime of the commodity is random which is sum of two variables namely natural life and extended life. This extended life of commodity occurs mainly due to cold storage facilities, humidity, chemical treatment etc. So the lifetime of the commodity is to be approximated with an additive exponential distribution having the probability density function of the form

$$f(t) = \frac{e^{-\frac{t}{\theta_1}} - e^{-\frac{t}{\theta_2}}}{\theta_1 - \theta_2}, \theta_1 > \theta_2 \text{ and}$$

$t \geq 0$.

Moreover the extensive research works have already been done by many researchers on inventory model by assuming a constant demand rate. Later the time dependent demand rates have been investigated by various researchers like Donaldson [9], Jalan et al [10], Mandal and Pal[11] etc are noteworthy.

For these sort of situations, efforts have been made to analyze an inventory model with additive exponential lifetime and linear trended demand function of time allowing shortages which is fully backlogged.

Finally a numerical example has been presented to illustrate the theory and sensitivity study of the optimal solution to changes in the parameters has been examined and discussed.

Assumptions and Notations:

The mathematical models are developed under the following assumptions and notations:

(i) Replenishment size is constant and replenishment rate is infinite.

(ii) Lead time is zero.

(iii) T is the fixed length of each production cycle.

(iv) C_h is the inventory holding cost per unit per unit time.

(v) C_o is the ordering cost/order.

(vi) C_s is the shortage cost unit per unit time.

(vii) TC is the average total cost per unit time.

(viii) The instantaneous rate of deterioration of the on-hand inventory is

$$\theta(t) = \frac{e^{-\frac{t}{\theta_1}} - e^{-\frac{t}{\theta_2}}}{\theta_1 e^{-\frac{t}{\theta_1}} - \theta_2 e^{-\frac{t}{\theta_2}}}, \theta_1 > \theta_2; t \geq 0$$

(xi). The demand rate R(t) is assumed as R(t)= a +b t, where a and b are positive constants. Here a stands for initial demand rate and b for the positive trend in demand.

(x). Shortages are allowed and fully backlogged.

(xii). There is no repair or replacement of the deteriorated items.

Formulation and Solution of the Model:

Let Q be the total amount of inventory produced or purchased at the beginning of each period and after fulfilling backorders let us assume we get an amount S(>0) as initial inventory. Due to reasons of market demand and deterioration of the items, the inventory level gradually diminishes during the period (0, t_1) and ultimately falls to zero at t = t_1. Shortages occur during time period (t_1, T) which are fully backlogged. Let I(t) be the on-hand inventory at any time t. The differential equations which the on-hand inventory I(t) governed by the following :

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -R(t), 0 \le t \le t_1 \quad (1)$$

And $$\frac{dI(t)}{dt} = -R(t), t_1 \le t \le T \quad (2)$$

The initial condition is $I(0) = S$ and $I(t_1) = 0$ (3)

Putting the values of $\theta(t) = \dfrac{e^{-\frac{t}{\theta_1}} - e^{-\frac{t}{\theta_2}}}{\theta_1 e^{-\frac{t}{\theta_1}} - \theta_2 e^{-\frac{t}{\theta_2}}}$ and R(t) = a + bt, and then solving the equations (1) and (2) using the initial condition (3) and neglecting the higher powers of $\frac{1}{\theta_1}$ and $\frac{1}{\theta_2}$, we get

$$I(t) = S(1 - \frac{t^2}{2\theta_1\theta_2}) - (at + \frac{b}{2}t^2 - \frac{a}{3\theta_1\theta_2}t^3 - \frac{b}{8\theta_1\theta_2}t^4), \ 0 \le t \le t_1 \quad (4)$$

And $$I(t) = a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2), \ t_1 \le t \le T \quad (5)$$

Since I(t1) = 0, we get from equation (4) the following neglecting higher order terms of $\frac{1}{\theta_1}$ and $\frac{1}{\theta_2}$

$$S = at_1 + \frac{b}{2}t_1^2 + \frac{a}{6\theta_1\theta_2}t_1^3 + \frac{b}{8\theta_1\theta_2}t_1^4 \quad (6)$$

The number of items backlogged at the beginning of the period is

$$Q - S = \int_{t_1}^{T}(a + bt)dt$$

Or, $$Q = aT + \frac{b}{2}T^2 + \frac{a}{6\theta_1\theta_2}t_1^3 + \frac{b}{8\theta_1\theta_2}t_1^4 \text{ (using (6))} \quad (7)$$

Therefore the average total cost per unit time is given by

$$TC(S, t_1) = \frac{C_o Q}{T} + \frac{C_h}{T}\int_0^{t_1} I(t)dt - \frac{C_s}{T}\int_{t_1}^{T} I(t)dt \quad (8)$$

Now substituting the expressions for Q and I(t) given by the equations (7), (4) and (5), then eliminating S and integrating we get,

$$TC(t_1) = \frac{C_o}{T}(aT + \frac{b}{2}T^2 + \frac{a}{6\theta_1\theta_2}t_1^3 + \frac{b}{8\theta_1\theta_2}t_1^4) + \frac{C_h}{T}(\frac{a}{2}t_1^2 + \frac{b}{3}t_1^3 + \frac{a}{12\theta_1\theta_2}t_1^4 + \frac{b}{15\theta_1\theta_2}t_1^5)$$

$$-\frac{C_s}{T}[a(t_1T - \frac{T^2}{2}) + \frac{b}{2}(t_1^2T - \frac{T^3}{3}) - \frac{a}{2}t_1^2 - \frac{b}{3}t_1^3] \quad (9)$$

For minimum, the necessary condition is $\frac{dTC(t_1)}{dt_1} = 0$

This gives

$$\frac{bC_h}{3\theta_1\theta_2}t_1^4 + \frac{1}{\theta_1\theta_2}(\frac{bC_o}{2} + \frac{aC_h}{3})t_1^3 + \{\frac{aC_o}{2\theta_1\theta_2} + b(C_h + C_s)\}t_1^2 + \{a(C_h + C_s) - bC_sT\}t_1 - C_saT = 0$$

Or, $$Lt_1^4 + Mt_1^3 + Nt_1^2 + Ot_1 + P = 0 \qquad (10)$$

Where $L = \frac{bC_h}{3\theta_1\theta_2}$, $M = \frac{1}{\theta_1\theta_2}(\frac{bC_o}{2} + \frac{aC_h}{3})$, $N = \frac{aC_o}{2\theta_1\theta_2} + b(C_h + C_s)$

$$O = a(C_h + C_s) - bC_sT\,,\; P = -C_saT$$

Since L > 0, M > 0, N > 0, P < 0, then there must exist one positive real root of the equation whenever the sign of O be.

For minimum the sufficient condition $\frac{d^2TC(t_1)}{dt_1^{\,2}}$ >0 would be satisfied.

Let $t_1 = t_1^*$ be the optimum value of t_1.

The optimal values Q^* of Q, S^* of S and TC^* of TC are obtained by putting the value $t_1 = t_1^*$ from the expressions (7), (6) and (9).

Numerical Example:

To illustrate the developed inventory model, let the values of parameters be as follows:
C_o = $5; C_h = $10; C_s = $2; θ_1 = 5; θ_2 = 3; a = 5; b = 2; T = 10 years
Solving the equation (10) with the help of computer using the above parameter values, we find the following optimum outputs
t_1^* = 2.38 year; Q^* = 152.02 units; S^* = 19.59 units and TC^* = Rs 1396.87
It is checked that this solution satisfies the sufficient condition for optimality.

Sensitivity Analysis and Discussion.

We now study the effects of changes in the system parameters C_o, C_h, C_s, θ_1, θ_2, a and b on the optimal total cost (TC^*), optimal ordering quantity (Q^*) and optimal on-hand inventory (S^*) in the present inventory model. The sensitivity analysis is performed by changing each of the parameters by – 50%, – 20%, +20% and +50%, taking one parameter at a time and keeping remaining parameters unchanged. The results are furnished in table A.

Table A: Effect of changes in the parameters on the model

Changing parameter	% change in the system parameter	% change in		
		Q^*	S^*	TC^*
C_o	-50	0.23	6.28	2.19
	-20	0.07	2.04	0.32
	+20	-0.09	-2.71	-1.09
	+50	-0.20	-6.02	-1.79
C_h	-50	1.85	76.31	4.32
	-20	0.82	20.67	3.55
	+20	-0.48	-15.42	-5.45
	+50	-0.82	-30.12	-11.25
C_s	-50	-1.13	-49.52	-58.35
	-20	-0.61	-20.11	-25.10
	+20	0.78	19.86	25.43
	+50	2.32	50.84	64.66

a	-50 -20 +20 +50	-16.69 - 6.68 6.68 16.69	-32.31 -12.91 12.91 32.26	-28.22 -11.29 11.29 28.22
b	-50 -20 +20 +50	-33.30 -13.32 13.33 33.31	-17.71 - 7.10 7.04 17.66	-21.78 - 8.71 8.71 21.78
θ_1	-50 -20 +20 +50	0.31 0.11 -0.12 -0.24	-10.26 - 3.11 1.58 4.19	-19.59 - 6.39 4.15 10.20
θ_2	-50 -20 +20 +50	0.31 0.11 -0.12 -0.24	-10.26 - 3.11 1.58 4.19	-19.59 - 6.39 4.15 10.20

Analyzing the results of table A, the following observations may be made:

(i) TC^* and S^* increase or decrease with the increase or decrease in the values of the system parameters C_s, a, b, θ_1 and θ_2. On the other hand TC^* and S^* increase or decrease with the decrease or increase in the values of the system parameters C_o and C_h. The results obtained show that TC^* and S^* are very highly sensitive to changes in the value of C_s and also sensitive to the changes of C_h, a and b.

(ii) Q^* increases or decreases with the increase or decrease in the values of the system parameters C_s, a and b. On the other hand it increases or decreases with the decrease or increase in the values of the system parameters C_o, C_h, θ_1 and θ_2. However Q^* is very sensitive to changes in the values of a and b whereas it has low sensitivity towards changes in C_s, C_o, C_h, θ_1 and θ_2.

From the above analysis, it is seen that a and b are critical parameters in the sense that any error in the estimation of a and b results in significant errors in the optimal solution. Hence, proper care must be taken to estimate a and b. Along with estimation of C_h needs adequate attention.

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