

# A Brief Didactic Theoretical Review on Convolutional Neural Networks, Deep Belief Networks and Stacked Auto-Encoders

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**Abstract**—This paper presents a brief theoretical review on deep neural network architectures, deep learning procedures, as well as some of its possible applications. The paper focuses on the most common networks structures: Convolutional Neural Network (CNN), Deep Belief Network (DBN) and Stacked Auto-encoders (SA). The building blocks which enabled the construction of deeper networks such as Rectified Linear Unit (ReLU) and softmax activation functions, convolution filters, restricted Boltzmann machines and autoencoders, are explained in the beginning and middle sections of the paper. A few examples of hybrid systems are also presented at the last sections of the paper. The paper concludes with some considerations on the state-of-art work and on the possible future applications enabled by deep neural networks.

**Index Terms**— Autoencoder, Boltzmann Machine, Convolutional Neural Network, Deep Learning Review.

## I. INTRODUCTION

Neural networks algorithms have been applied to a wide variety of complex tasks, in areas ranging from computer vision, speech recognition, text translation, system identification and control, among others.

The greatest advantage of this algorithm lies on their ability to learn from a set of examples, without the need for defining a set of explicit rules for a given task. After learning how to solve a given problem, an artificial neural network would generally perform in the same level or better than a rule-based algorithm for the same task, especially for very abstract problems such as in computer vision.

While neural networks were shown to theoretically be able to represent any nonlinear function [1], in practice neural networks were limited in depth and by long training times. What allowed neural networks to achieve the high level of performance seen today was the development of a series of techniques for training deep networks in the past decade. This set of techniques is what is now known as deep learning.

This paper presents a brief theoretical review on deep

neural network's structures, training procedures, and enumerates some of its possible applications. The paper focuses on presenting a general description on the inner workings of the most common deep architectures, namely the Deep Belief Networks (DBN), Stacked Autoencoders (SA) and Convolutional Neural Networks (CNN).

In terms of structure, these three topologies can be decomposed in fundamental blocks, such as the ReLU and softmax activation functions, convolution filters, restricted Boltzmann machines and autoencoders. These blocks, along with the associated architectures, are described in the middle sections of the paper.

A few examples of hybrid systems are also presented at later sections of the paper. The paper concludes with some considerations on the state-of-art work and on the possible future applications enabled by deep neural networks.

## II. BASIC CONCEPTS

### A. Artificial Neuron Structure

An Artificial Neural Network (ANN) is a parallel computational structure loosely inspired on real neural networks, capable of learning from large amounts of data. The network is trained to generate a set of outputs from the inputs presents on the training data. Thus, an ANN can act as a universal approximator of nonlinear functions [1].

These networks are composed of several small units, called neurons or nodes, grouped in multiple sequential layers. Each neuron in a layer receives signals from neurons localized in other layers or from the network's input itself. The neuron then responds by emitting a signal of its own, propagating the information forward to the next layers in the network.

The output signal  $y_n$  fired by a neuron as a response to an input vector  $\mathbf{x}_n$  is described by:

$$y_n = \sigma_n(\mathbf{w}_n \mathbf{x}_n + b_n) \quad (1)$$

Here,  $\mathbf{w}_n$  and  $b_n$  are the connection weight vector and activation bias respectively. The mathematical function  $\sigma_n$  is a nonlinear function called "activation function" and describes the response of the neuron to its collective input.

Historically,  $\sigma_n$  used to be simple linear functions (such as in the original perceptron [2]) and sigmoid functions, but with the popularization of deeper networks, less computationally expensive options such as Rectified Linear Unit (ReLU) started to be employed. Fig. 1 shows a plot of some of the commonly used activation functions.

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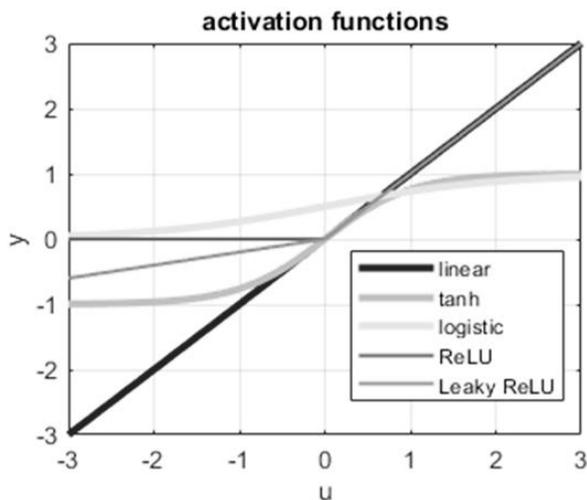


Fig. 1 – Common activation functions.

A special activation function is the “softmax” function, which normalizes the sum of outputs of all neurons of the previous layer to the interval of 0 to 1. The softmax output generated by a neuron of raw output  $x_i$  is given by:

$$Softmax(x)_i = \frac{e^{x_i}}{\sum_{j=1}^n e^{x_j}} \quad (2)$$

Where  $n$  is the number of possible classes considered by the Softmax layer.

This is useful for classification problems, where it is necessary to calculate the probability of the input to belong to a given  $i$  class, among all possible  $n$  classes [3]. For example, assume that a softmax layer receives an input  $x_i > 0$ , while all other inputs  $x_k$  are zero. This will cause the layer to increase the probability of the  $i$  class and reduce the probability of all other  $k$  classes. The output of the network for this scenario is shown in Fig. 2.

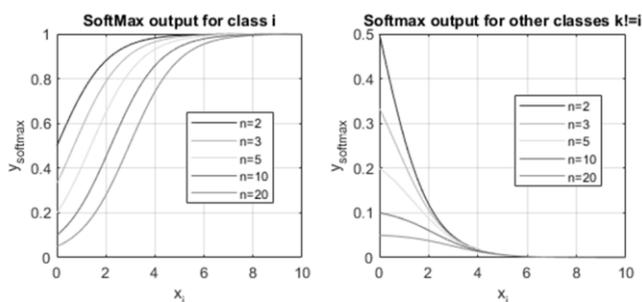


Fig. 2 – Softmax activation function.

### B. Feed-Forward and Recurrent Networks

Neural Networks can be categorized in two groups depending on information flow direction: feedforward networks and recurrent networks.

Feedforward networks are designed so that input information is propagated layer to layer in a unidirectional manner. There are no feedback connections between the more advanced layers to the previous layers in the network. This is appropriate for time invariant problems, as the network is not required to keep track of past events. An example is the multi-layer perceptron (MLP)[4], Convolutional [5] and Deep Belief networks [6]. An example of feedforward neural network is shown at the left side of Fig. 3.

A recurrent network (also called feedback network) allows information to also flow backwards in the network, allowing the network to remember past states. This creates a nonlinear dynamical system [7] which can be trained to contextualize and retain some of the information already processed by the network, making the network more appropriate to tackle time variant problems. This added complexity comes with a cost. As the ANN’s behaves as a highly nonlinear dynamical system, more complex learning algorithms are required to ensure the network’s stability [8]. Some examples of dynamical ANN are the Hopfield [4], Elman[9] and the Long Short-Term Memory (LSTM)[10] networks. An example of a recurrent neural network is shown at the right side of Fig. 3.

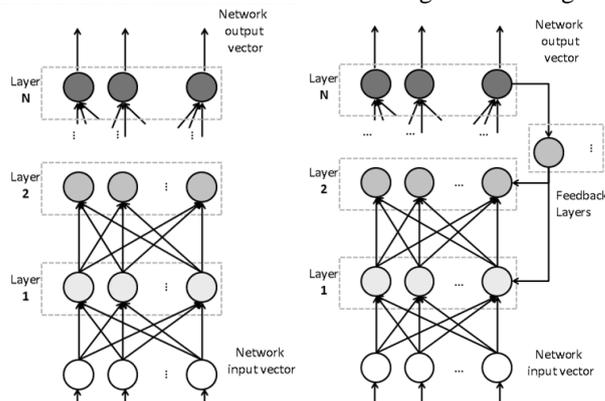


Fig. 3 – Example of feedforward and recurrent neural networks.

### C. Training as a nonlinear parameter estimation task

The training procedure of an ANN, in which the networks learn to replicate a given function from data, can be thought as a parameter estimation problem. An ANN learns by changing the weights of the connections of its neurons, so as to minimize an estimation error (such as Mean Square Error, MSE) between the correct output and the output generated by the network. Due to the nonlinearity present in activation functions, direct methods such as least squares estimation cannot be applied, and therefore, iterative methods are required[11].

The most common method for adjusting the weights in a neural network is the backpropagation algorithm and its variations, such as the Levenberg-Marquardt method [12]. The algorithm performs gradient descent over an error-measuring function, by applying the chain rule over each layer to find the appropriate adjustment for every weight present on the network.

For shallow networks with one or two hidden layers, it is possible to use backpropagation algorithms over the entire network, but for deeper architectures, the gradient either fades away over layers (referred as “vanishing gradient problem”) or increases indefinitely (referred as “exploding gradient problem”), with the former problem especially prevalent on recurrent networks[13].

Another issue is data availability and reliability. For problems where data is often readily available but just as often mislabeled (for example, user generated content in video streaming websites), unsupervised learning methods (non-reliant on labeled data) are required.

In some other problems, training data is limited or fewer than the number of parameters to be adjusted. This enables the network to simply memorize the training data, generating an

exact copy of the output during training, rather than generalizing for the underlying model. This problem is referred as “overfitting” in the literature [13] [14].

To prevent these problems, several different components for building deeper nets were designed, such as restricted Boltzmann machines, autoencoders and convolution kernels, providing the foundation of what is now known as “deep learning”. A common trait shared by all of these structures is that they provide means for extracting salient features contained in input data, generating a more abstract set of inputs to be processed by the next block. The training at this point is in most cases unsupervised. The final blocks provide a highly abstract preprocessed data, which is then used for generating the network’s output. The learning in this final stage is supervised, that is, uses the labeled data in the training set. The following sections describe these structures in more detail.

### III. DEEP BELIEF NEURAL NETWORKS

#### A. Boltzmann Machines

A Boltzmann Machine (BM) is a network model introduced by Hinton, Sejnowski and Ackley in 1985 [15]. This structure resembles a Hopfield network, in the sense that both are capable of learning the internal distribution of a training set. However, a Restricted Boltzmann Machine (RBM) uses a stochastic activation function based on the Boltzmann distribution, and its training is inspired on thermodynamic principles.

This network comprises a visible input layer and a hidden layer, with each neuron being connected to all other neurons in the network. The visible layer contains neurons whose states can be directly overridden by an input signal, while hidden layer comprises neurons which can only be accessed by the visible neurons. This topology is shown at the left side of Fig. 4.

Each  $i$ -th neuron has two states, “on” ( $s_i = 1$ ) or “off” ( $s_i = 0$ ). The network’s internal energy  $E$  is given by:

$$E = -\sum_{i < j} w_{ij} s_i s_j + \sum_i \theta_i s_i \quad (3)$$

Where  $\theta_i$  is bias of the  $i$ -th neuron, and  $w_{ij}$  is the connection weight between the  $i$ -th and  $j$ -th neurons. The energy difference  $\Delta E_i$  between “off” and “on” states of the  $i$ -th neuron is given by:

$$\Delta E_i = \sum_j w_{ij} s_j - \theta_i \quad (4)$$

The probability  $p_i$  of the  $i$ -th neuron setting its state to “on” follows the Boltzmann distribution, hence the network’s name.

$$p_i = \frac{1}{(1 + e^{-\Delta E_i/T})} \quad (5)$$

Where  $T$  is analogous to system temperature. It can be noticed that (3) (4) and (5) model a system steadily losing energy, eventually reaching a thermal equilibrium state.

BM training has the goal of minimizing the difference between the probability distribution of the state of visible neurons at thermal equilibrium,  $P$ , and the probability distribution of the training samples,  $P^+$ . The difference between two distributions can be measured by the Kullback-Leibler’s (KL) divergence,  $G$ :

$$G = \sum_v P^+(V) \ln \left( \frac{P^+(V)}{P(V)} \right) \quad (6)$$

Where  $V$  denotes each one of the visible input neurons. Fortunately, the gradient of  $G$  in relation to any given weight  $w_{ij}$  has a very simple form:

$$\frac{\partial G}{\partial w_{ij}} = -\frac{1}{T} (p_{ij}^+ - p_{ij}^-) \quad (7)$$

Where  $p_{ij}^+$  is the probability of both  $i$ -th and  $j$ -th being “on” at thermal equilibrium, with all input neurons overridden by a training signal, while  $p_{ij}^-$  denotes the probability of the same event happening without the influence of the training signal.

This indicates that a simple rule can be applied to change the weights. Assuming a step size of  $\varepsilon$ , the variation on weights  $\Delta w_{ij}$  and on biases  $\Delta \theta_i$  is given by:

$$\Delta w_{ij} = \varepsilon (p_{ij}^+ - p_{ij}^-) \quad (8)$$

$$\Delta \theta_i = \varepsilon (p_i^+ - p_i^-) \quad (9)$$

While (8) and (9) are fairly simple learning rules, the excessive complexity of the BM’s topology causes the learning to be inefficient for complex problems. Likewise, the training requires that the BM is simulated until it reaches thermal equilibrium, which increases training complexity [16].

An improvement on training performance can be achieved restricting BM’s topology, creating a so-called restricted Boltzmann Machine (RBM), with the most common restriction being eliminating connections between neurons in the same layer [17]. This RBM is shown at the right side of the Fig. 4.

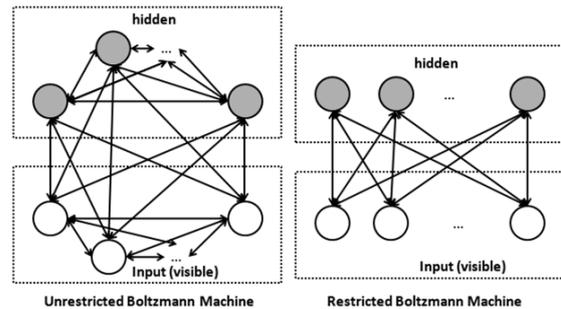


Fig. 4 – Unrestricted and Restricted Boltzmann Machines

The neuron state determination process is also simplified as a result of the topology restriction. The probability of a given neuron (visible or hidden) being active, given the current state of the neurons on the opposite layer, can now be written similarly to (1).

$$p(v_i = 1 | \mathbf{h}) = \sigma(a_i + \sum_j h_j w_{ij}) \quad (10)$$

$$p(h_j = 1 | \mathbf{v}) = \sigma(b_j + \sum_i v_i w_{ij}) \quad (11)$$

Where  $p(v_i = 1)$  and  $p(h_j = 1)$  are the probability of a visible or a hidden layer being on the active state,  $w_{ij}$  is the connection weight between neurons  $i$  and  $j$ , and  $a_i$  and  $b_j$  are biases for the visible and hidden neurons respectively. In the RBM’s case,  $\sigma$  is a logistic function.

The learning function can also be simplified, as to not depend on simulating the machine to its thermal equilibrium. Instead, the RBM can be simulated for a  $k$  number of cycles, with the state of all neurons (including those on the hidden layer) being sampled for the learning rule.

Initially, the visible neurons in the RBM are overridden by with an input vector contained in the training set, so that all hidden neurons change their state accordingly to (11). The state of all neurons is represented by  $\langle v_i h_j \rangle_{data}$ . The

visible neurons are then unclamped, so that the network is free to reconstruct the input vector using the information stored in the hidden layers. The neurons are then updated sequentially, with their states being set accordingly to (10) and (11). This procedure is called Gibbs's sampling. After  $k$  cycles simulating the network by Gibbs's sampling, the state of the neurons  $\langle v_i h_j \rangle_{rebuild}$  is then sampled. The weight adjustment is given by:

$$\Delta w_{ij} = \varepsilon (\langle v_i h_j \rangle_{data} - \langle v_i h_j \rangle_{rebuild}) \quad (12)$$

This algorithm is called Contrastive Divergence, or CD- $k$  [18]. It is much faster learning algorithm, as it does not require simulating the RBM to its equilibrium point. Even though the learning rule in (12) does not follow the divergent  $G$  defined on (6), it still provides a sufficiently precise approximation for training the RBM, even for a single simulation cycle ( $k=1$ ) [19].

### B. Deep Belief Networks (DBN)

Deep Belief Networks (or more commonly Bayesian networks) are statistical models that represent a set of variables along with their conditional dependencies. This network can infer the probability of an event being caused by one of several possible known causes. An example of this structure is shown in Fig. 5, with each event's dependency on others is marked with arrows.

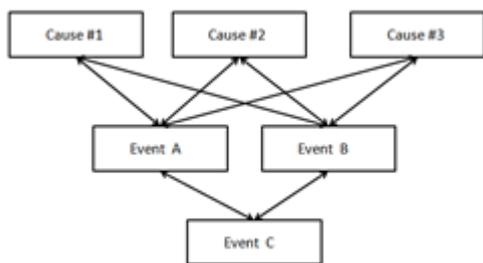


Fig. 5 – Belief network general structure.

This network can infer variables, parameters and hidden variables from the distribution of a given event. This capacity for inference and general structure has been perceived as similar to those found on Boltzmann Machines in the 1980's [14], but due to the inherent complexity of BM's topology and its training, this similarity could not be further explored until two decades later.

The creation of RBM's in the late 1980's [16] and the development of faster algorithms for their training in early 2000's [18] led to development of deep belief nets (DBN) composed of stacked RBMs. The proposed structure is seen on Fig. 6.

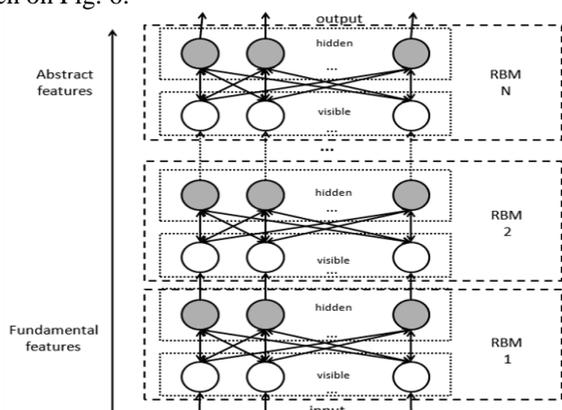


Fig. 6 – Stacked RBMs used on a DBN.

The structure presented in Fig. 5 shows a stack of RBMs. The visible layer of the bottom RBM is used as the input layer of the DBN, with the hidden layer acting as input for the next RBM. All RBMs are connected in the same way, forming a sequential structure up to hidden layer of the last RBM, which provides the output of the stack.

Each one of the RBMs are trained separately in an unsupervised manner, from the bottom RBM's to the top. After finishing training an RBM, their hidden neurons will register some of the most salient characteristics contained on its training inputs. The output of the trained RBM is then used as a training input for the next RBM in the stack, and so on. As a result, each RBM extracts a feature present in the training data, with higher layers extracting increasingly more abstract information [20].

After training all RBMs in the stack, the output from the top layer can be used to activate a classifier network using the softmax function. The training in this stage is now supervised, as the labeled data associated with the features extracted by the stack [21].

## IV. STACKED AUTOENCODERS

### A. Autoencoders

Another building block which can be stacked to form a deep network is the autoencoder, also known in literature as an auto-associator. An auto-encoder is a neural network designed for creating an efficient representation of its input data, often with reduced dimensionality. Like the RBM, it can be used for extracting features from an unlabeled set of data, although their training procedures are quite different [22].

The traditional autoencoder is a simple network trained to regenerate its own input as its output, hence the structure's name. Learning can be done through the standard backpropagation algorithm, in the same way as the MLP. As the training goal is to learn its own input value, it does not require labeled data, making it a form of unsupervised learning.

After the training is completed, the hidden layers of the autoencoder will have learned a useful encoding of the data, based on the distribution of the training data. Likewise, the output layer will also learn a form to decode and regenerate the original data from its encoding [23].

A common issue of the traditional autoencoder's training is that the network has the possibility of learning the identity function, preventing the autoencoder from learning a more useful representation of the data.

To avoid this problem, a few variations of the autoencoder have been designed, such as denoising autoencoders and sparse autoencoders. Denoising autoencoders explicitly introduce noise in the input's training set, so that the network is trained to filter the noise present in the input [24]. The sparse autoencoder introduces an additional penalty during training to constrain hidden neurons activations and weights to a set of predetermined values, preventing the network from learning the identity [25]. Both techniques force the autoencoder to rely on features contained in the training data.

A simpler solution for the issue is by having a smaller number of hidden neurons than inputs in the autoencoder, creating an under-complete system. This prevents the network from learning the identity function, although at the cost of learning less features compared with over-complete autoencoders [21].

An illustrated example of a MLP network trained as an autoencoder is shown on Fig. 7. After training the MLP either traditionally or using any of the variants presented, the autoencoder's input  $x$  is encoded by the hidden layer, creating an encoding function  $x' = f_{encoding}(x)$ . The output layer approximately reconstructs the original input as  $y \cong x$ .

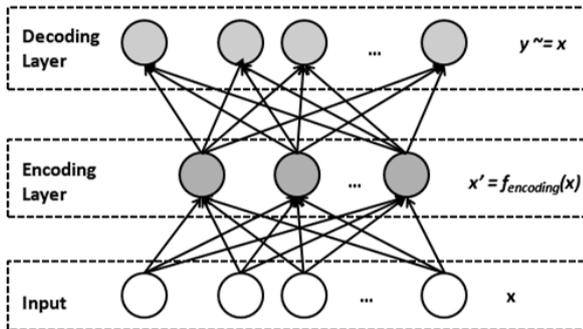


Fig. 7 – Example of an autoencoder

### B. Stacked Autoencoders

Autoencoders can be stacked to create a single deep network, similarly to how RBMs are used on a DBN. The first autoencoder is trained to encode and regenerate the network's input. After training, the decoding layer is discarded, and the hidden layers' output (the encoded information  $x'$ ) is used as input to train the next autoencoder. This procedure is repeated for each autoencoder in succession. After all autoencoders in the stack have been trained, the stack is then connected to the input of a fully connected network, which will generate the output of the network. Finally, all layers are then fine-tuned using supervised training [13].

Fig. 8 shows the structure of a deep network formed by the stacked autoencoders (SA). While this is very similar to DBN in structure, SA composed of traditional auto-encoders has slightly inferior results compared to DBN [26] due to the CD-k algorithm being a better approximation of log-likelihood gradient than the reconstruction error gradient, while the denoising variation of the autoencoder performed similarly to DBN [13].

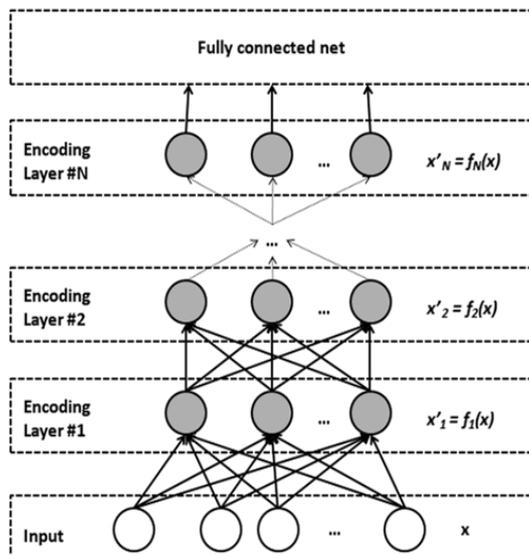


Fig. 8 – Stacked autoencoder structure

## V. CONVOLUTIONAL NEURAL NETWORKS (CNN)

### A. CNN development history

Both methods for creating the deep neural nets presented previously, DBN and SA, make use of greedy layer-wise training to avoid the vanishing gradient problem. Until 2006, there were no means for training deep networks with fully connected layers.

However, a specialized type of deep network that could be trained traditionally was developed two decades prior to the appearance of DBNs. These networks are the Convolutional Neural Networks (CNN or alternatively ConvNet). They have a fanned-out structure, which simplifies its training.

Convolutional neural networks follow an architecture loosely inspired on the visual system of living beings. The works of Hubel and Wiesel, at the end of the 1950s and through the 1960s, gave start to this line of research by presenting the response generated by the striate cortex of cats to several moving stimuli of light [27].

In their experiment, it was found that some isolated neurons (referred by the authors as "simple" cells) fired only when subject to image edges in very specific positions and orientations, while some other neurons with larger receptive fields (referred as "complex" neurons) reacted specifically to more complex stimuli, while being insensitive to changes in position. These properties had been verified in the nervous system of cats [26] and monkeys [28].

The concept that an image could be processed by several parallel processing units, each one absorbing increasingly abstract features of the image inspired the development of the "neocognitron" network by K. Fukushima, which is considered to be the first example of CNN in literature [29]. The neocognitron is composed of several sequential modules, with each module comprising two layers of neurons. The first layer contains units that resemble the simpler neurons described by Hubel and Wiesel, which are responsible for extracting edges from the image. The second layer contains units that resemble the complex neurons, which are less sensitive to position. The complex neurons' outputs feed the next module, creating a serialized structure similar to DBNs and SA. Another similarity with modern networks is that the Neocognitron uses unsupervised training to learn features present in the data.

The work inspired the design of many other networks based on this hierarchical architecture, such as the convolutional network developed by LeCun in 1989 for recognizing hand-written digits [30].

This network accepts as input an image of 16x16 pixel and contains three hidden layers. A single output layer classifies the ten possible digits in the input image.

The first hidden layer has 12 convolution kernels, each one composed of 64 neurons, which scans the image for local features. By performing spatial convolution of the input with each kernel, twelve feature maps of 8x8 pixels are obtained.

The second layer has 12 convolution kernels, each one with 16 neurons, creating a 4x4 pixel feature map. The third layer had 30 neurons and processed the more abstract features contained in the image. Backpropagation was applied to learn all connection and kernel weights.

Further work on this network led to the development of LeNet-5 by the same author in 1998 [31], which had a profound impact in the field of computer vision. LeNet-5 has 7 layers and accepts a 32x32 pixel image as its input.

The first layer extracts 6 feature maps of 28x28 pixels,

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obtained by performing convolution of the input with a 5x5 pixel kernel. The second layer subsamples the feature maps into smaller feature maps of 14x14 pixels, by subdividing the feature maps into groups of 2x2 pixels and averaging their values.

The third layer extracts 16 feature maps of 10x10 pixels, by performing convolutions with 5x5 pixel kernels. The 16 feature maps are then subsampled by the fourth layer in the same way as the second layer, generating 16 feature maps of 5x5 pixels.

The fifth layer extracts 120 feature maps of 1x1 pixels, by convoluting its input with 5x5-pixel kernels. Due to the size of the resulting feature map (composed of a single pixel), this layer acts as a fully connected layer.

The sixth layer has 84 neurons. This number has been chosen so that its output can be rearranged to form a 7x12-pixel image. The seventh and final layer classifies the network's input, through Euclidean Radial Basis Functions (RBF) units. There is one RBF unit for each possible class, with the centers hyper-parameters of each RBF purposely defined to compare the image generated by the sixth layer with all of the possible classes.

LeNet 5 was superior to many other character identification algorithms of its time. This topology based on sequential convolutions and sub sampling layers is still used in modern CNNs. In fact, the best algorithms for object identification on computer vision are based on CNNs, with examples such as GoogleLeNet, a 22-layer deep CNN which achieved a classification error of about 6% on ImageNet's Large-Scale Visual Recognition Challenge in 2014 [32][33]. The use of GPUs for training CNNs, a technique introduced in the 2000's, greatly reduced learning times [34][35], further popularizing this architecture.

### B. CNN structure

Current CNNs are composed of several layers reducing in size, with each layer applying a form of compression on its input data. The neurons on these layers are not fully connected to the other layers, greatly reducing the number of weights to be adjusted by training. Data is reduced in size from layer to layer, until the final layers. The final layers are usually densely connected, so as to use the compressed data to perform classification or regression. An example of typical CNN structure used for classifying images is shown in Fig. 9.

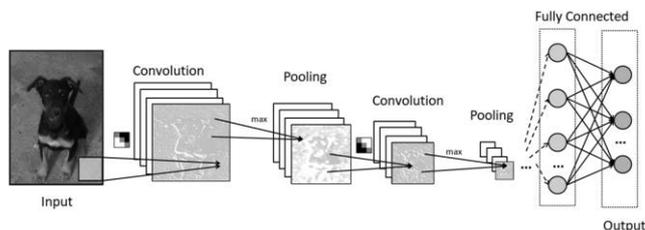


Fig. 9 – Typical CNN structure for image classification

The initial compression stage is composed of two types of layers, convolution layers and pooling layers. Convolution layers are the main building block in CNNs. The layer's input is swept by a filter (the convolution kernel) designed to extract a specific feature. This extraction is done by performing cross-correlation (convolution over a mirrored kernel) of the input with the kernel, centered on the first element of the input. After each convolution, the

kernel advances a number of elements on the input before executing the next convolution. The number of elements advanced is called "stride". This procedure is repeated until the entire input is swept by the kernel. A ReLU function may be used after each convolution to increase nonlinearity in activation [36].

The second type of layer on the compression stage of CNNs is the pooling layer. These are usually placed after a number of convolution layers. Pooling refers to down sampling the input by partitioning it into smaller groups of elements, and then performing a nonlinear function over the subgroup elements, called the pooling function. The output of the pooling function for each subgroup is used as the layer's output. Historically, averaging and *tanh* was used as pooling function as seen in LeNet-5 [30], but recent CNNs use *max*, as it can converge faster and have better generalization in practice [37].

Fig. 10 shows the results of an example image convoluted by four 3x3 pixel kernels, followed by a 2x2 pixel max pooling. The white squares represent an element of value 1, gray squares represent 0 and black squares represent -1. Notice that each kernel extracts a different set of edges from the image.

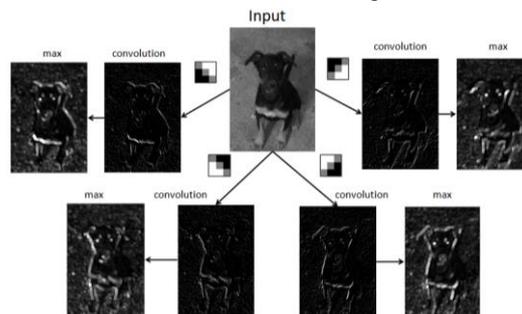


Fig. 10 – Image processed by convolutional kernels and max operations.

The second stage performs generates the output of the network using the data acquired by the first stage. The final layers are fully connected, with large numbers of adjustable parameters. To improve learning speed and preventing neuron saturation, ReLU is often used as activation function. Another problem caused by the large number of weights is overfitting, as the number of examples is often outnumbered by the number of parameters in several applications.

A recent developed technique for preventing overfitting is "dropout". This technique consists in randomly deactivating neurons during training with probability  $p$ . This forces the network to not rely on a few specific neurons, therefore improving network robustness. After training, all neurons may be used normally, but their outputs (or their corresponding weights) should be multiplied by a factor  $1-p$  to compensate for the deactivation procedure [38].

## VI. HYBRID NETWORKS

### A. Generative Adversarial Networks (GAN)

The three networks presented previously can be combined with other topologies and artificial intelligence techniques in order to create a larger hybrid network. This is done to allow domain-specific networks to function beyond their original design purposes and limitations. The most common hybrids combine CNN architectures with other technologies, justified by the CNN's simplicity and impact in the computer vision community.

A recent example is Generative Adversarial Networks (GAN) created in 2014 [39]. This network combines two smaller networks acting in opposing roles, as a generative network and a discriminator, often in a form of zero-sum game. Differently from the majority of other deep networks, which are focused on classification problems, GANs are primarily used as a generative model, and are often used in practice for generating synthetic images [40].

The GAN's generative network,  $G$ , is trained to generate data samples that match the distribution present on the training set, similarly to how a Boltzmann machine is trained to generate new data. The second network, the discriminator  $D$ , is trained to identify the synthetic data created by the generative network among the real examples contained in the training set. This discriminator is often a CNN.

Both networks are trained alternately, in a form so that the loss function for  $G$  and  $D$  can be rewritten to represent a minmax game. Training is often done using gradient descent algorithms, but due to the nature of the game between  $G$  and  $D$ , the algorithm may fail to converge, as improvement of one network may incur a decrease on the adversary performance [41].

This procedure drives both networks into becoming better at generating and detecting the synthetic data. The training proceeds until the synthetic data created by  $G$  is undistinguishable from the training data.

### B. RBM –CNN hybrids

Due to its generative capability and feature extraction properties, RBMs are often paired with CNNs and other Artificial Intelligence (AI) techniques.

A structure proposed in [42] is a CNN-RBM hybrid, for face verification in wild conditions. The last stages of CNN can be connected to stack of RBMs, creating a form of DBN on top of the convolutional stages. The method is shown to have comparable results with other high precision methods, with the advantage of not requiring hand-made filters for extracting features and strong alignment.

Another example of a CNN-RBM hybrid is shown in [43] for speech recognition. The structure presented in this work is a CNN that uses RBMs as the convolutional filters, creating a stochastic convolution layer. The RBMs can be pretrained to learn the correct filters. This improves network performance if the training set is small.

### C. Autoencoder hybrids

Autoencoders hybrids are not as common as RBM and CNN hybrids, as the classical forms of the autoencoder lack some of the interesting properties of the former topologies. However, a recent variation of the autoencoder called variational autoencoder (VAE) can act as generative model, which is a desirable property for creating hybrid systems.

A VAE combines variational inference and deep learning in a direct probabilistic graphical model. Rather than directly mapping the input values like in the classical autoencoder's training, the VAE's training aims to model the parameters of the training set distribution, treating the set of inputs as probabilistic values. The training tries to minimize the KL divergence between the training input set and the reconstructed set.

After training, a VAE will learn not only the mean value of a given input, but also its variance. Thus, it is able to

regenerate inputs through stochastic procedures [44]. This property can be used in conjunction with a CNN and recurrent models to generate text as presented in [45]. Moreover, a VAE can act as generative network for use in a GANs, as reported in [46].

## VII. CONCLUSION

This paper has described the basic building blocks which constitute what is now known as Deep Learning, as well as some of the most common network architectures that can be built with those blocks.

The increase of depth allowed neural networks to achieve much higher levels of precision as a function approximator compared to their shallow counterparts. While any shallow networks capable of approximating a function given enough neurons, the number of neurons required for a precise approximation increases exponentially as a result of the curse of dimensionality. The addition of extra layers in the network leads to a higher level of abstraction in data processing, and therefore higher precision, as seen on all of the architectures shown in the paper.

As a result of this increase on precision, deep neural networks have been used as solution for many problems for which there's no closed analytical solution, with many networks achieving much better performance over traditional rule-based algorithms. With the increase on data availability and hardware capacity, it should be expected that many of the rule-based algorithms used today for tackling complex problems to be improved or even be superseded by neural-network based approaches in the near future.

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## A Brief Didactic Theoretical Review on Convolutional Neural Networks, Deep Belief Networks and Stacked Auto-Encoders

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