Vibrational Study of Beams by Incorporating Geometric nonlinearity of the structures

S. Gurumoorthy, L. Bhaskara Rao

Abstract— **The nonlinear vibration of 3D beams is studied for different end conditions of beams (such as clamped, Pinned and free etc.) and by considering the geometric nonlinearity into account. Various cross sections of the beams are taken and different materials are considered for the analysis. The beams are studied using P-Version, H-Version and HP-Version finite element method. Timoshenko and Bernoulli – Euler beam models are used for the analysis. The principle of virtual work is used for the deriving the equations of motion. Bending linear natural frequencies of beams with different materials and different end conditions are calculated. The significance of warping is evaluated for different cross sectional dimensions of the rectangular beams. Nonlinear static analysis is performed for the beams for the different loads (Point loads and moment etc) with different number of shape functions. Twist angle for the different dimensions of the rectangular beams is calculated (for moment load) with and without including the linearization of the trigonometric terms in strain – displacement relations. A comparative study is done for with and without considering the higher order parameters in the direct strain equations. Comparison between the two different beams models is done. The consequences of quadratic parameters of the displacement along the longitudinal direction are studied. Beam's dynamic response due to the harmonic excitation is calculated with considering the bending torsion coupling. This kind of study will be useful while validating some of the automotive structures also.**

Index Terms— **Beam, end conditions, nonlinear, Vibration**

I. Introduction

Beams are the structures, which are having one dimension as larger one as compared with the other two dimensions. Beam models are regularly used in design, because they can offers precious insight into structure's behavior. The beam modeling can be divided as linear & non-linear modeling. In mechanical systems, a number of types of nonlinearities are present. The general nonlinearities are geometric, material and inertial.

S.Bagheri et al [11] explored the responses of a buckled clamped-clamped beam. In order to derive the equations of motion, two mathematical approaches called He's Variational Approach and Laplace Iteration Method are employed in this research. Based on the results comparison, it is known that for other nonlinear oscillations the above two methods can be easily extended to it is concluded that above two methods can be extensively suitable in physics and engineering.

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S. Stoykov , P. Ribeiro [12] investigated the nonlinear vibrations of beams by taking into account the geometric nonlinearity. The cross section of the beam is taken as rectangular. In this study it is assumed that the beam may experience torsional, bending and longitudinal deformations in any plane. P-version finite element method is used. Both Euler – Bernoulli beam theory and Timoshenko's beam theory are used. Green's strain tensor is considered in to the study. Principle of virtual work is used to derive the equation of motion.

Li-Qun Chena, Xiao-Dong Yang. B [13] studied the nonlinear free transverse vibration of a beam which is axially moving . Newton's second law is used to derive the partial differential equation which governs the transverse vibration of the beam. Method of multiple scales used to two equations to evaluate nonlinear natural frequencies. The axial speed, nonlinear term and the order of mode are the main parameters which causes the difference between the two models

Iacob Borş et al., [14] investigated the beam's free vibrations under axial load conditions. In this study geometric nonlinearity is also considered. In this study it is considered that the beam has continuous mass. This problem is included into a system which is having ∞ dynamical degree of freedom. Mode shapes and natural frequencies are found out based on homogeneous equations of vibrations. Those equations are solved by using the separations of variables method.

Ozgur Turhan , Gokhan Bulut [15] explored the rotating beam's nonlinear bending vibrations . In the form of an integral-partial differential equation, the equation of motion is obtained. Perturbation analyses are carried out to obtain the natural frequencies, which are amplitude dependent. For these analyses, both 1DOF and 2DOF models are used.

Tai Ping Chang [16] studied the fixed-fixed beam's nonlinear vibration behavior. The nonlinear behavior is studied by considering vibrating magnetic field and Oscillating axial load. The transverse magnetic force, transverse magnetic couple, axial force, uniform translation spring force, transverse surface force and the damper are considered in the system. For deriving the equation of motion, Hamilton's principle is adopted under certain hypo –theses. To attain the solutions, Galerkin's method is utilized

In the present study, geometric nonlinear vibrations of 3D beams with different end conditions (such as fixed, free and pinned etc.,) is studied. Different types of cross sectional beams with different materials are considered for the analysis. The beams, which experience bending, torsional and longitudinal deformations in space, are studied using H-

Method, P-Method and HP- method which exists in the finite element analysis.

II. PROBLEM DESCRIPTION

Two main beam models are considered in this study. One of the beam models is Bernoulli- Euler beam theory [2-5] and another one is Timoshenko's theory for flexure [1].

Symmetrical cross section such as rectangular section is considered for the nonlinear vibration analysis. By applying the principle of virtual work, equation of motion is derived. Based on Green's strain tensor [6-7[], shear strain and axial strain is derived by considering the geometric nonlinearity. Natural frequencies are calculated by using P-Version, H-Version & HP-Version of FEM [8-9].

Both bending and torsional natural frequency are calculated by using different version of FEM. Nonlinear static analysis has been done to find out the transverse displacement & angular displacement of the beam for various types of loads like point loads, moment, uniform distributed load and uniform varying load.

Static deformation of different types of beams (Fixed, pinned and guided etc.) with different cross sections are studied by including & neglecting the higher order terms which are appearing in the direct strain equation of the beams. Comparison of Timoshenko theory & Bernoulli-Euler theory is carried out by using P-Version FEM.

A difference between the beams models has been found out for the various magnitudes of loads . The consequences of quadratic parameters of the displacement along the longitudinal direction are studied for the beam which is having the cross section as rectangle. Beam's dynamic response due to the harmonic excitation, is calculated with considering the bending torsion coupling. Steady state time response of the beam is also done. All the above mentioned analysis has been done for different materials like aluminum, concrete & polymer.

Natural frequency of the beam was found out by using ANSYS [10]. In analysis, BEAM 189 type of element is used. BEAM189 is a type of element which is appropriate for analyze the slender and thick beam structures. This element is formulated based on the Timoshenko's beam theory.

The effects of shear deformation are also incorporated. This element is a 3node beam element and each node is having six degrees of freedom. It means that, each node is having 3 translations and 3 rotations. For large displacement, large rotation, and nonlinear applications, this type of element is mostly suited.

III. MATHEMATICAL FORMULATION

Formulation of the equation of motion is the first step in any vibration analysis problem .Two different methods are there in the equation of motion formulation. Analytical and vectorial .Newton's second and third laws are applied directly in the vectorial approach. The system is considered as a whole rather than as individual components in the analytical method. The equation of motion is derived based on variational approach, in the analytical method. Principle of virtual work, Lagrange's equation and Hamilton's principle are the some of the variational approaches.

To treat the partial differential equations, there are two methods. The partial differential equations and boundary conditions are treated numerically or analytically in the first method. The partial differential equations and boundary conditions are discretized either by weighted residual or variational method in the second method. Galerkin, sub domain and collocation are some of the most widely used weighted residual methods. Rayleigh Ritz is the mostly used variational method.

Element's shape function will be linear for H- Method. This phenomenon will be accounted in this method by altering the number of elements. More exact information is attained by increase the element numbers. In the P-Method, there is no need to alter the elements.

The element's shape function will be altered in order to make the element to have the capability to solve non-linear displacement functions. In P- method, more accurate information is obtained by increasing the complexity of the shape function. The P- method's accuracy will be changed based on the increase in the shape function's polynomial order.

IV. MODEL DETAILS

Beam with assorted cross sections are considered in this study. Beam length is taken as 0.58m and dimensional details for rectangular beam cross sections are shown in Fig 1 .The units are in m.

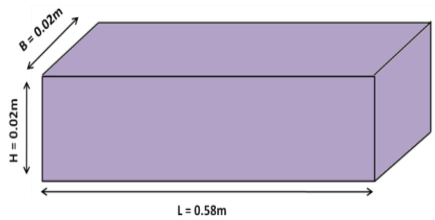

Fig 1. Cross section details

V. RESULTS

A beam of 0.58 X 0.02 X 0.002 m dimension [12] has been taken for the analysis. Comparison of natural frequency results for clamped-free beam condition and for different material by using H- method is tabulated below.

In H – methods, the no of elements used for the analysis are 2, 6,10,20,25 & 30.

Table 1. Natural frequency values for a Clamped – free beam condition for Aluminum

Mode shapes	2 ele	6 ele	10 ele	20 ele	25 ele	30 ele
1	4.82	4.82	4.82	4.82	4.82	4.82
2	31.99	30.23	30.21	30.21	30.21	30.21
3	48.24	48.16	48.16	48.16	48.16	48.16
4	107.55	85.06	84.64	84.58	84.58	84.58
5	252.97	169.21	166.21	165.76	165.74	165.73

Table 2. Natural frequency values for a Clamped – free beam condition for steel

Mode shapes	2 ele	6 ele	10 ele	20 ele	25 ele	30 ele
1	4.94	4.93	4.93	4.93	4.93	4.93
2	32.78	30.97	30.95	30.95	30.95	30.95
3	49.42	49.34	49.34	49.34	49.34	49.34
4	110.19	87.15	86.72	86.66	86.66	86.66
5	270.21	173.86	170.29	169.83	169.81	169.8

Table 3. Natural frequency values for a Clamped – free beam condition for Lead

Mode shapes	2 ele	6 ele	10 ele	20 ele	25 ele	30 ele
1	1.06	1.05	1.05	1.05	1.05	1.05
2	7.02	6.64	6.63	6.63	6.63	6.63
3	10.59	10.57	10.57	10.57	10.57	10.57
4	23.62	18.68	18.59	18.58	18.58	18.58
5	55.10	37.16	36.50	36.40	36.40	36.40

Table 4. Natural frequency values for a Clamped – free beam condition for Rubber

Mode shapes	2 ele	6 ele	10 ele	20 ele	25 ele	30 ele
1	1.39	1.39	1.39	1.39	1.39	1.39
2	9.20	8.69	8.69	8.69	8.69	8.69
3	13.87	13.85	13.85	13.85	13.85	13.85
4	30.93	24.46	24.32	24.32	24.32	24.32
5	70.46	48.65	47.79	47.66	47.66	47.65

Harmonic analysis results for the clamped – Free beam condition for different materials are shown below

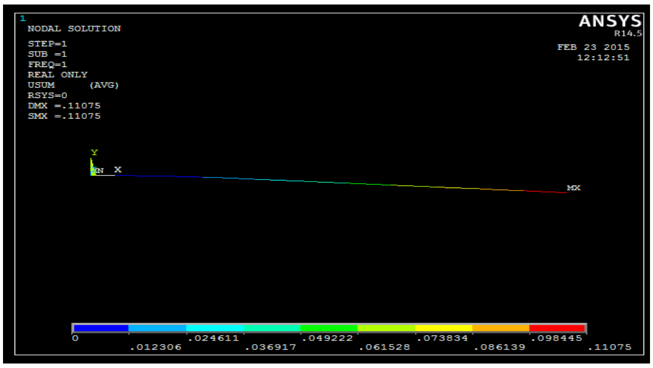

Fig 2. Displacement plot for Lead

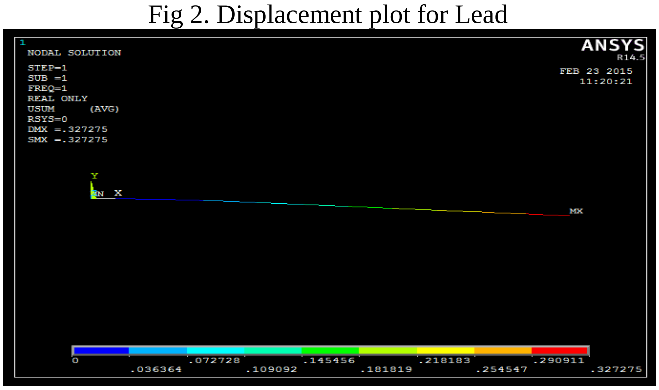

Fig 3. Displacement plot for Steel

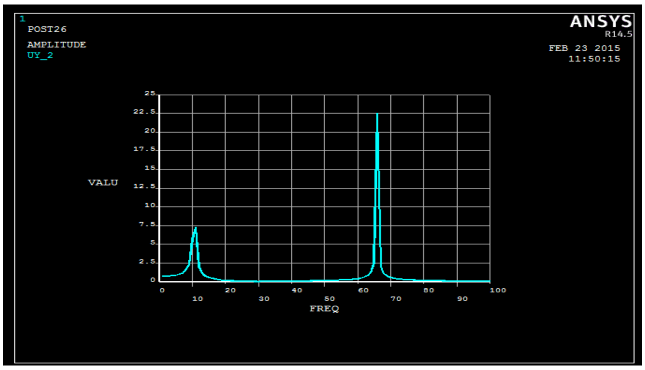

Fig 4. Frequency Vs Displacement plot for Lead

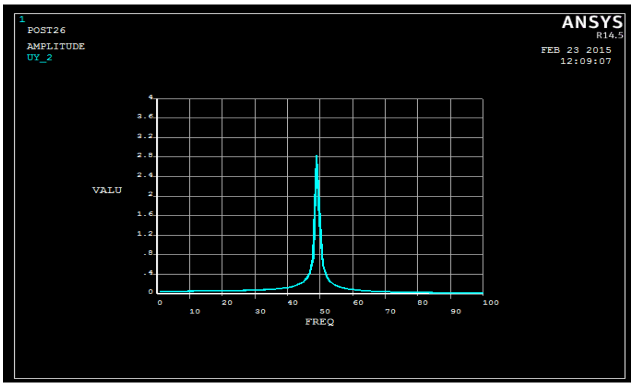

Fig 5. Frequency Vs Displacement plot for Steel

Like this modal, static and dynamic analysis was done for the different end conditions of the beam (both ends simply supported, simply supported – clamped, guided-guided etc. and for the different materials also.

VI. SUMMARY

Isotropic beam models which are vibrating in space and having symmetrical cross sections are analyzed. Natural frequencies, longitudinal and rotational displacement for different types of loads are calculated for beams with different end conditions and with different materials. All the above calculations are done by using H- Version FEM. The same calculation and the equation of motion derivation are under progress for P – Version and HP – Version FEM.

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