International Journal of Engineering and Technical Research (IJETR)
ISSN: 2321-0869 (O) 2454-4698 (P), Volume-10, Issue-1, January 2020

Radiation and MHD effects on an oscillatory flow of a Jeffrey fluid in a circular tube

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***Abstract*— In this paper, we studied the effects of heat transfer and MHD on oscillatory flow of Jeffrey fluid in a circular tube with radiation. The expressions for the velocity field and temperature field are obtained analytically. The effects of various emerging parameters on the velocity field and temperature field studied in detail with the help of graphs**.

***Index Terms*— Jeffrey fluid, Reynolds number, Prandtl number, Grashof number, Nusselt number**.

I. Introduction

The effect of heat transfer on unsteady MHD oscillatory flow of fluid in a tube is encountered in a wide range of engineering and industrial applications such as molten iron flow, recovery extraction of crude oil, geothermal systems. Many chemical engineering processes like metallurgical and polymer extrusion processes involve cooling of a molten liquid being stretched in a cooling system. Some polymers fluids like polyethylene oxide and polysobutylene solutions in a cetane, having better electromagnetic properties are normally used as cooling liquid as their flow can be regulated by external magnetic fields in order to improve the quality of the final product. Also, the radiative heat transfers is an important factor of thermodynamics of very high temperature systems such as electric furnaces, solar collectors, storage of nuclear wastes packed bed catalytic reactors, satellites, steel rolling, cryogenic engineering etc. The study of such flow under the influence of magnetic field and heat transfer has attracted the interest of many investigators and researchers. Unsteady and oscillatory flow of viscous fluids in locally constricted, rigid, axisymmetric tubes at low Reynolds number has been studied by Ramachandra Rao and Devanathan (1973), Hall(1974), Chaturani and Upadyaya (1978) and Schneck and Ostrach (1975). Haldar (1987) have considered the oscillatory flow of a blood through an artery with a mild constriction. Ogulu and Abbey (2005) have studied the effect of heat transfer on the motion of blood in a diseased artery has been modeled under the optically thin fluid assumption. Several other workers, Misra and Singh (1987), Ogulu and Alabraba (1992), Tay and Ogulu (1997) and Elshahed (2003), to mention but a few, have in one way or the other modeled and studied the flow of blood through a rigid tube under the influence of pulsatile pressure gradient. Arunakumari et al. (2012) investigated the influence of heat transfer on MHD oscillatory flow of Jeffery fluid in a Channel. Unsteady Flow of a Jeffrey Fluid in an elastic tube with stenosis was investigated by Sreenadh et al (2012).

In view of these, we studied the effects of heat transfer and MHD on oscillatory flow of Jeffrey fluid in a circular tube with radiation. The expressions for the velocity field and temperature field are obtained analytically. The effects of various emerging parameters on the velocity field and temperature field studied in detail with the help of graphs.

II. .MATHEMATICAL FORMULATION

We consider an oscillatory flow of a Jeffrey fluid in a heated uniform cylindrical tube of constant radius R in the presence of uniform magnetic field B_0 . The wall of the tube is maintained at a temperature T_w . We choose the cylindrical coordinates (r,θ,z) such that $r=0$ is the axis of symmetry. The flow is considered as axially symmetric and fully developed. The geometry of the flow is shown in Fig. 1.

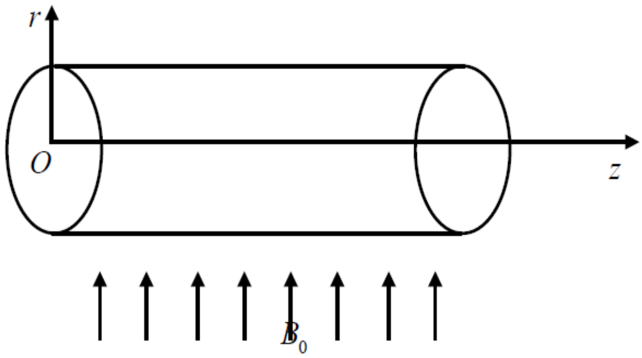

Fig. 1 The physical model of the

Fig. 1 The physical model of the problem

The equations governing the flow (Kavitha and Ramakrishna Prasad, 2012) are given by

$$\rho\frac{\partial w}{\partial t}=-\frac{\partial p}{\partial z}+\frac{\mu}{1+\lambda_1}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)-\sigma B_0^2 w+\rho g\beta\left(T-T_\infty\right) \quad (1)$$

$$\rho c_p\frac{\partial T}{\partial t}=k_0\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)-\frac{\partial q}{\partial r} \quad (2)$$

where ρ is the fluid density, λ_1 is the ratio of relaxation to retardation times, μ is the fluid viscosity, p is the pressure, w is the velocity component in z - direction, g is the acceleration due to gravity, σ is electrical oconductivity, β

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coefficient of thermal expansion, T is the temperature, k_0 is the thermal conductivity and c_p is the specific heat at constant pressure. Following Cogley et al. (1968), it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q}{\partial y} = 4\alpha^2\left(T_\infty - T\right) \qquad (3)$$

here α is the mean radiation absorption coefficient.

The appropriate boundary conditions are

$$w = 0,\ \ T = T_w \quad \text{at} \quad r = R$$

$$\frac{\partial w}{\partial r} = 0,\ \ T = T_\infty \quad \text{at} \quad r = 0 \qquad (4)$$

Introducing the following non-dimensional variables

$$\bar{r} = \frac{r}{R},\ \bar{z} = \frac{z}{R},\ \bar{t} = \frac{w_0 t}{R},\ \ \alpha^2 = \frac{\rho R^2}{\mu},\ \ \lambda = \frac{R}{w_0}$$

$$\bar{w} = \frac{w}{w_0},$$

$$\bar{p} = \frac{p - p_w}{\mu},\ \ \theta = \frac{T - T_\infty}{T_w - T_\infty}$$

$$\Pr = \frac{\mu c_p}{k_0},\ \ \mathrm{Re} = \frac{\rho w_0 R}{\mu},$$

into the Eqs. (1) and (2), we get (after dropping bars)

$$\mathrm{Re}\frac{\partial w}{\partial t} = -\lambda\frac{dp}{dz} + \frac{1}{1+\lambda_1}\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right] - M^2 w + \frac{Gr}{\mathrm{Re}}\theta$$

(5)

$$\Pr\mathrm{Re}\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} + N^2\theta$$

(6)

where $\Pr$ is the Prandtl number, $M = RB_0\sqrt{\frac{\sigma}{\mu}}$ is the Darcy number and Re is the Reynolds number.

The corresponding non-dimensional boundary conditions are

$$w = 0,\ \ \theta = 1 \quad \text{at} \quad r = 1$$

$$\frac{\partial w}{\partial r} = 0,\ \ \frac{\partial \theta}{\partial r} = 0 \quad \text{at} \quad r = 0 \qquad (7)$$

III. SOLUTION

It is fairly unanimous that, the pumping action of the heart results in a pulsatile blood flow so that we ca represent the pressure gradient (pressure in the left ventricle) as

$$-\frac{dp}{dz} = p_0 e^{iwt} \qquad (8)$$

and flow variables expresses as

$$\theta(y,t) = \theta_0(r)e^{iwt} \qquad (9)$$

$$w(y,t) = w_0(r)e^{iwt} \qquad (10)$$

Substituting Eqs. (8) - (10) into Eqs. (5) and (6) and solving the resultant equations subject to the boundary conditions in (7), we obtain

$$\theta_0 = \frac{I_0(\Omega r)}{I_0(\Omega)} \qquad (11)$$

$$w_0 = \frac{Gr}{\mathrm{Re}}\frac{(1+\lambda_1)}{(\beta_1^2+\Omega^2)}\left[\frac{I_0(\beta_1 r)}{I_0(\beta_1)} - \frac{I_0(\Omega r)}{I_0(\Omega)}\right] + \frac{\lambda p_0}{\beta_1^2}(1+\lambda_1)\left[1 - \frac{I_0(\beta_1 r)}{I_0(\beta_1)}\right]$$

(12)

Here $\Omega^2 = i\omega\Pr\mathrm{Re} - N^2$, $\beta_1^2 = \left(M^2 + i\omega\mathrm{Re}\right)(1+\lambda_1)$ and $I_0(x)$ is the modified Bessel function of first kind of order zero.

Thus the temperature distribution and the axial velocity are given by

$$\theta = \frac{I_0(\Omega r)}{I_0(\Omega)}e^{i\omega t} \qquad (13)$$

$$w = \left(\frac{Gr}{\mathrm{Re}}\frac{(1+\lambda_1)}{(\beta_1^2+\Omega^2)}\left[\frac{I_0(\beta_1 r)}{I_0(\beta_1)} - \frac{I_0(\Omega r)}{I_0(\Omega)}\right] + \frac{\lambda p_0}{\beta_1^2}(1+\lambda_1)\left[1 - \frac{I_0(\beta_1 r)}{I_0(\beta_1)}\right]\right)e^{i\omega t}$$

(14)

The skin friction on the boundary of the tube is is given by

$$\tau = \left.\frac{\partial u}{\partial r}\right|_{r=1} = \left(\frac{Gr}{\mathrm{Re}}\frac{(1+\lambda_1)}{(\beta_1^2+\Omega^2)}\left[\frac{\beta_1 I_1(\beta_1)}{I_0(\beta_1)} - \frac{\Omega I_1(\Omega)}{I_0(\Omega)}\right] + \frac{\lambda p_0}{\beta_1^2}(1+\lambda_1)\left[1 - \frac{\beta_1 I_1(\beta_1)}{I_0(\beta_1)}\right]\right)e^{i\omega t}$$

(15)

The rate of heat transfer coefficient in terms of Nusselt number on the boundary of the tube is

$$Nu = \left.\frac{\partial \theta}{\partial r}\right|_{r=1} = \frac{\Omega I_1(\Omega)}{I_0(\Omega)}e^{i\omega t} \qquad (16)$$

IV. DISCUSSION OF THE RESULTS

Fig. 2 shows the effects of material parameter λ_1 on w for $M = 1$, $N = 1$, $p = 1$, $\omega = 10$, $\lambda = 0.5$, $Pr = 0.7$, $Gr = 1$, $\mathrm{Re} = 1$ and $t = 0.1$. It is observed that, the axial velocity w increases at the axis of tube with increasing material parameter λ_1, while it decreases near the tube wall with increasing λ_1.

Effects of Hartmann number M on w for $\lambda_1 = 0.3$, $N = 1$, $p = 1$, $\omega = 10$, $\lambda = 0.5$, $Pr = 0.7$, $Gr = 1$, $\mathrm{Re} = 1$ and $t = 0.1$ is shown in Fig. 3. It is found that the axial velocity w decreases with increasing Hartmann number M.

Fig. 4 depicts the effects of Prandtl number Pr on w for $\lambda_1 = 0.3$, $N = 1$, $p = 1$, $\omega = 10$, $M = 1$, $\lambda = 0.5, Gr = 1, \mathrm{Re} = 1$ and $t = 0.1$. It is noted that, the axial velocity w increases on increasing Prandtl number Pr.

Effects of radiation parameter N on w for $\lambda_1 = 0.3$, $p = 1$, $\omega = 10$, $Gr = 1$, $\lambda = 0.5$, $M = 1$, $Pr = 0.7, \mathrm{Re} = 1$ and $t = 0.1$ is presented in Fig. 5. It is observed that, the axial velocity w increases with increasing N.

Fig. 6 depicts the effects of Grashof number Gr on w for $\lambda_1 = 0.3$, $N = 1$, $p = 1$, $\omega = 10$, $M = 1$, $\lambda = 0.5, \mathrm{Pr} = 0.7, \mathrm{Re} = 1$ and $t = 0.1$. It is noted that, the axial velocity w increases at the axis of tube on increasing Grashof number Gr, while it decreases near the tube wall with increasing Gr.

The effect of Reynolds number Re on w for $N = 1$, $\lambda_1 = 0.3$, $p = 1, \omega = 10$, $\lambda = 0.5$, $M = 1$, $Pr = 0.7, \mathrm{Re} = 1$ and $t = 0.1$ is shown in Fig. 7. It is found that, the axial velocity w decreases with an increase in Reynolds number Re.

Fig. 8 shows the effect of λ on w for $\lambda_1 = 0.3$, $N = 1$, $M = 1$, $Pr = 0.7$, $Gr = 1$, $p = 1$, $\omega = 10$, $\mathrm{Re} = 1$ and $t = 0.1$. It is observed that, the axial velocity w increases on increasing λ.

Effect of oscillating parameter ω on w for $M = 1$, $p_0 = 1$, $\lambda_1 = 0.5$, $\lambda = 0.5$, $Pr = 0.7$, $Gr = 1$, $N = 1$, $\mathrm{Re} = 1$ and $t = 0.1$ is depicted in Fig. 9. It is found that, the axial velocity w oscillates with ω.

Fig. 10 shows the effect of Prandtl number Pr on θ for $\omega = 10$, $N = 1$, $\mathrm{Re} = 1$ and $t = 0.1$. It is found that, the temperature θ decreases with increasing Prandtl number Pr.

Effect of radiation parameter N on θ for $\omega = 10$, $Pr = 0.7$, $\mathrm{Re} = 1$ and $t = 0.1$ is shown in Fig. 11. It is noted that, the temperature θ increases with increasing N.

Fig. 12 depicts the effect of Reynolds number Re on θ for $\omega = 10, Pr = 0.7$, $N = 1$ and $t = 0.1$. It is observed that, the temperature θ decreases with increasing Reynolds number Re.

Effects of oscillating parameter ω on θ for $Pr = 0.7$, $N = 1$, $\mathrm{Re} = 1$ and $t = 0.1$ is depicted in Fig. 13. It is found that, the temperature θ increases with increasing ω.

Table-1 shows the effects of various physical parameters like λ_1, Gr, Re, λ, M, Pr and N on the skin friction τ at the tube wall $r = 1$. It is found that, the skin friction increases for increasing the parameters λ_1, Gr, Re and λ, while it decreases M, Pr and N. $r = 1$. It is observed that, the Nusselt number increases with increasing Pr and Re, while it decreases with increasing N.

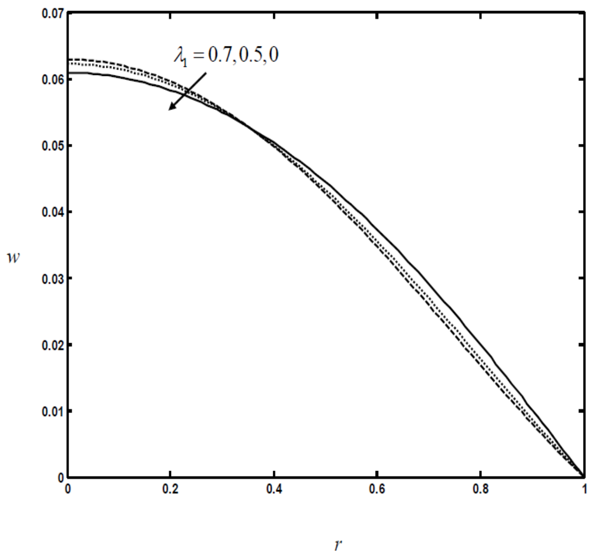

Fig. 2. Effects of material parameter λ_1 on w for $M = 1$, $p_0 = 1$, $\omega = 10, \lambda = 0.5$, $Pr = 0.7$, $Gr = 1$, $N = 1$, $\mathrm{Re} = 1$ and $t = 0.1$.

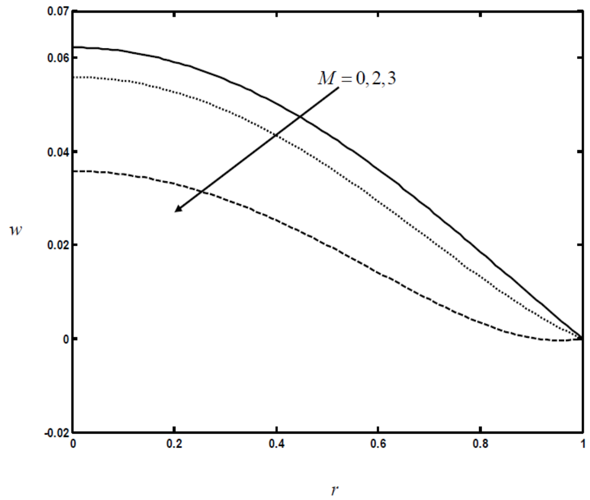

Fig. 3. Effects of Hartmann number M on w for $\lambda_1 = 0.5$, $p_0 = 1$, $\omega = 10$, $\lambda = 0.5$, $Pr = 0.7$, $Gr = 1$, $N = 1$, $\mathrm{Re} = 1$ and $t = 0.1$.

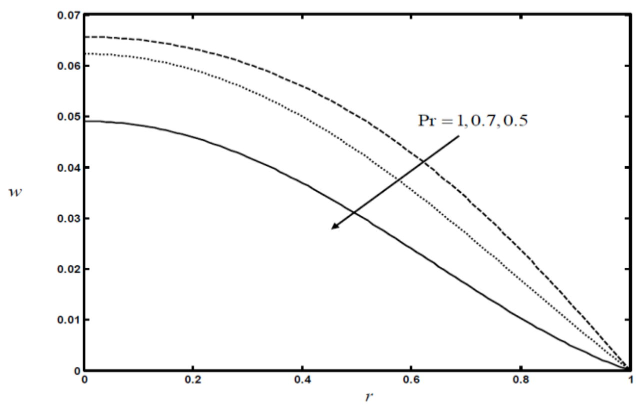

Fig. 4. Effects of Prandtl number λ_1 on w for $M=1$, $p_0=1$, $\omega=10$, $\lambda=0.5$, $\lambda_1=0.5$, $Gr=1$, $N=1$, $\mathrm{Re}=1$ and $t=0.1$.

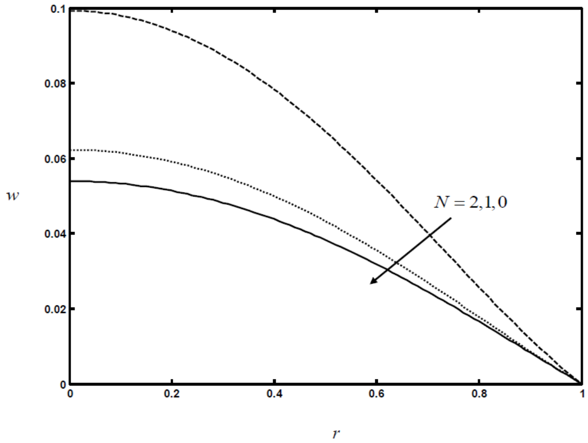

Fig. 5. Effects of radiation parameter N on w for $M=1$, $p_0=1$, $\omega=10$, $\lambda=0.5$, $Pr=0.7$, $Gr=1$, $\lambda_1=0.5$, $\mathrm{Re}=1$ and $t=0.1$.

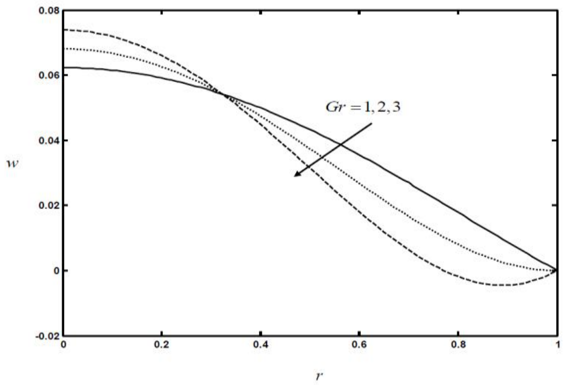

Fig. 6. Effects of Grashof number Gr on w for $M=1$, $p_0=1$, $\omega=10$, $\lambda=0.5$, $Pr=0.7$, $\lambda_1=0.5$, $N=1$, $\mathrm{Re}=1$ and $t=0.1$.

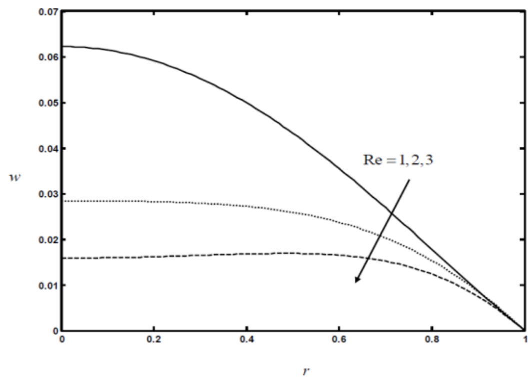

Fig. 7. Effects of Reynolds number Re on w for $M=1$, $p_0=1$, $\omega=10$, $\lambda=0.5$, $Pr=0.7$, $Gr=1$, $N=1$, $\lambda_1=0.5$ and $t=0.1$.

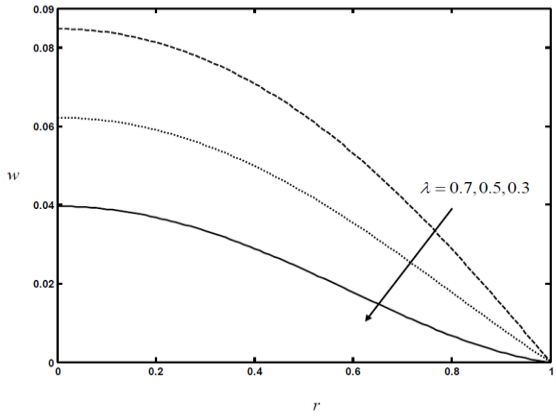

Fig. 8. Effects of pressure constant λ on w for $M=1$, $p_0=1$, $\omega=10$, $\lambda_1=0.5$, $Pr=0.7$, $Gr=1$, $N=1$, $\mathrm{Re}=1$ and $t=0.1$.

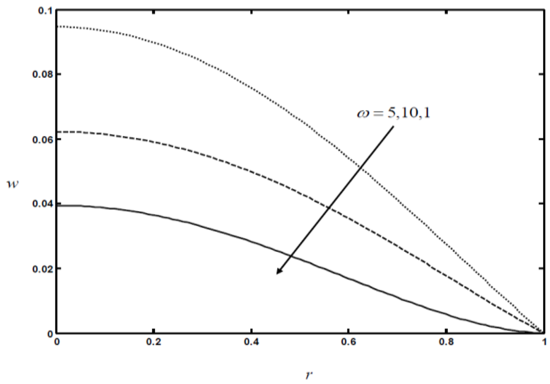

Fig. 9. Effects of oscillating parameter ω on w for $M=1$, $p_0=1$, $\lambda_1=0.5$, $\lambda=0.5$, $Pr=0.7$, $Gr=1$, $N=1$, $\mathrm{Re}=1$ and $t=0.1$.

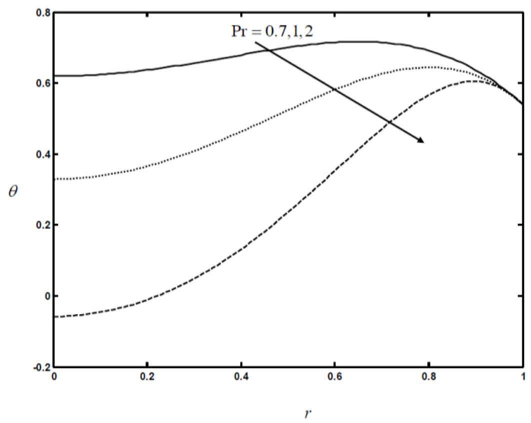

Fig. 10. Effects of Prandtl number Pr on θ for $\omega=10$, $N=1$, $\mathrm{Re}=1$ and $t=0.1$.

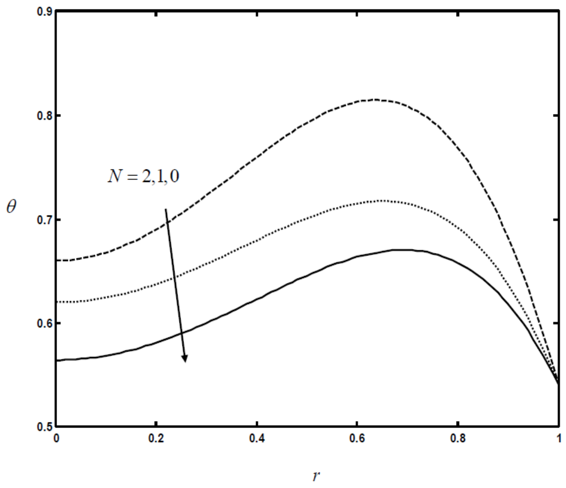

Fig. 11. Effects of radiation parameter N on θ for $\omega = 10$, $Pr = 0.7$, $\mathrm{Re} = 1$ and $t = 0.1$.

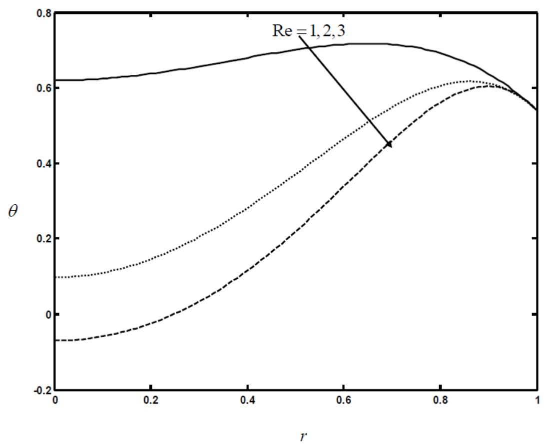

Fig. 12. Effects of Reynolds number Re on θ for $\omega = 10$, $Pr = 0.7$, $N = 1$ and $t = 0.1$.

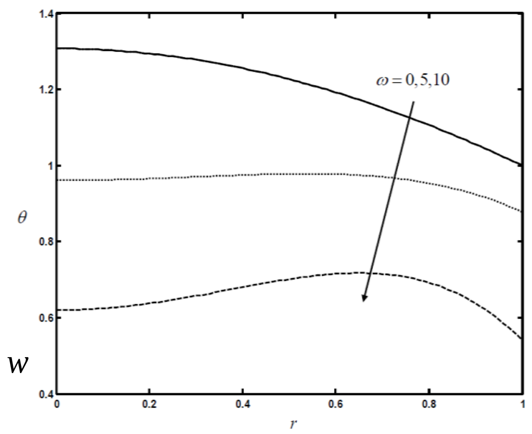

Fig. 13. Effects of oscillating parameter ω on θ for $Pr = 0.7$, $N = 1$, $\mathrm{Re} = 1$ and $t = 0.1$.

Table – 1: The Skin friction τ at the tube wall $r = 1$

λ_1	M	Pr	N	Gr	Re	λ	τ
0.5	1	0.7	1	1	1	0.5	-0.1118
0.7	1	0.7	1	1	1	0.5	-0.1072
0.5	2	0.7	1	1	1	0.5	-0.1189
0.5	1	1	1	1	1	0.5	-0.1308
0.5	1	0.7	2	1	1	0.5	-0.1609
0.5	1	0.7	1	2	1	0.5	-0.0809
0.5	1	0.7	1	1	2	0.5	-0.0814
0.5	1	0.7	1	1	1	0.7	-0.1072

Table – 2: The Nusselt number Nu at the tube wall $r = 1$

Pr	N	Re	Nu
0.7	1	1	1.2113
1	1	1	1.6620
0.7	2	1	1.1200
0.7	1	2	2.0789

V. CONCLUSIONS

In view of these, we studied the effects of heat transfer and MHD on oscillatory flow of Jeffrey fluid in a circular tube with radiation. The expressions for the velocity field and temperature field are obtained analytically. It is found that the velocity w increases with increasing $\lambda_1, \mathrm{Pr}, N, Gr, \omega$ and λ, while it decreases with increasing M and Re; the temperature increases with increasing Pr, N and ω, while it decreases with increasing Re; the skin friction increases for increasing the parameters λ_1, Gr, Re and λ, while it decreases M, Pr and N and the Nusselt number increases with increasing Pr and Re, while it decreases with increasing N.

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  International Journal of Engineering and Technical Research (IJETR) ISSN: 2321-0869, Volume-3, Issue-4, April 2015
- Effect of Heat Transfer on Oscillatory Flow of a Fluid through a Porous Medium in a Channel with an Inclined Magnetic Field

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