Simulation of Robust DWT-SVD Transform Domain Based Digital Image Watermarking Technique

Shubham Arya, Mr. Pratyush Tripathi

Abstract— The purpose of this paper is to improve the robustness of traditional image watermarking based on singular value decomposition (SVD) by using optimization-based quantization on multiple singular values in the wavelet domain. In this work, we divide the middle-frequency parts of discrete-time wavelet transform (DWT) into several square blocks and then use multiple singular value quantizations to embed a watermark bit. To minimize the difference between original and watermarked singular values, an optimized-quality formula is proposed. First, the peak signal-to-noise ratio (PSNR) is defined as a performance index in a matrix form. Then, an optimized-quality functional that relates the performance index to the quantization technique is obtained. Finally, the Lagrange Principle is utilized to obtain the optimized-quality formula and then the formula is applied to watermarking. Experimental results show that the watermarked image can keep a high PSNR and achieve better bit-error rate (BER) even when the number of coefficients for embedding a watermark bit increases.

Index Terms—SVD, DWT, PSNR, BER

I. INTRODUCTION

With the rapid development of activity on the internet, much digital information is widely spread. Digital watermarking was developed to hide digital information and protect the copyright of multimedia signals, like audio, images, etc. Due to the fact that discrete-time wavelet transform (DWT) provides a useful platform, numerous DWT-based algorithms for digital watermarking have been proposed in recent years. Watermarking in the spatial domain [1-11] is usually more vulnerable than watermarking in the frequency domain [12–29] with the same embedding capacity due to the fact that spatial-domain methods are generally fragile to image-processing operations and other attacks [23-25]. The spatial-domain singular value decomposition (SVD) for image watermarking was first introduced by Liu et al. [8]. In this paper, the authors used a spread-spectrum technique to embed a watermark by modifying the singular values of the host image in the spatial domain. Some authors embedded watermark to U and V components to increase embedding capacity [9, 10] while Ghazy et al. [11] presented a blockbyblock SVD-based image-watermarking scheme to increase embedding capacity. However, the robustness of SVD-based image watermarking in the spatial domain is low. In recent years, many image-watermarking techniques combine DWT and SVD to achieve better transparency and robustness [17, 18, 24, 25]. Bao et al. [17] proposed a novel, yet simple, image-adaptive watermarking scheme for image authentication by applying a simple quantization index-

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modulation process on each single singular value of the blocks in the wavelet domain. Their watermarking scheme is blind and is robust against JPEG compression but extremely sensitive to malicious manipulation such as filtering and random noising. Ganic et al. [18] applied SVD to all details, approximating part of the DWT and watermark image to increase embedding capacity. Gaurav and Balasubramanian [24] embedded a watermark into the reference image by modifying the singular value of the reference image using the singular values of the watermark. The robustness is slightly enhanced. However, the computation is significantly increased. Lai and Tsai [25] reduced the computation in [24] by directly embedding the watermark into the singular values in the wavelet domain. In this work, we first divide the DWT middle frequency parts LH3 and HL3 into several square blocks to have high embedding capacity. Unlike the traditional spread-spectrum technique on single singular values [24, 25], we use multiple singular value quantizations to embed a watermark bit. It does not only keep a high embedding capacity but also achieves strong robustness against median filtering. On the other hand, an optimized quality formula is proposed by minimizing the difference between original and watermarked singular values. First, the peak signal-to-noise ratio (PSNR) is defined as a performance index in matrix form. Then, an optimized quality functional that relates the performance index to the quantization technique is obtained. Finally, the Lagrange Principle is utilized to obtain the optimized quality formula; then, the formula is applied to watermarking. Experimental results show that the watermarked image can keep a high PSNR and achieve a better bit error rate (BER) even when the number of coefficients for embedding a watermark bit increases. In particular, the robustness against median filtering is significantly improved.

This paper is organized as follows. In Section II, we review some mathematical preliminaries. Section III introduces the proposed watermark embedding and extraction. In Section IV, we rewrite PSNR as a performance index. An optimized-quality equation that relates the performance index to the quantization constraint is proposed, and the Lagrange Principle is used to solve the optimized-quality problem. The solution is utilized to embed the watermark, and we discover a very good result; the watermark is extracted without the original image. In Section V, we present some experiments to test the performance of the proposed scheme. Finally, conclusions are drawn in Section VI.

II. PRELIMINARIES

In this section, some related steps for the proposed image watermarking scheme are reviewed.

DISCRETE-TIME WAVELET TRANSFORM (DWT)

The wavelet transform is obtained by a single prototype function which is regulated with a scaling parameter and shift parameter [28–31]. The discrete normalized scaling and wavelet basis function are defined as follows:

$$\varphi_{j,l}(t) = 2^{l/2} \varphi \left(2^{j} t - \tau \right) \tag{1}$$

$$\psi_{j,l}(t) = 2^{l/2} \psi \left(2^{l} t - \tau \right)$$
⁽²⁾

where j and τ are the dilation and translation parameters; from this, one can require that the sequence

$$\{0\} \subset \cdots \subset V_1 \subset V_0 \subset V_{-1} \subset \cdots \subset L^2(\mathbb{R})$$
⁽³⁾

Forms a mutiresolution analysis of L2(\mathbb{R}) and that the subspaces $\cdot \cdot \cdot$, W1, W0, W₋₁, $\cdot \cdot \cdot$ are the orthogonal differences of the above sequence; that is, Wj is the orthogonal complement of Vj inside the subspace Vj – 1. Then, the orthogonality relations follow from the existence of sequences $h = {h\tau}{\tau} \in \mathbb{Z}$ and $g = {g\tau}{\tau} \in \mathbb{Z}$ that satisfy the following identities:

$$h_{\tau} = \varphi_{0,0}, \varphi_{-1,\tau} \text{ and } \varphi(t) = \sqrt{2} \sum_{\tau \in \mathbb{Z}} h_{\tau} \varphi(2t - \tau)$$

$$g_{\tau} = \psi_{0,0}, \phi_{-1,\tau} \text{ and } \psi(t) = \sqrt{2} \sum_{\tau \in \mathbb{Z}} g_{\tau} \varphi(2t - \tau)$$
(4)
(5)

where $h = {h\tau} \tau \in \mathbb{Z}$ and $g = {g\tau} \tau \in \mathbb{Z}$ are, respectively, the sequence of low-pass and high-pass filters. In this paper, we use a Haar scaling function and wavelet to transform the host image into the orthogonal DWT domain by three-level decomposition. A method to implement DWT is a filter bank that provides perfect reconstruction. DWT has local analysis of frequency in the space and time domains, and it obtains image multi-scale details step by step. If the scale becomes smaller, every part gets more accurate and ultimately all image details can be focalized accurately. If DWT is applied to an image, it will produce high-frequency parts, middle-frequency parts, and a lowest-frequency part. Figure 1 shows the procedure of applying one-level DWT to an image. In order to guarantee both image quality and robustness, this study embeds the watermark into the middle frequency parts LH3 and HL3 in DWT level-three.

SINGULAR VALUE DECOMPOSITION (SVD)

The singular value decomposition of a matrix A with size $m \times n$ is given by

$$A = UDV^T$$
⁽⁶⁾

Where, U and V are orthogonal matrices, and $D = \text{diag}(\lambda i)$ is a diagonal matrix of singular values λi , $i = 1, 2, \dots$, which are arranged in decreasing order. The columns of U are the left singular vectors, and the columns of V are the right singular vectors of image A.

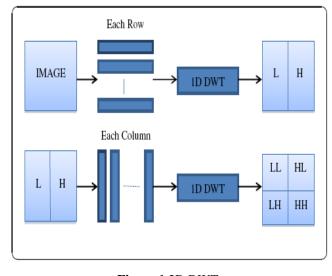


Figure 1 2D DWT

III. OPTIMIZATION SOLVER

To find the extreme of the matrix function, some optimization methods are summarized in [29–31]. The operations of the matrix function are first shown as follows:

Theorem 1: If W is a $k \times k$ constant matrix, and X^A is a $k \times 1$ column vector with k unknown variables, then

$$\frac{\partial \mathbf{W} \mathbf{X}}{\partial \widehat{\mathbf{X}}} = \mathbf{W}$$

Theorem 2: If X is a $k \times 1$ constant vector and ^X is a $k \times 1$ column vector with k unknown variables, then

(7)

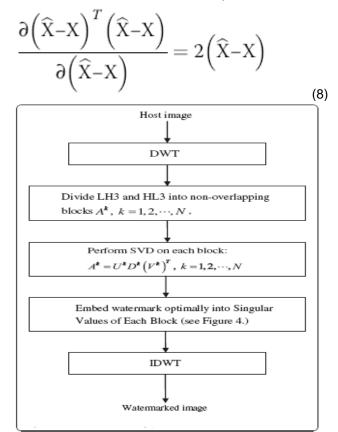


Figure 2 Watermark Embedded Process

$$\nabla f(\widehat{X}) = \frac{\partial f}{\partial \widehat{X}} = \left[\frac{\partial f}{\partial \widehat{X}_1} \frac{\partial f}{\partial \widehat{X}_2} \cdots \frac{\partial f}{\partial \widehat{X}_k}\right]^2$$

Now we consider the problem of minimizing (or maximizing) the matrix function $f(X^{A})$ subject to a constraint $bX^{1/4}$ 0. This problem can be described as follows:

minimize
$$f(\widehat{X})$$
 9(a)

subject to $g(\widehat{X}) = 0$ 9(b)

Theorem 3: Suppose that g is a continuously differentiable function of $\hat{\mathbf{X}}$ on a subset of the domain of a function f. Then if $\hat{\mathbf{Xo}}$ minimizes (or maximizes) f ($\hat{\mathbf{X}}$)subject to the Constraint ($\hat{\mathbf{X}}$); $\nabla f(\hat{\mathbf{Xo}})$ and ; $\nabla g(\hat{\mathbf{Xo}})$ are parallel.

That is, if $b_X0 \neq 0$, then there exists a scalar ξ such that

$$\nabla f(\hat{\mathbf{X}}_{0}) = \xi \nabla g(\hat{\mathbf{X}}_{0}).$$
(10)
Watermarked image
DWT
Group the optimized singular values of
SVD in each block
$$\widehat{\boldsymbol{\beta}}_{j}^{k} = \left| \sum_{i=1}^{r} \hat{\lambda}_{i}^{k} / q + 0.5 \right| \mod 2$$
Watermark $\hat{\mathbf{B}}$

Figure 3 Watermark extraction process

$$H\left(\widehat{\mathbf{X}},\xi\right) = f\left(\widehat{\mathbf{X}}\right) + \xi g\left(\widehat{\mathbf{X}}\right)$$

Then the original problem (9) becomes a function H $(\widehat{\mathbf{X}}\xi)$ which has no constraint. The necessary conditions for existence of the extreme of function H $(\widehat{\mathbf{X}}\xi)$ are:

$$\frac{\partial H}{\partial \xi} = 0, \tag{12}$$

$$\frac{\partial H}{\partial \hat{X}} = 0.$$

IV. PROPOSED OPTIMIZATION-BASED DWT-SVD WATERMARKING SCHEME

(13)

The proposed watermarking scheme is introduced in this section. The watermark is extracted without the original image.

WATERMARK EMBEDDING

The proposed embedding process as shown in Fig. 2 is summarized as follows.

(1)Use three-level DWT to decompose the original image A into four subbands (i.e., LL3, LH3, HL3, and HH3).

(2)Divide LH3 and HL3 into non-overlapping blocks Ak, $k = 1, 2, \dots, N$.

(3) Apply SVD to each block, i.e.,

$$A^{k} = U^{k} D^{k} (V^{k})^{T}, \quad k = 1, 2, \cdots, N$$
 (14)

where ${\sf k}$ represents the number of blocks in LH3 and HL3.

(4)Watermark $B = \{\beta_i\}$ randomly generated using a binary PN sequence is embedded by modifying

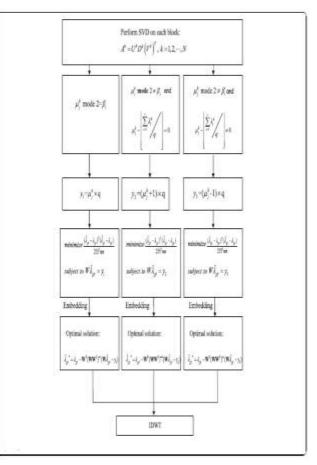


Figure 4 Optimal Embedding

Singular values $\lambda \lambda_i^k$, $i = 1, \dots, r$ of the matrix A^K as follows: Let

(11)

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 $\mu_j^k = \left\lfloor \sum_{i=1}^r \lambda_i^k \middle/_q + \frac{1}{2} \right\rfloor,$

WATERMARK EXTRACTION

To detect the watermark, DWT is first performed and then the optimized singular values of SVD in each block are grouped. The embedded binary bits are extracted by using the following rule.

$$\hat{\beta}_{j}^{k} = \left\lfloor \sum_{i=1}^{r} \hat{\lambda}_{i}^{k} \right/_{q} + 0.5 \right\rfloor \mod 2,$$

Figure 3 shows the detailed process of the proposed watermark extraction.

V. OPTIMIZATION OF PSNR ON SINGULAR VALUES

Generally, the quality of a watermarked image is evaluated by the peak signal-to-noise ratio (PSNR). Since a tradeoff exists between image quality measured by PSNR and robustness measured by BER, a scalar parameter ξ is applied to connect the PSNR and the quantization equation to optimize the tradeoff in this section. The details are in the following:

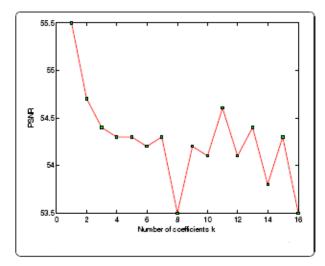


Figure 5 The relation b/w PSNR and number of coefficients

TABLE 1

The PSNR. If I(i, j) and $\overline{I}(i, j)$ are the values of the original and the corresponding modified pixel in the original image I and watermarked image \overline{I} , then is defined as

$$PSNR = -10 \log_{10} \left(\frac{\sum_{i=1}^{m} \sum_{j=1}^{n} (\bar{I}(i,j) - I(i,j))^{2}}{255^{2} mn} \right)$$
16

where m and n represent the height and width of the host image. Based on the watermark embedding of singular values, PSNR is expressed as

$$PSNR = -10 \log_{10} \left(\frac{\left\| \hat{\lambda}_{D^k} - \lambda_{D^k} \right\|_2^2}{255^2 mm} \right)$$
(17)

For the optimization of the watermarked image quality,

Eq. 17 is rewritten as a performance index:

$$f\left(\hat{\lambda}_{D^{k}}\right) = \frac{\left\|\hat{\lambda}_{D^{k}} - \lambda_{D^{k}}\right\|_{2}^{2}}{255^{2}mn}$$
(18)

Or,

(15)

$$f\left(\hat{\lambda}_{D^{k}}\right) = \frac{\left(\hat{\lambda}_{D^{k}} - \lambda_{D^{k}}\right)^{2} \left(\hat{\lambda}_{D^{k}} - \lambda_{D^{k}}\right)}{255^{2} mn}$$
(19)

minimize
$$\frac{\left(\hat{\lambda}_{D^{k}} - \lambda_{D^{k}}\right)^{T} \left(\hat{\lambda}_{D^{k}} - \lambda_{D^{k}}\right)}{255^{2} mn} \qquad 20(a)$$

subject to
$$W\lambda_{D^k} = y_1$$
 20(b)

To embed the watermark B, we need to solve the optimization problem (20). By Theorem 3, we set the Lagrange multiplier λ to combine (20a) and (20b) into a matrix function:

$$H\left(\hat{\lambda}_{D^{k}},\xi\right) = \frac{\left(\hat{\lambda}_{D^{k}}-\lambda_{D^{k}}\right)^{\mathsf{T}}\left(\hat{\lambda}_{D^{k}}-\lambda_{D^{k}}\right)}{255^{2}mm} + \xi\left(\mathbb{W}\hat{\lambda}_{D^{k}}-y_{1}\right)$$

(21)

(26)

which has no constraint. Since 2552 mn is a constant, we redefine $H_{\lambda}Dk$; ξ_{as} follows:

$$H(\hat{\lambda}_{D^{k}},\xi) = (\hat{\lambda}_{D^{k}} - \lambda_{D^{k}})^{\mathsf{T}} (\hat{\lambda}_{D^{k}} - \lambda_{D^{k}}) + 255^{2} mn\xi (\mathbb{W}\hat{\lambda}_{D^{k}} - y_{1})$$

$$(22)$$

$$\frac{\partial H}{\partial \hat{\lambda}_{D^{k}}} = 2\left(\hat{\lambda}_{D^{k}} - \lambda_{D^{k}}\right) + 255^{2} mn \xi W^{\mathsf{T}} = 0$$
23(a)

$$\frac{\partial H}{\partial \xi} = W \hat{\lambda}_{D^k} - y_1 = 0$$
(23(b))

Multiplying (23a) by W, we observe that $2(W\hat{\lambda}_{rk} - W\lambda_{rk}) + 255^2 mn \mathcal{E}WW^{\mathsf{T}} = 0$

$$2\left(w_{D}k^{2}-w_{D}k\right)+255\ mm,\ w\ w\ =0$$
(24)

Since W^ λ Dk ¹/₄ y1 from (23b) and 255² mn is a scalar, we rewrite (24) as

$$\left(y_1 - W\hat{\lambda}_{D^k}\right) + \frac{255^2mn}{2}\xi WW^{\mathsf{T}} = 0$$

(25)

Some operations yield the optimal solution for parameter ξ as

$$\xi^* = \frac{2}{255^2 \, mn} \left(\mathbf{W} \mathbf{W}^{\mathsf{T}} \right)^{-1} \left(\mathbf{W} \hat{\lambda}_{D^k} - y_1 \right)$$

Replacing Eq. 26 with Eq. 23a yields the optimal embedded singular values

$$\hat{\lambda}_{D^{k}}^{*} = \lambda_{D^{k}} - \frac{255^{2}mn}{2} \xi^{*} W^{\mathsf{T}}$$
$$= \lambda_{D^{k}} - W^{\mathsf{T}} (WW^{\mathsf{T}})^{-1} (W\lambda_{D^{k}} - y_{1})$$
(27)

By using y2 instead of y1 yields the optimal embedded singular values

$$\hat{\lambda}_{D^{\star}}^{*} = \lambda_{D^{\star}} - W^{\mathsf{T}} (WW^{\mathsf{T}})^{-1} (W\lambda_{D^{\star}} - y_{2})$$
(28)

VI. RESULS

This section presents experimental results that indicate the performance of the proposed image-watermarking scheme. Forty host images including the four images, Lena, Jet, Peppers, and Cameraman, each a size of 512×512 , are decomposed into three levels by applying DWT; then, the watermark is embedded into the LH3 and HL3 coefficients. Figure 5 shows that the watermarked image can keep a high and stable PSNR (almost 54.5 dB) even when the number of coefficients for embedding a watermark bit increases. This feature indicates the proposed optimization embedding formula using Lagrange principle. In order to compare with the SVD-based method [25], PSNR is fixed to be 55 dB. Table 1 shows the comparison of the embedding capacity under fixed PSNR = 55. Figure 6 shows the original images, and Figs. 7 and 8 show the watermarked images obtained with different parameters.

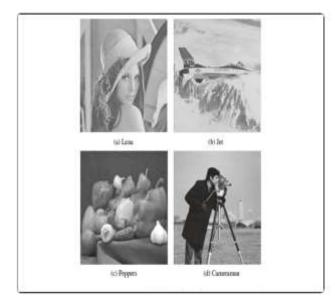


Figure 6 Original images

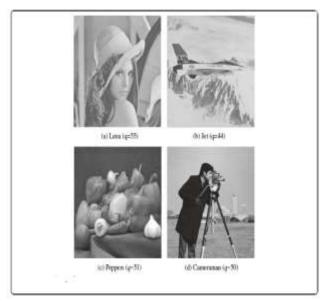


Figure 7 Watermarked image for k=4

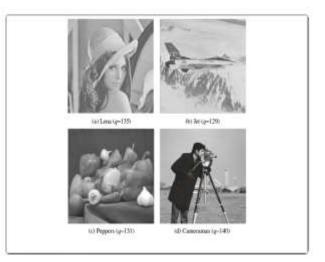


Figure 8 Watermarked image for k=8

To evaluate the robustness of the proposed method, the 40 images including the four images, Lena, Jet, Peppers, Cameraman, are tested. After the embedding process, four attacks are adopted to test the robustness of the embedded watermark in cases k = 4 and k = 8. The robustness is measured by BER defined by

$$BER = \frac{B_{error}}{B_{tatal}} \times 100\%$$

Where,

 B_{error} and B_{total} denote the number of error bits and the number of total bits, respectively. The method proposed herein is compared with the SVD based method using the spread-spectrum technique in the DWT domain [25].

VII. CONCLUSION

This study improved the robustness of traditional SVD based image watermarking by using optimization-based quantization on multiple singular values in the wavelet domain. Experimental results show that the watermarked image can keep a high PSNR and achieve a better BER even when the number of coefficients for embedding a watermark bit increases. In particular, the robustness against JPEG compression, Gaussian noise, and median filtering is significantly improved. The future work is the consideration of improving robustness against rotation.

REFERENCES

- [1] F Hartung, M Kutter, Multimedia watermarking techniques. Proceedings of the IEEE, 1999, p. 87
- [2] M Alghoniemy, AH Tewfik, Geometric distortion correction in image watermarking, in Proceedings SPIE Security and Watermarking of Multimedia Contents II 3971, 2000, pp. 82–89
- [3] M Alghoniemy, AH Tewfik, Progressive quantized projection watermarking scheme, in Proceedings 7th ACM International Multimedia Conference, Orlando, FL, 1999, pp. 295–298
- [4] B Chen, GW Wornell, Quantization index modulation: a class of provably good methods for digital watermarking and information embedding. IEEE Trans. Inf. Theory 47, 1423–1443 (2001)
- [5] P Kumswat, K Attakitmongcol, A Striaew, A new approach for optimization in image watermarking using genetic algorithms. IEEE Trans. Signal Process. 53(12), 4707–4719 (2005)
- [6] MU Celik, G Sharma, AM Tekalp, E Saber, Lossless generalized-LSB data embedding. IEEE Trans. Image Process. 14(2), 253–266 (2005)

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- [7] LC Lin, YB Lin, CM Wang, Hiding data in spatial domain images with distortion tolerance. Elsevier: Computer Standards and Interfaces 31, 458–464 (2009)
- [8] R Liu, T Tan, An SVD-based watermarking scheme for protecting rightful ownership. IEEE Transactions on Multimedia 4(1), 121–128 (2002)
- [9] C-C Chang, P Tsai, C-C Lin, SVD-based digital image watermarking scheme. Pattern Recogn. Lett. 26(10), 1577–1586 (2005)
- [10] KL Chung, WN Yang, YH Huang, ST Wu, YC Hsu, On SVD-based watermarking algorithm. Application. Math. Comput. 188, 54–57 (2007)
- [11] RA Ghazy, NA El-fishawy, MM Hadhoud, MI Dessouky, FEA El-Samie, An efficient block-by-block SVD-based image watermarking scheme, in 2007 Radio Science Conference, Cairo, 2007, pp. 1–9
- [12] SF Lin, SC Shie, JY Guo, Improving the robustness of DCT-based image watermarking again JPEG compression. Elsevier: Computer Standards and Interfaces 32, 57–60 (2010)
- [13] CC Lin, PF Shiu, High capacity data hiding scheme for DCT-based images. J. Inform. Hiding. Multimedia. Signal. Process. 1(3), 220–240 (2010)
- [14] H Qaheri, A Mustafi, S Banerjee, Digital watermarking using ant colony optimization in fractional fourier domain. J. Inform. Hiding. Multimedia. Signal. Process. 1(3), 179–189 (2010)
- [15] CH Manuel, GU Francisco, NM Mariko, HM Pérez-Meana, Robust hybrid color image watermarking method based on DFT domain and 2D histogram modification. Springer: Signal Image and Video Processing 8(1), 49–63 (2014)
- [16] L Xiao, H Wu, Z Wei, Multiple digital watermarks embedding in wavelet domain with multiple-based number. J. Computer. Aided. Design. Computer. Graphics. 15(2), 200–204 (2003)
- [17] P Bao, X Ma, Image adaptive watermarking using wavelet domain singular value decomposition. IEEE Transactions on Circuits and Systems for Video Technology 15(1), 96–102 (2005)
- [18] E Ganic, AM Eskicioglu, Robust embedding of visual watermarks using DWT-SVD. J. Electronic. Imaging. 14(4), 1–13 (2005)
- [19] M Sharkas, B Youssef, N Hamdy, An adaptive image-watermarking algorithm employing the DWT. the 23th National Radio Science Conference, 2006, pp. 14–16
- [20] CT Li, Reversible watermarking scheme with image-independent embedding capacity. IEEE Proceedings on Vision, Image, and Signal Processing 152(6), 779–786 (2006)
- [21] CV Serdean, MK Ibrahim, A Moemeni, MM Al-Akaidi, Wavelet and multiwavelet watermarking. IET Image Process. 1(2), 223–230 (2007)
- [22] OZ Azza, M Achraf, B Ammar, Wavelet domain watermark embedding strategy using TTCQ quantization. J. Computer Sci. Network Security. 7(6), 165–170 (2007)
- [23] N Li, X Zheng, Y Zhao, H Wu, S Li, Robust algorithm of digital image watermarking based on discrete wavelet transform. International Symposium on Electronic Commerce and Security, 2008
- [24] B Gaurav, R Balasubramanian, A new robust reference watermarking scheme based on DWT-SVD. Elsevier: Computer Standards and Interfaces, 2009, pp. 1–12
- [25] CC Lai, CC Tsai, Digital image watermarking using discrete wavelet transform and singular value decomposition. IEEE Trans. Instrum. Meas. 59(11), 3060–3063 (2010)
- [26] S-T Chen, H-N Huang, C-Y Hsu, Optimization-based image watermarking scheme in the wavelet-domain, in 2010 Fourth International Conference on Genetic Evolutionary Computing, ShenZhen, China, 2010, pp. 671–674
- [27] K Loukhaoukha, JY Chouinard, MH Taieb, Optimal image watermarking algorithm based on LWT-SVD via multi-objective and colony optimization. J. Inform. Multimedia. Signal. Process. 2(4), 303–319 (2011)
- [28] A Mishra, C Agarwal, A Sharma, P Bedi, Optimized gray-scale image watermarking using DWT- SVD and firefly algorithm. Elsevier: Expert Systems with Applications 41, 7858–7867 (2014)
- [29] S-T Chen, H-N Huang, W-M Kung, C-Y Hsu, Optimization-based image watermarking with integrated quantization embedding in the wavelet domain. Springer: Multimedia Tools and Applications, 2015. doi:10.1007/s11042-015-2522-8
- [30] S-T Chen, G-D Wu, H-N Huang, Wavelet-domain audio watermarking scheme using optimisation-based quantisation. IET Proceedings on Signal Processing 4(6), 720–727 (2010)
- [31] S-T Chen, H-N Huang, C-C Chen, K-K Tseng, S-Y Tu, Adaptive audio watermarking via the optimization point of view on

wavelet-based entropy. Elsevier: Digital Signal Processing, 2013, pp. 971–980

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