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Manufacturing Cost and Strength Reliability Effects on the Optimal Design of Hand Prosthesis Mechanisms

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Abstract— **The middle finger mechanism of a hand prosthesis is optimally designed with respect to five design criteria: motion posture error, mechanical error, maximum driving torque, strength reliability and manufacturing cost of the mechanism. It is shown that the optimal design variables obtained are different from one design criterion to another when optimized individually. Otherwise, the introduction of the global manufacturing cost to the multiobjective optimization of the first three criteria on the one hand and of the first four criteria on the other hand, gave the same optimum variables obtained when minimising simultaneously only the three first criteria. Therefore, the global manufacturing cost has a real impact on the whole process of the mechanism optimization due to the stochastic nature of the trajectory of the cutting tools during the whole process of manufacturing.**

Index Terms— **hand prosthesis, manufacturing cost, mechanisms, optimal design, optimization criteria, strength reliability.**

I. Introduction

Assuming that all fingers of a natural hand, except the thumb, are functionally and mechanically similar during the generation of the various modes of a basic gripping of the hand, a simple mechanism with one degree of freedom is often used [1,2,3,4] in order to proceed to a multiobjective optimization of the middle finger mechanism only.

Among many mechanisms and criteria used by many authors to optimize the middle finger, we choose the across four-bar mechanism usually optimized by the means of motion posture error, maximum driving torque [1,2,3,4] and mechanical error due to dimensional tolerances [1,2] which have a significant effect to that mechanism. In order to obtain a realistic optimal design, two additional criteria, namely strength reliability and manufacturing cost, will be also used and their effect in the optimization process will be studied.

In a first approach, each criterion will be optimized individually. Then, the three first criteria will be optimized simultaneously. Strength reliability and manufacturing cost will be added individually and finally, we will carry out the optimization of all the five criteria simultaneously.

II. MIDLE FINGER DESCRIPTION

A. *Finger model*

The type of movement achieved by a natural hand is a balance-balance one. Therefore, the planar one degree of freedom and across four-bar mechanism used to model the finger of the prosthesis is also a balance-balance mechanism (fig. 1 [2,5]). It is equipped with the following elements:

- a driving system consisting of a cable winded around a pulley.
- a fixed bar QM on the palm of hand;
- a driving bar AM activated by the driving system and allowing phalanges to bend by the means of the driven bar QB and the junction bar AB;
- a return spring allowing not only to maintain a minimum of tension in the cable, but also to make sure of the finger's return to its opening or operating position when the cable is unwinded.

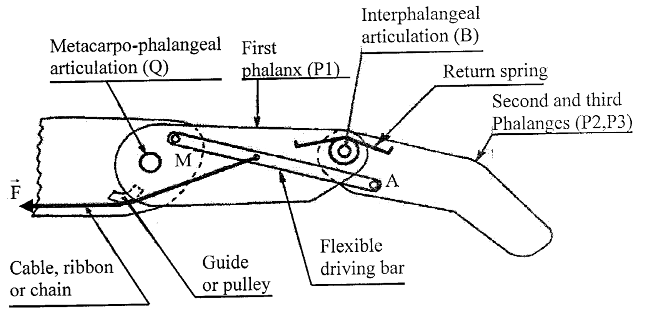

Figure 1: Across four-bar mechanism of the middle finger

B. *Description of the middle finger's mechanism*

Fig. 2 shows the middle finger's parameters related to the across four-bar mechanism equipped with its driving system.

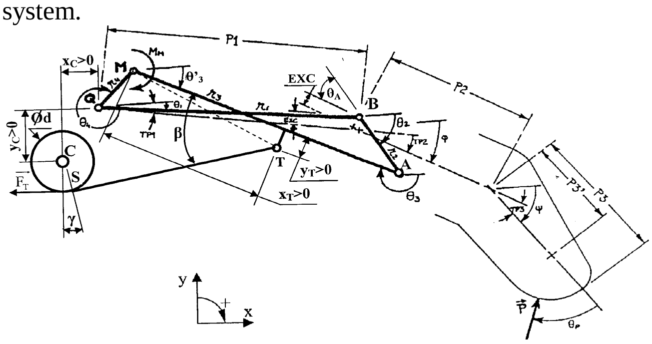

Figure 2: Middle finger's parameters

P3': effective length of the third phalanx P3;

EXC: eccentricity of the articulated joint between the first and second phalanges;

r_1: length of driven bar QB related to first phalanx;

r_2: length of junction bar AB related to second phalanx;

r_3, r_4: respectively lengths of driving bar AM and fixed bar QM;

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TP1, φ, ψ: bending angles of phalanges P1, P2 and P3 with respect to horizontal palm plan;

TP2, TP3: bending angles of phalanges P2 and P3 with respect to P1 and P2;
θ_1, θ_2, θ_3, θ_4: respectively angles of bars QB, AB, AM and QM with respect to palm plan;
θ_A: angle between junction bar AB and second phalanx axis;
x_T, y_T: connection T coordinates with respect to M;
x_C, y_C: pulley center coordinates with respect to Q;
d: pulley diameter;
β: traction angle; γ: position of contact point S between pulley and cable;
P: applied force to the finger end with respect to the third phalanx P3;
F_T: tension in the cable;
M_M: necessary driving torque applied to the driving bar AM;

C. Mechanism synthesis

At any mechanism position i, Freudenstein's relation [1,2,4,5] between θ_{1i} and θ_{2i} is :

$$k_1 \cos(\theta_{1i}-\theta_4)+k_2\cos(\theta_{2i}-\theta_4)-k_3=\cos(\theta_{1i}-\theta_{2i}) \quad (1)$$

$$\text{with } \begin{cases} k_1=\dfrac{r_4}{r_2};k_2=\dfrac{r_4}{r_1};k_3=\dfrac{r_1^2+r_2^2-r_3^2+r_4^2}{2r_1r_2} \\ r_1=\sqrt{P1^2+EXC^2} \\ \theta_{1i}=TP1_i-arctg\left(\dfrac{EXC}{P1}\right) \\ \theta_{2i}=\theta_A+TP1_i+TP2_i \end{cases}$$

Assuming that r_1, θ_A, θ_4, 7 positions $TP1_i$ and $TP2_i$ are known, the mechanism synthesis is then performed using the least square method [1,2,4].

III. Optimization of the middle finger's mechanism

A. Mechanism parameters

The force P applied at the end of middle finger's is equal to 45N and makes an angle θ_P=90° with the phalanx P3 axis. The maximum tension developed in the cable is F_{Tmax}=400N. Table 1 gives overall dimensions of middle finger's mechanism with DP1, DP2 and WR3 which are respectively diameters of phalanges P1, P2 and width of links.

Table 1: Linear and angular dimensions of middle finger's mechanism [1,2]

(mm)	DP1	DP2	WR3	P1	P2	P3'	EXC	
	15	13	5	43.5	29	13.5	2	
( )°	$TP1_1$	$TP1_2$	$TP1_3$	$TP1_4$	$TP1_5$	$TP1_6$	$TP1_7$	
	18	29	40	51	61	73	84	
( )°	$TP2_1$	$TP1_2$	$TP2_3$	$TP2_4$	$TP2_5$	$TP2_6$	$TP2_7$	TP3
	14.274	18.027	24.022	32.142	41.416	55.649	75.670	30

B. Design variables

From the mechanism parameters and with the aim to introduce the driving system in the optimization process, vector X of the independent design variables is:

$$X=[x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9] \\ =[\theta_A,r_2,r_3,r_4,\theta_4,t_m,x_T,y_T,d] \quad (2)$$

with t_m= thickness of different bars and $\theta_A=\theta_{2i}-(TP2_i+TP1_i)$. Table 2 gives design variables bounds.

Table 2: Design variables bounds related to middle finger mechanism [1,2]

(mm)	r_2	r_4	θ_4(°)	t_m	x_T	y_T	d
min	5	5	360	0.794	10	-5	10
max	15	7.5	380	1.588	20	5	17

C. Objective functions

Five performance criteria are considered in the process of the mechanism optimization:

- motion posture error or structural error on the bending angle of the second phalange that would be minimized to ensure that the prosthetic finger should pass by the prescribed positions [1,2,4];
- mechanical error on the bending angle of the second phalange, due to dimensional tolerances and clearances on the articulations, that would be minimized[1,2];
- maximum shaft driving torque that would be minimized and able to counter-balance a grasping force applied at the end of the middle finger [1,2,4];
- strength reliability of the mechanism that would be maximized to ensure the mechanism strength during grasping, holding or pinching operations;
- manufacturing cost of the mechanism that would be minimized to evaluate its effect on the design variables;

Objective function related to the structural error on bending angle TP2

$$f_1(x)=\frac{1}{6}\sum_{i=1}^{7}(\theta_{2ic}-\theta_{2id})^2 \quad (3)$$

where θ_{2id} = θ_A + $TP1_i$ + $TP2_i$ = x_1 + $TP1_i$ + $TP2_i$ is the desired angle and θ_{2ic} the computed or real angle.

Objective function related to the mechanical error on bending angle TP2

$$f_2(x)=\frac{1}{6}\sum_{i=1}^{7}(\Delta\theta_{2i})^2 \quad (4)$$

with $\Delta\theta_{2i}=3\sigma_{\theta_{2i}}$ where standard deviation is :

$$\sigma_{\theta_{2i}}=\sqrt{2\sum_{j=1}^{10}\left(\frac{\partial\theta_{2i}}{\partial a_j}\sigma_{a_j}\right)^2}$$

$$=0.001\sqrt{2\left[\left(\frac{\partial\theta_{2i}}{\partial r_1}r_1\right)^2+\sum_{j=1}^{9}\left(\frac{\partial\theta_{2i}}{\partial x_j}x_j\right)^2\right]}$$

and $\sigma_{a_j}=0.001a_j$

Objective function related to the maximum shaft driving torque

$$f_3(x) = \max(M_P) \quad (5) \quad \text{with}$$

$$M_P = \frac{M_M d}{2(x_T \sin\beta + y_T \cos\beta)} = \frac{x_9 M_M}{2(x_7 \sin\beta + x_8 \cos\beta)}$$

M_P is the driving torque applied to the shaft while M_M is torque reduced to point M of the driving bar [1,3,6], assuming that frictional forces at joints are negligible.

Objective function related to the strength reliability of the mechanism

Since the failure of any link causes the breakdown of the whole mechanism, the across-four bar mechanism is comparable to a system made up of four components assembled in a serial configuration. Therefore, its strength reliability is the product of the reliability of driven, junction and driving bars R_1, R_2, R_3 [6,8] given by the mathematical expression:

$f_4(x) = - R_1.R_2.R_3$ (6) The minus sign is used to minimize the objective function $f_4(x)$ in order to maximize the strength reliability of the mechanism. The strength reliability of any element is $R = \left[1 - \left(\frac{1.29}{|u|}\right)^{7.8125}\right]$ [8]. Assuming that the maximum stress ρ induced in a bar and the strength S of material used are lognormally distributed, the statistical parameter is $u = -\frac{\overline{\ln\rho} - \ln S}{\sqrt{\sigma^2_{\ln\rho} + \sigma^2_{\ln S}}}$ [1,2,8] where

$$\overline{\ln a} = \ln \bar{a} - 0.5\sigma^2_{\ln a}, \quad \sigma^2_{\ln a} = \ln\left[\left(\frac{\sigma_a}{\bar{a}}\right)^2 + 1\right]$$

$$\sigma_a = 0.001\left[\sqrt{2\left(\left(\frac{\partial a}{\partial r_1} r_1\right)^2 + \sum_{j=1}^{9}\left(\frac{\partial a}{\partial x_J} x_J\right)^2\right)}\right].$$

$\bar{a}$ is the mean value of a, σ_a is the standard deviation of stress or strength a. Using the maximum shear theory, $\rho = \tau_{max}$ is the maximum shear stress induced in element and $S = S_y/2$ where S_y= 1345MPa is the yield strength of material used. The maximum shear stress ρ induced in each element is determined from the free body diagram of each bar:

<u>Junction bar AB</u> (fig. 3)

$$\rho_2 = \tau_{\max 2} = \frac{|\sigma_{x'}|}{2} = \frac{1}{2t_m DP2}\left(|N| + \frac{6|M_{f\max}|}{DP2}\right) \quad (7)$$

with

$$\begin{cases} N = -P[\cos\theta_P \cos(\psi - \varphi) - \sin\theta_P \sin(\psi - \varphi)] \\ M_{f\max} \\ = -P[P_3' \sin\theta_P + P_2(\cos\theta_P \sin(\psi - \varphi) + \sin\theta_P \cos(\psi - \varphi))] \end{cases}$$

$$F_{BX} = \frac{C_7}{C_1};\ F_{BY} = C_7;\ F_{AX} = F_{BX} + C_2;\ F_{AY} = F_{BY} - C_3$$

$$\text{with} \begin{cases} C_1 = tg\theta_1;\ C_2 = P_x = -P\cos(\theta_P + \psi); \\ C_3 = P_y = P\sin(\theta_P + \psi);\ C_4 = r_2 \sin\theta_2; \\ C_5 = r_2\cos\theta_2;\ C_7 = C_1 \dfrac{C_6 - C_2C_4 - C_3C_5}{C_4 - C_1C_5}; \\ C_6 = P_y(P3'\cos\psi + P2\cos\varphi - EXC\sin\varphi) + \\ \quad P_x(P3'\sin\psi + P2\sin\varphi + EXC\cos\varphi) \end{cases}$$

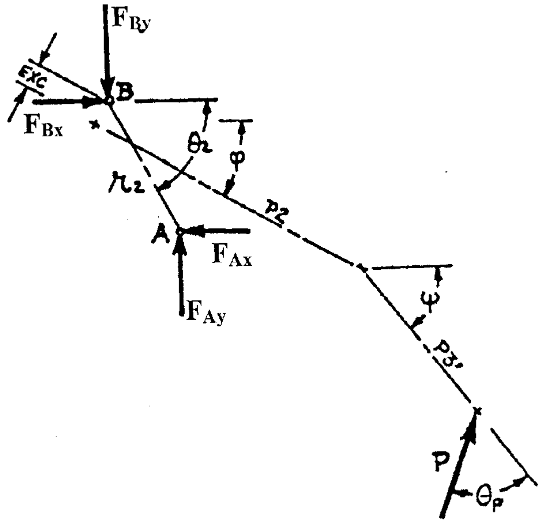

Figure 3: Free body diagram of junction bar AB

<u>Driven bar QB</u> (fig. 4)

$$\rho_1 = \tau_{\max 1} = \frac{|\sigma_{x'}|}{2} = \frac{\sqrt{F^2_{BX} + F^2_{BY}}}{2t_m DP1} \quad (8)$$

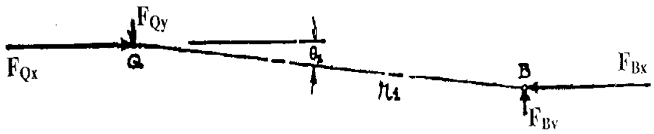

Figure 4: Free body diagram of driven bar QB

<u>Driving bar AM</u> (fig. 5)

$$\rho_3 = \tau_{\max 3} = \frac{|\sigma_{x'}|}{2} \begin{cases} \text{if } F_{MX'} \le F_{AX'} \\ = \dfrac{1}{2t_m WR3}\left(F_{AX'} + \dfrac{6|M_{f\max}|}{WR3}\right) \\ = \dfrac{1}{2x_6 WR3}\left(F_{AX'} + \dfrac{6|M_{f\max}|}{WR3}\right) \\ \text{if } F_{MX'} \ge F_{AX'} \\ = \dfrac{-1}{2t_m WR3}\left(F_{MX'} + \dfrac{6|M_{f\max}|}{WR3}\right) \\ = \dfrac{-1}{2x_6 WR3}\left(F_{MX'} + \dfrac{6|M_{f\max}|}{WR3}\right) \end{cases} \quad (9)$$

$$\text{if } (y_T \le 0 \text{ and } F_{MY'} \le 0) \text{ or } (y_T \ge 0 \text{ and } F_{MY'} \ge 0)$$

$$|M_{f\max}| = |x_T|F_{MY'}|| = x_7|F_{MY'}|$$

$$if\ \left(y_T \le 0\ and\ F_{MY'} \ge 0\right)\ or\ \left(y_T \ge 0\ and\ F_{MY'} \le 0\right)$$

$$\left|M_{f\max}\right| = x_T\left|F_{MY'}\right| + \left|y_T\right|F_T\cos\beta = x_7\left|F_{MY'}\right| + \left|x_8\right|F_T\cos\beta$$ wi

$$F_T = \frac{-r_3F_{AY'}}{\left(x_T\sin\beta + y_T\cos\beta\right)} = \frac{-r_3F_{AY'}}{\left(x_7\sin\beta + x_8\cos\beta\right)};$$

th $$F_{MY'} = -F_T\sin\beta - F_{AY'}$$

$$F_{MX'} = F_T\cos\beta - F_{AX'};\ F_{AX'} = F_{AX}\cos\theta_3^{'} + F_{AY}\sin\theta_3^{'};$$

$$F_{AY'} = F_{AY}\cos\theta_3^{'} - F_{AX}\sin\theta_3^{'}$$

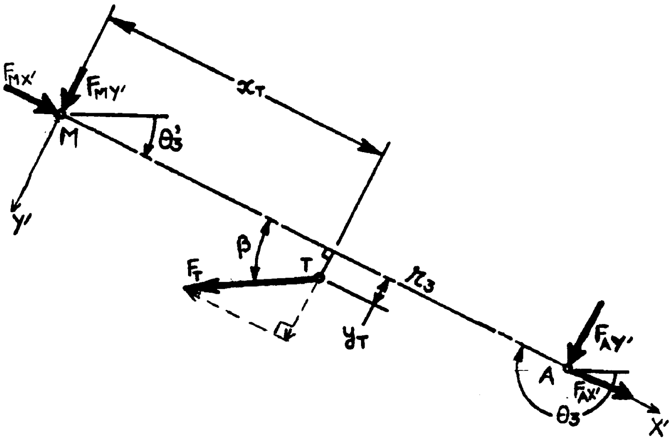

Figure 5: Free body diagram of driving bar AM

Objective function related to the global manufacturing cost

The assembly drawing of the middle finger (fig. 6) is done according to parameters of the mechanism described in section II.B and taking into account the width WR3 of the driving link, the diameters DP1 of the first phalanx and DP2 of the second phalanx.

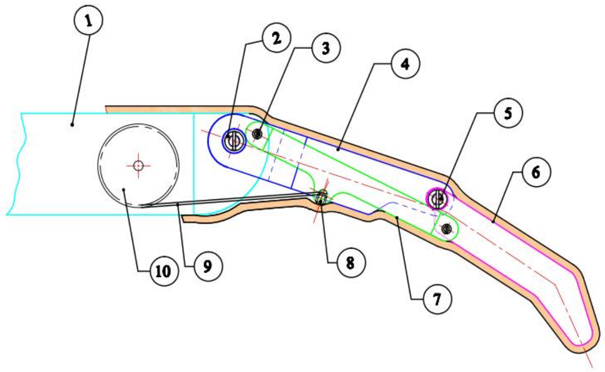

Figure 6: Assembly drawing of the middle finger with its across four-link mechanism

1. Palmer frame 6. Junction bar
2. Guiding screw 01 7. Driving bar
3. Guiding screw 02 8. Pressure screw
4. Involved bar 9. Cable
5. Guiding screw03 10. Pulley

<u>Basic assumptions</u>

- Stainless steel pieces have overall fixed dimensions 2x5xL and 2x11.5xL where L is the length;
- Unit manufacturing cost is expressed in terms of manufacturing time and machine tool hourly cost;
- Because of the programmable machine tool used, machine hourly cost is constant;
- Unit manufacturing costs of machining operations such as turning, drilling, boring, facing and undercutting are equal;
- Each manufacturing dimension is obtained in only one final operation;
- Machining linear velocity V is in m/min;
- Tools store room is equipped with necessary cutting tools used on the programmable machine tool;
- Thermal deformations of machine-piece-tool set are stabilized.

<u>Global manufacturing cost</u>

The present economic constraints impose to the enterprises of production, the maximal profit that is the production at a minimal cost and at a minimal time [11,12,13]. The advanced methods of machining as the use of the manufacturing Computer Aided Design, improved the productivity of the machining operation meaningfully. The prostheses are manufactured in very small set, therefore, the manufacturing Computer Aided Design is appropriate for this purpose [14].

Let's consider :

C_g: The global manufacturing cost of across four-bar mechanism ;

$C_{(i)}$: The global manufacturing cost of the ith bar of the structure of the mechanism, where i=1,2,3 which corresponds respectively to the involved bar, driving bar and junction bar ;

$C_{u\ (ij)}$: The manufacturing unitary cost of the jth Manu6 facturing operation on the ith bar of the mechanism, where j=1,2,3,4,5 corresponding respectively to facing milling, lateral milling, drilling, counter boring and tapping ;

$D_{(ij)}$: Cutting tools used relatively at ith bar and jth manufacturing operation (Milling cutter 2 cuttings Ø20, cutter 2 cuttings Ø6, Drill Ø2, Drill Ø4, piloting Milling cutter to counter boring Ø2x6 and Machining tapping M2);

$L_{(ij)}= L_{s(ij)}= L_{c(ij)}= L_{p(ij)}= L_{l(ij)}= L_{t(ij)}$: Tools trajectories relatively to the facing milling, lateral milling, drilling, counter boring and tapping of the the ith bar of the mechanism and the jth manufacturing operation ;

$OP_{(i)}=2(X_{0(i)}+Y_{0(i)})=XY_{(i)}$: Starting machine programme = Starting piece programme of the ith bar of the mechanism ;

V, N, f, k, C : respectively linear speed, mass-production set, advance in turn/tooth, Taylor constant function of material of the cutting tool, Taylor constant function of material of the working piece (NF E 66-505) ;

T_1, T_2, T_3 : Holes 1,2, and 3 ;

A, F, R, L, E, S : respectively technical amortization, financial expenses, maintenance expenses and of repair, expenses of local or of clutter, expenses of energy and wage costs and social.

C_m : hourly machine cost (Advanced Manufacturing Computer Aided Design)

C_s : Cost cutting tools ;

C_p, C_T, C_{Tm}, C_o : respectively sum of the expenses related directly to the preparation, cutting duration, out of cutting time and the cutting tool ;

Ap : global rate of exploitation of the preparation section, general expenses and labour ;

<u>Expression of the unit cost of machining</u> ($C_{u(ij)}$)

The production to the minimal cost and to the maximal profit called on the mathematical models translating the laws of wear of the tools according to the cutting times also named simplified Taylor's law [15]. The manufacturing unitary cost of the j^th manufacturing operation on the i^th bar of the mechanism is $C_{u(ij)} = C_p + C_T + C_{Tm} + C_0$ with

$$C_p = \frac{Ts}{N} \times \frac{Ap}{60};\ C_T = \frac{C_m}{60} \times \frac{L_{(ij)} \times \pi \times D_{(ij)}}{10^3 \times V \times f};$$

$$C_{Tm} = \frac{C_m}{60} \times \left(\sum_{j=1}^{3} T_j + T_{tm} \right);$$

$$C_o = C_S \left(\frac{L_{(ij)} \times \pi \times D_{(ij)}}{10^3 \times f \times C^{-K}} V^{-K-1} - \frac{1}{N} \right);$$

$$C_m = A + F + R + L + E + S$$

The final expression of the cost of a machining pass (turning, drilling, milling…) is:

$$C_{u(ij)} = C_F + \pi L_{(ij)} D_{(ij)} M$$

$$with \begin{cases} M = \left[\left(\dfrac{C_m}{6 \times 10^4 \times V \times f} \right) + \left(\dfrac{C_S}{10^3 \times f \times C^{-K}} V^{-K-1} \right) \right] \\ C_F = C_p + C_{Tm} - \dfrac{C_S}{N} \end{cases}$$

<u>Manufacturing cost of the junction bar</u>

Fig. 7 shows the manufacturing drawing of junction bar. It is manufactured in four basic operations : facing milling $L_{s(21)}$, lateral milling $L_{c(22)}$, drilling $L_{p(23)}$, and counter boring $L_{l(24)}$. The unit manufacturing cost related to each of these operations are :

$$C_{u(31)} = C_F + \pi L_{s(31)} D_{(31)} M;\ C_{u(32)} = C_F + L_{c(32)} D_{(32)} M;$$

$$C_{u(33)} = C_F + L_{p(33)} D_{(33)} M;\ C_{u(34)} = C_F + L_{l(34)} D_{(34)} M;$$

Therefore, the manufacturing cost of the junction bar is :

$$C_3 = 4C_F + \pi \left(L_{s(31)} D_{(31)} + L_{c(32)} D_{(32)} + L_{p(33)} D_{(33)} + L_{l(34)} D_{(34)} \right) M$$

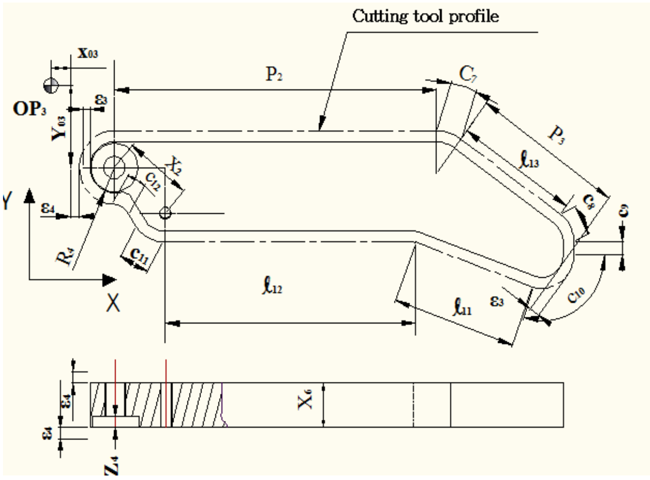

Figure 7: Junction bar

<u>Manufacturing cost of the driving bar</u>

Fig. 8 shows the manufacturing drawing of driving bar. The trajectories of the cutting tools are established according to the working drawings [7]. This bar **1** is manufactured in four basic operations : facing milling $L_{s(11)}$, lateral milling $L_{c(12)}$, grilling $L_{p(13)}$, counter boring $L_{l(14)}$. The unit manufacturing cost related to each of these operations are :

$$C_{u(11)} = C_F + \pi L_{s(11)} D_{(11)} M;\ C_{u(12)} = C_F + L_{c(12)} D_{(12)} M;$$

$$C_{u(13)} = C_F + L_{p(13)} D_{(13)} M;\ C_{u(14)} = C_F + L_{l(14)} D_{(14)} M;$$

Th

erefore, the manufacturing cost of the driving bar is :

$$C_1 = 4C_F + \pi \left(L_{s(11)} D_{(11)} + L_{c(12)} D_{(12)} + L_{p(13)} D_{(33)} + L_{l(14)} D_{(14)} \right) M$$

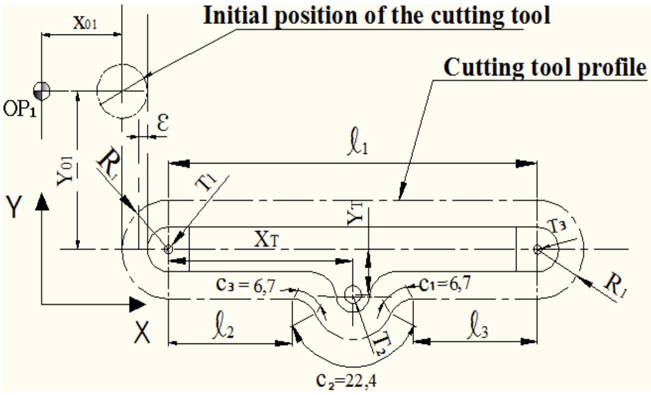

Figure 8 : Driving bar

<u>Manufacturing cost of the driven bar</u>

Fig. 9 shows the manufacturing drawing of driven bar. It is manufactured in five basic operations : facing milling $L_{s(21)}$, lateral milling $L_{c(22)}$, drilling $L_{p(23)}$, counter boring $L_{l(24)}$, tapping $L_{t(25)}$. The unit manufacturing cost related to each of these operations are:

$$C_{u(21)} = C_F + \pi L_{s(21)} D_{(21)} M;\ C_{u(22)} = C_F + L_{c(22)} D_{(22)} M;$$

$$C_{u(23)} = C_F + L_{p(23)} D_{(23)} M;\ C_{u(24)} = C_F + L_{l(24)} D_{(24)} M;$$

$$C_{u(25)} = C_F + L_{t(25)} D_{(25)} M;$$

Therefore, the manufacturing cost of the driven bar is:

$$C_2 = 5C_F + \pi \begin{pmatrix} L_{s(21)} D_{(21)} + L_{c(22)} D_{(22)} \\ + L_{p(23)} D_{(33)} + L_{l(24)} D_{(24)} + L_{l(25)} D_{(25)} \end{pmatrix} M$$

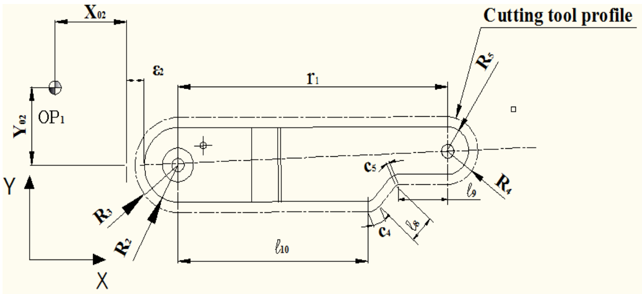

Figure 9: Driven bar

<u>Global manufacturing cost of the mechanism *Cg*</u>

The global manufacturing cost of the mechanism is

$$C_g = C_1 + C_2 + C_3 = 13C_F + \pi M \times LD \qquad (10)$$

With

$$LD = \left[\begin{pmatrix} L_{s(11)} D_{(11)} + L_{c(12)} D_{(12)} + L_{p(13)} D_{(13)} + L_{l(14)} D_{(14)} \\ + L_{s(21)} D_{(21)} + L_{c(22)} D_{(22)} + L_{p(23)} D_{(23)} + L_{l(24)} D_{(24)} \\ + L_{l(25)} D_{(25)} + L_{s(31)} D_{(31)} + L_{c(32)} D_{(32)} + L_{p(33)} D_{(33)} \\ + L_{l(34)} D_{(34)} \end{pmatrix} \right] C_g$$

is in US DOLLARS and the dimensions of all mechanism pieces are in millimeter.

While taking into account the dimensional tolerances Δr_j to the bars, clearances Rc_j at the articulations and an addimensionnal variable k [2] :

$$\frac{\Delta r_j}{3} = \frac{\Delta Rc_j}{3} = kr_j = 0.001 r_j \Rightarrow \Delta r_j = 3.10^{-3} r_j \qquad (11)$$

T ables 3, 4a and 4b give respectively constant expressions and bars profile coordinates used in the global manufacturing cost Cg.

Table 3 : Constant expressions

Tools Trajectories	Constants
$L_{s11}=Q_1+2x_3$	$Q_1=4(\varepsilon_1+R_1)$
$L_{c12}= Q_2+ x_3$	$Q_2=2\pi R_1 + l_2+ l_3+c_1+ c_2+ c_3$
$L_{p13}= Q_4+ Z_1+Z_3$	$Q_3=OP_1 + 3\ \varepsilon_1$
$L_{l14}= Q3+2(x_7+x_8)+3x_6$	$Q_4=OP_2 + Z_1+Z_3$
	$Q_5=OP_2+4\varepsilon_2+2R_2+ l_4+ l_5$
$L_{s21}=Q_5+r_1$	$+ Z_s+ l_6$
$L_{c22}= Q_6+ r_1$	$Q_6= OP_2+2\varepsilon_3+\pi\ R_3+ \pi\ R_4$
$L_{p23}= Q_{11}+Q_{12}+2Q_{13}$	$+ l_9 + c_5+ l_8+ c_4+ l_{10}$
$+2Zp +x_4$	$Q_7= OP_2+ 2\varepsilon_3 +2\ \varepsilon_4+2R_2$
$+2\ (r_1+x_6)$ *with*	$Q_8= OP_2+2\varepsilon_3$
$x_4=X_4[cos\ (2\pi- \Theta_4)$	$+2R_2+2(\varepsilon_4+Z_2)$
$+sin(2\pi-\Theta_4)]$ *and*	$Q_9= OP_2+ 2\varepsilon_2+2R_2+2\varepsilon_4$
$\Theta_4=x_5$	$Q_{10}= OP_2+ 2\varepsilon_2 +2\ \varepsilon_4+2Z_4$
$L_{l24}=Q_7+Q_8+2Q_9+2Zp$	$Q_{11}= OP_2+ 2\varepsilon_3 +2\ \varepsilon_4+2R_2$
$+ x_4 +2\ (r_1+x_6)$	$Q_{12}= OP_2+2\varepsilon_3 + 2R_2$
$L_{t25} = Q_{10}+2Z_4$	$+2(\varepsilon_4+ Z_2)$
	$Q_{13}= OP_2+ 2\varepsilon_2+2R_2+2\varepsilon_4$
$L_{s31}= Q_{14}$	$Q_{14}= OP_3+2(2\varepsilon_3+R_4+P_2+ P_3)$
$L_{c32}= Q_{15}$	$Q15= OP_3+2\varepsilon_4+\pi R_4+P_2$
$L_{p33}=Q_{16}+Q_{17}+2(x_6+x_2)$	$+ c_7+ c_8+ c_9+ c_{10}+ c_{11}$
$+2x_6$	$+ c_{12}+ l_{13}$
$L_{l34} =2x_6+Q_{18}$	$Q_{16}= OP_3+ 2\varepsilon_3+ 2R_4+ 2\varepsilon_4$
	$Q_{17}= OP_3+ 2\varepsilon_3+ 2R_4+ 2\varepsilon_4$
	$Q_{18}= OP_3+ 2\varepsilon_3+ 2R_4+ 2\varepsilon_4$

Table 4a : Bars profile coordinates

Bars	Operations	relatives profile coordinates								
Driving	Facing milling Lateral milling Drilling Counter boring	X_{01}	Y_{01}	l_1	l_2	l_3	c_1	c_2	c_3	
		30,2	35,2	x_3	29,6	29,6	6,7	22,4	6,7	
		ε_1	t_m	x_T	y_T	r_3	X_{10}		R_1	
		3	x_6	x_7	x_8	x_3	19,2		11	
Involved	Facing milling Lateral milling Drilling Counter boring Tipping	Z_1	Z_2	r_3	ε_2	R_2	l_4	l_5	l_6	R_s
		1	1,2	x_3	5	11	62,1	5,6	5,6	7
		R_3	R_4	ε_3	c_4	c_5	l_8	l_9	l_{10}	
		10	4,2	5,6	14	0,8	62,1	62,1	0,8	
		X_{02}	Y_{02}	Z_2	Z_{p4}	Z_4	x_4	x_6		
		23,2	22,8	2	7	1	x_4	x_6		

Table 4b : Bars profile coordinates

Bars	Operations	relatives profile coordinates									
Junction	Facing milling Lateral milling Drilling Counter boring	X_{03}	Y_{03}	ε_4	R_4	c1	c7	c8	c9	c10	c11
		5,7	5,7	3	9,9	7,5	8,5	3,8	16,8	35,5	10,9

Objective functions related to multiobjective optimization

$f_6(x) = [f_1(x), f_2(x), f_3(x)]$;
$f_7(x) = [f_1(x), f_2(x), f_3(x), f_4(x)]$;
$f_8(x) = [f_1(x), f_2(x), f_3(x), f_5(x),]$;
$f_9(x) = [f_1(x), f_2(x), f_3(x), f_4(x), f_5(x)]$.

Each of these multiobjective functions consists of a set of objective functions which are simultaneously optimized.

D. Constraint functions

These functions, expressed in terms of design variables are equality constraints $h_i(x) = 0$ and inequality constraints $g_i(x) \leq 0$ according to Matlab optimization toolbox R2015a.

Equality constraints

Using Freudenstein's relation and least square method, three equality constraints [1,2,4,5] related to mechanism synthesis with reference to seven positions given in table 1 are obtained. Otherwise, one additional equality constraint is introduced by the means of security factor related to the design of the driving bar.

Inequality constraints

Eighteen inequality constraints [1,2] used are respectively related to transmission angle, mechanism dead points, restriction of driving link to be within the phalanges P1 and P2, maximum value of cable tension, maximum driving shaft rotation, connection cable T keeping inside phalanx P1 during a cycle of mechanism's closure, cable keeping within palm and finger.

In addition to design bounds given in table 2, angle θ_{21} must to be included between 20° and 115°[1,2]; therefore θ_A is bounded by -12.274° and 82.726°.

IV. RESULTS AND DISCUSSION

A. Optimization results and interpretation

From three different starting points, each of objective functions $f_1(x)$, $f_2(x)$, $f_3(x)$, $f_4(x)$ and $f_5(x)$, subjected to four equality and eighteen inequality constraints, was optimized, taking into account design variables bounds, by the means of a nonlinear programming algorithm developed in Matlab optimization toolbox R2015a.

Using the optimum values of each single objective function, the multiobjective optimization of $f_6(x)$, $f_7(x)$, $f_8(x)$, and $f_9(x)$ subjected to the same constraints was carried out by the means of Matlab goal attainment method [1,2] developed in Matlab optimization toolbox R2015a.

Table 5: Optimum design variables and functions

	$f_1(^O)^2$	$f_2(^O)^2$	f_3 (Nmm)	f_4	f_5 (US $)	f_6	f_7	f_8	f_9
	0.1595	**13.6892**	**1307.978**	**0.9999**	**494.845**				
$x_1(^O)$	82.726	73.409	72.257	82.518	82.726	78.193	82.152	82.063	82.063
x_2(mm)	5.000	5.148	5.000	5.065	5.604	5.060	5.094	5.060	5.060
x_3(mm)	37.153	36.741	36.954	37.046	36.232	37.035	36.987	37.035	37.035
x_4(mm)	5.856	7.003	6.872	5.973	6.724	6.019	6.057	6.019	6.019
$x_5(^O)$	375.063	380.000	380.000	375.415	377.091	375.680	375.739	375.680	375.680
x_6(mm)	0.828	0.794	0.807	0.846	0.794	0.851	0.848	0.851	0.851
x_7(mm)	18.738	20.000	20.000	19.727	20.000	20.000	20.000	20.000	20.000
x_8(mm)	4.800	4.800	5.000	4.800	4.472	5.000	5.000	5.000	5.000
x_9(mm)	16.000	16.000	10.000	16.000	10.000	10.000	10.000	10.000	10.000

The optimum values of design variables are shown in table 5 for each objective function including multiobjective functions.

As shown in table 5, the optimum values of design variables are different from one single objective function to another. Otherwise, mechanical error f_2 generally induces highest values of optimum design variables while their lowest values are obtained with manufacturing cost f_5. It is also shown that multiobjective functions f_6, f_7, f_8 and f_9 optimization allow generally to obtain intermediate values of these extreme design variables. Therefore, strength reliability and mainly manufacturing cost have a considerable effect on the optimal design of the middle finger's mechanism. In fact, results obtained from multiobjective functions f_6, f_8 and f_9 are the same and highlight the manufacturing cost role.

Fig. 10 give optimum values of each objective function used in each of optimization problems related to single objective and multiobjective functions.

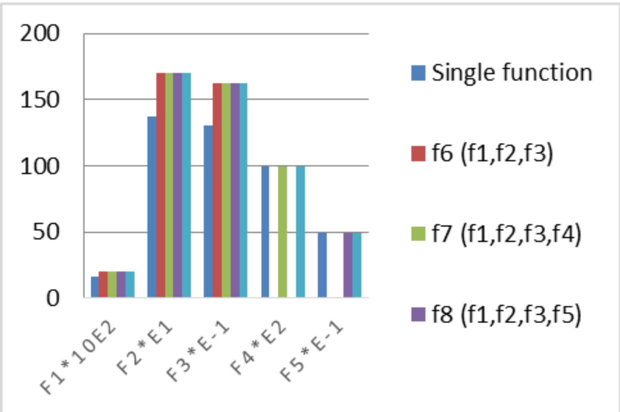

Figure10: Optimum values of objective functions

It can be shown that:

- Optimum values of single objective function optimization are smaller than those obtained by the means of multiobjective function optimization, except those related to reliability and manufacturing cost which are a little different;
- From one multiobjective optimization to another, optimum values of each single function remain the same whereas the fact that optimum values of design variables related to optimization function f_6 are different from those of optimization functions f_7, f_8 and f_9.

B. Discussion

In the previous studies [1,2], optimal design variables obtained from one multiobjective optimization to another are different. In the present work, the global manufacturing cost, included in any combination of the simultaneous optimization, enabled to obtain the same optimum variables design.

Otherwise, the optimal global manufacturing cost obtained in this study is about 495 US Dollars for the middle finger mechanism versus 419 US Dollars proposed by Ventimiglia [16] which includes complete middle finger assembly. Therefore, it will be necessary to evaluate the really cost of our complete middle finger assembly.

V. CONCLUSION

The aim of this study was to introduce into the optimization process of the crossed four-bar link mechanism of hand prosthesis, two new performance criteria, namely the reliability strength of the whole structure and the manufacturing cost of the mechanism. These criteria were added to those related to the mean quadratic error of the bending angle of the second joint of the middle finger, the mechanical error due to dimensions tolerances and the maximum driving torque applied to the driving shaft. This study permits us to confirm that, through the five different optimal mechanisms obtained, each criterion used here generates a particular design, translating its importance in the optimization process.

Otherwise, because of the influence of each of these criteria on the mechanism design, their simultaneously optimization showed that the global manufacturing cost, included in any combination of the multiobjective optimization, enabled to obtain the same optimum variables design. Finally, the strength reliability of the driving is always equal to the unit. That translated the fact that, the constraints induced inside each bar, remain less than the maximum material constraints.

References [17], [18] and [19], related to physic-mechanical, chemical and mineralogical characterization of Materials clay of the Far North Region of Cameroon, will be helpful to produce ceramic material in order to obtain moulded prosthetic fingers in which the manufactured mechanisms will be inserted.

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