Plane wave reflection in micropolar transversely isotropic thermo-visco-elastic half space under GN type-II and type-III

Rajani Rani Gupta, Raj Rani Gupta

Abstract— **Present article deals with the reflection of wave's incident at the surface of transversely isotropic micropolar visco elastic medium under the theory of thermoelasticity of type-II and type-III. The wave equations are solved by imposing proper conditions on components of displacement, stresses and temperature distribution. It is found that there exist four different waves viz., quasi-longitudinal displacement(qLD)wave, quasi transverse displacement(qTD)wave, quasi transverse microrotational (qTM)wave and quasi thermal wave (qT). Amplitude ratio of these reflected waves are presented, when different waves are incident. Numerically simulated results have been depicted graphically for different angle of incidence with respect to frequency. Some special cases of interest also have been deduced form the present investigation.**

Index Terms—**Micropolar, Reflection, Amplitude Ratios, Viscoelastic.**

I. INTRODUCTION

Depending upon the mechanical properties, the materials of earth have been classified as elastic, viscoelastic, sandy, granular, microstructure etc. Some parts of earth may supposed to be composed of material possessing micropolar/granular structure instead of continuous elastic materials. To explain the fundamental departure of microcontinuum theories from the classical continuum theories, the former is a continuum model embedded with microstructures to describe the microscopic motion or a non-local model to describe the long range material interaction. This extends the application of the continuum model to microscopic space and short-time scales.

Material is endowed with microstructure, like atoms and molecules at microscopic scale, grains and fibers or particulate at mesoscopic scale. Homogenization of a basically heterogeneous material depends on scale of interest. When stress fluctuation is small enough compared to microstructure of material, homogenization can be made without considering the detailed microstructure of the material. However, if it is not the case, the microstructure of material must be considered properly in a homogenized formulation [1], [2]. The concept of microcontinuum, proposed by Eringen [1], can take into account the microstructure of material while the theory itself remains still in a continuum formulation. The first grade microcontinuum consist a hierarachy of theories, such as, micropolar, microstretch and micromorphic, depending on how much micro-degrees of freedom is incorporated. These microcontinuum theories are believed to be potential tools to characterize the behavior of material with complicated microstructures.

The most popular microcontinuum theory is micropolar one, in this theory, a material point can still be considered as infinitely small, however, there are microstructures inside of this point. So there are two sets of variable to describe the deformation of this material point, one characterizes the motion of the inertia center of this material point; the other describes the motion of the microstructure inside of this point. In micropolar theory, the motion of the microstructure is supposed to be an independently rigid rotation. Application of this theory can be found in [1] and [3].

Recently, the theory of thermoelasticity without energy dissipation, which provides sufficient basic modifications to the constitutive equation to permit the treatment of a much wider class of flow problems, has been proposed by Green and Naghdi [4] (called the GN theory). The discussion presented in the above reference includes the derivation of a complete set of governing equations of the linearized version of the theory for homogeneous and isotropic materials in terms of displacement and temperature fields and a proof of the uniqueness of the solution of the corresponding initial mixed boundary value problem. Chandrasekharaiah and Srinath [5] investigated one-dimensional wave propagation in the context of the GN theory.

The aim of the present paper is to study the reflection of waves in transversely isotropic micropolar viscoelastic medium under the theory of thermoelasticity of type-II and type-III. The propagation of waves in micropolar materials has many applications in various fields of science and technology, namely, atomic physics, industrial engineering, thermal power plants, submarine structures, pressure vessel, aerospace, chemical pipes and metallurgy. The graphical representation in given for amplitude ratios of various reflected waves for different incident waves at different angle of incidence i.e. for $\theta = 30^o, 45^o\ and\ 90^o$.

II. BASIC EQUATIONS

The basic equations in dynamic theory of the plain strain of a homogeneous, transversely isotropic micropolar viscoelastic

Manuscript received June 09, 2016.
Rajani Rani Gupta, Department of Mathematics & Applied Sciences, Middle East College, Muscat, Oman, (e-mail: dr.rajanigupta@gmail.com).
Raj Rani Gupta, Department of CS & IT, Mazoon College, Muscat, Oman, (e-mail: raji.mmec@gmail.com).

medium following Eringen [1] and Green and Naghdi [4] in the theory of thermoelasticity of type-II and type-III in absence of body forces, body couples and heat sources are given by:

Balance laws

$$t_{ij,j} = \rho \ddot{u}_i \quad (1)$$

$$m_{ik,i} + \varepsilon_{ijk} t_{ij} = \rho j \ddot{\phi}_k , \quad (2)$$

Heat conduction equation

$$K^*_{kl} \dot{T}_{,kl} + \kappa_{kl} T_{,kl} = \rho c^* \frac{\partial^2 T}{\partial t^2} + T_0 \frac{\partial^2}{\partial t^2} \beta_{kl} u_{l,k} \quad (3)$$

The constitutive relations can be given as:

$$t_{kl} = A_{klmn} E_{mn} + G_{klmn} \Psi_{mn} - \beta_{kl} T,$$
$$m_{kl} = A_{lkmn} E_{mn} + G_{mnlk} \Psi_{mn} \quad (4)$$

Where

$$A_{ijkl} = A'_{ijkl} + A''_{ijkl} \frac{\partial}{\partial t},$$
$$B_{ijkl} = B'_{ijkl} + B''_{ijkl} \frac{\partial}{\partial t},$$
$$G_{ijkl} = G'_{ijkl} + G''_{ijkl} \frac{\partial}{\partial t},$$

deformation and wryness tensor are defined by

$$e_{kl} = u_{l,k} + \varepsilon_{lkm} \phi_m , \quad \Psi_{kl} = \phi_{l,k} , \quad (5)$$

In these relations, we have used the following notations: ρ is the density, ε_{lkm} permutation symbol, u_i components of displacement vector, ϕ_i components of microrotation vector, t_{kl} components of the stress tensor, m_{kl} components of the couple stress tensor, e_{kl} components of micropolar strain tensor, κ_{kl} are the characteristic constants of the theory, c^* is specific heat at constant strain, K^*_{kl} is the thermal conductivity, $\beta_{kl} = A_{klmn} \alpha_{mn}$ are the thermal elastic coupling tensor, α_{mn} are the coefficient of linear thermal expansion.

III. Formulation of the Problem

In present case we consider homogeneous, transversely isotropic micropolar viscoelastic medium under the theory of thermoelasticity of type-II and type-III, initially in an undeformed state and at uniform temperature T_o. We take the origin of coordinate system on the plane surface with x_3 axis pointing normally into the half-space, which is thus represented by $x_3 > 0$. We restricted our analysis to the two dimensional problem by assuming the displacement vector $\vec{u}$ and microrotation vector $\vec{\phi}$ as

$$\vec{u} = (u_1, 0, u_3), \vec{\phi} = (0, \phi_2, 0) \quad (6)$$

With the aid of equation (6), the field equations (1)-(4) for transversely isotropic micropolar viscoelastic medium under the theory of thermoelasticity of type-II and type-III reduce to:

$$A_{11} \frac{\partial^2 u_1}{\partial x_1^2} + A_{55} \frac{\partial^2 u_1}{\partial x_3^2} + (A_{13} + A_{56}) \frac{\partial^2 u_3}{\partial x_1 \partial x_3} + K_1 \frac{\partial \phi_2}{\partial x_3} - \beta_1 \frac{\partial T}{\partial x_1} = \rho \frac{\partial^2 u_1}{\partial t^2}, \quad (7)$$

$$A_{66} \frac{\partial^2 u_3}{\partial x_1^2} + A_{33} \frac{\partial^2 u_3}{\partial x_3^2} + (A_{13} + A_{56}) \frac{\partial^2 u_1}{\partial x_1 \partial x_3} + K_2 \frac{\partial \phi_2}{\partial x_1} - \beta_3 \frac{\partial T}{\partial x_3} = \rho \frac{\partial^2 u_3}{\partial t^2}, \quad (8)$$

$$B_{77} \frac{\partial^2 \phi_2}{\partial x_1^2} + B_{66} \frac{\partial^2 \phi_2}{\partial x_3^2} - X \phi_2 + K_1 \frac{\partial u_1}{\partial x_3} + K_1 \frac{\partial u_3}{\partial x_3} = \rho \frac{\partial^2 \phi_2}{\partial t^2}, \quad (9)$$

$$\kappa_1 \frac{\partial^2 \phi_2}{\partial x_1^2} + \kappa_3 \frac{\partial^2 \phi_2}{\partial x_3^2} + K_1^* \frac{\partial^2 \phi_2}{\partial x_1^2 \partial t} + K_3^* \frac{\partial^2 \phi_2}{\partial x_3^2 \partial t} = \rho C^* \frac{\partial^2 T}{\partial t^2} + T_o \frac{\partial^2}{\partial t^2} \left(\beta_1 \frac{\partial u_1}{\partial x_1} + \beta_3 \frac{\partial u_3}{\partial x_3} \right), \quad (10)$$

where

$$\beta_1 = A_{11}\alpha_1 + A_{13}\alpha_3, \beta_3 = A_{31}\alpha_1 + A_{33}\alpha_3, K_1 = A_{56} - A_{55},$$
$$K_2 = A_{66} - A_{56}, X = K_2 - K_1, \alpha_1, \alpha_3$$

are the coefficients of linear thermal expansion and we have used the notations $11 \to 1, 33 \to 3, 12 \to 7, 13 \to 6, 23 \to 4$ for the material constants.

For simplification we use the following non-dimensional variables:

$$x_i' = \frac{x_i}{L}, \quad u_i' = \frac{u_i}{L}, \quad t_{ij}' = \frac{t_{ij}}{\beta_1 T_o}, \quad m_{ij}' = \frac{m_{ij}}{L \beta_1 T_o}, \quad \phi_2' = \frac{\phi_2 A_{11}}{\beta_1 T_o},$$
$$T' = \frac{T}{T_0} \quad t' = \frac{c_1}{L} t, \quad c_1^2 = \frac{A_{11}}{\rho}. \quad (11)$$

where L is a parameter having dimension of length and c_1 is the longitudinal wave velocity of the medium.

IV. Solution of the Problem

Let $\vec{p}(p_1, 0, p_3)$ denote the unit propagation vector of the plane waves propagating in $x_1 x_3$-plane.

We seek plane wave solution of the equations of motion of the form

$$(u_1, u_3, \phi_2, T) = (\bar{u}_1, \bar{u}_3, \bar{\phi}_2, \bar{T}) e^{i\xi(p_1 x_1 + p_3 x_3 - ct)} \quad (12)$$

where ξ and k are respectively the phase velocity and the wave number the components $(\bar{u}_1, \bar{u}_3, \bar{\phi}_2, \bar{T})$ define the amplitude and polarization of particle displacement, microrotation and temperature distribution in the medium.

With the help of equations (11) and (12) in equations (7)-(10), we get four homogeneous equations in four unknowns. Solving the resultant system of equations for non-trivial solution we obtain,

$$Ac^8 + Bc^6 + Cc^4 + Dc^2 + E = 0, \qquad (13)$$

where

$A = b_4 - \frac{b_5}{\omega^2}, B = g_1 + \frac{g_6}{\omega^2}, C = g_3 + \frac{g_2 + g_4}{\omega^2} + \frac{g_4}{\omega^4}, D = g_5 + \frac{g_7}{\omega^2}, E = g_8, g_1 = b_1 - b'_2 - b_4 a_1,$

$g_2 = b_8 - b_2 - b_4 a_1 + b_{17} a_{14} + b_{12} a_{15} - a_1 b_5, g_3 = b'_6 - b_1 - b_3 + b'_2 a_1 + b'_{12} a_{10},$

$g_4 = b_6 + a_1 b_2 + a_{10}(b_{12} - b'_{13}) + a_{14}(b_{15} - b_{18} + b'_{15}) - a_{15} b_{18}, g_5 = b_7 + b_3 a_1 + b_9 a_{10} - a_1 b'_6$

$g_6 = b_5 a_1 - a_{15} b_{22}, \ g_7 = b_{14} a_{14} + b_{20} a_{15} - b_6 a_1 - a_{10} b_{10}, \ g_8 = -b_7 a_1 - b_{11} a_{10},$

$g_9 = b_{19} a_{15} - a_1 b_8 - a_{10} b_{13}, b_1 = d_7 a_8 d_{12}, b_2 = d_7(a_8 d_{11} + d_9 d_6) + a_7 a_4 d_{12}, b'_2 = a_2 a_9 d_{12},$

$b_3 = a_8(a_6 d_7 + a_2 d_{12}), b_4 = d_{12} d_7 a_9, b_5 = d_7 d_{11} a_9, b_6 = a_6 a_7 a_4 + a_8(a_2 d_{11} - a_3 a_5), b'_6 = a_2 a_6 a_9$

$b_7 = a_2 a_6 a_8, b_8 = a_9(a_2 d_{11} - a_3 a_5), b_9 = a_8 a_{11} d_{12}, b_{10} = a_{11}(a_8 d_{11} + a_6 d_9) + a_5 a_8 a_{12} - a_6 a_7 a_{13},$

$b_{11} = a_8 a_6 a_{11}, b_{12} = -a_7 a_{13} d_{12}, b_{13} = a_7 a_{13} d_{11},$

$b'_{13} = a_9(a_{11} d_{12} + a_5 a_{12}), b_{14} = a_8(a_1 a_3 + a_2 a_{12}),$

$b_{15} = a_7(a_4 a_{12} - a_3 a_{13}), b'_{15} = a_9(a_1 a_3 + a_2 a_{12}),$

$b_{16} = a_8 a_{12} d_7, b_{17} = -a_9 a_{12} d_7, b_{18} = d_{12}(a_4 a_{11} + a_2 a_{13}),$

$b_{19} = d_{11}(a_{14} a_4 + a_2 a_{13}) - a_6 a_{13} d_7 + a_5(a_4 a_{11} - a_3 a_{13}),$

$b_{20} = a_6(a_4 a_{11} + a_2 a_{13}), b_{21} = a_{13} d_{12} d_7, b_{22} = a_{13} d_7 d_{11},$

$a_1 = d_1 p_3^2 + p_1^2, a_2 = p_3^2 + p_1^2 d_4, a_3 = i\, p_1 d_{10},$

$a_4 = i\omega^2 \in_2 p_3, a_5 = i\, p_1 d_6, a_6 = p_3^2 + p_1^2 d_8, a_7 = -i p_3 d_{14},$

$a_8 = p_3^2(i\omega \in_1 -1) + p_1^2(i\omega \in_1 \bar{\kappa} - \bar{K}), a_9 = \varepsilon,$

$a_{10} = p_1 p_3 d_2, a_{11} = -p_1 p_3 d_5, a_{12} = i\, p_3 d_9, a_{13} = i\omega^2 \in_2 p_1 \bar{\beta},$

$a_{14} = i p_3 d_3, a_{15} = -i p_1 d_{13}, d_1 = \frac{A_{11}}{A_{55}}, d_2 = \frac{(A_{13} + A_{56})}{A_{11}},$

$d_3 = \frac{K_1 \beta_1 T_o}{A_{11}^2}, d_4 = \frac{A_{66}}{A_{33}}, d_5 = \frac{(A_{13} + A_{56})}{A_{33}},$

$d_6 = \frac{K_2 \beta_1 T_o}{A_{11} A_{33}}, d_7 = \frac{A_{11}}{A_{33}}, d_8 = \frac{B_{77}}{B_{66}}, d_9 = \frac{K_1 L^2 A_{11}}{B_{66} \beta_1 T_o},$

$d_{10} = -\frac{K_2 L^2 A_{11}}{B_{66} \beta_1 T_o}, d_{11} = \frac{X L^2}{B_{66}}, d_{12} = \frac{A_{11} j}{B_{66}}, d_{13} = \frac{\beta_1 T_o}{A_{11}},$

$d_{14} = \frac{\beta_3 T_o}{A_{33}}, d_{15} = \frac{B_{66}}{A_{11} L^2}, d_{16} = \frac{A_{65}}{A_{11}}, d_{17} = \frac{K_1}{A_{11}},$

$d_{18} = \frac{\beta_3 T_o}{A_{11}}, d_{19} = \frac{A_{13}}{A_{11}}, \bar{\beta} = \frac{\beta_1}{\beta_3}, \bar{K} = \frac{K_1^*}{K_3^*}, \in = \frac{\rho C^* c_1^2}{\kappa_3}$

$\in_1 = \frac{K_3^* c_1}{L \kappa_3}, \in_2 = \frac{\beta_3 c_1^2}{\kappa_3}, p_1 = \sin(\theta), p_3 = \cos(\theta),$ θ is the plane-wave incident angle with x_3-axis.

The roots of this equation gives four values of c^2. Four positive values of c will b-e the velocities of propagation of four possible waves. The waves with velocities c_1, c_2, c_3, c_4 correspond to four types of quasi waves. Let us name these waves as quasi-longitudinal displacement (qLD) wave, quasi transverse displacement (qTD) wave, quasi transverse microrotational (qTM) wave and quasi thermal wave (qT).

V. Reflection of waves

We consider a transversely isotropic micropolar viscoelastic medium under the theory of thermoelasticity of type-II and type-III occupying the region $x_3 \ge 0$. Incident qLD, qTD, qTM or qT waves at the interface will generate reflected qLD, qTD, qTM and qT waves in the half space $x_3 > 0$. The total displacements and temperature distribution are given by

$$(u_1, u_3, \phi_2, T) = \sum_{j=1}^{8} A_j (1, r_j, s_j, t_j) e^{iB_j}, \qquad (15)$$

where

$$B_j = \begin{cases} \omega(t - x_1 \sin e_j - x_3 \cos e_j)/c_j, & j = 1,2,3,4 \\ \omega(t - x_1 \sin e_j + x_3 \cos e_j)/c_j, & j = 5,6,7,8, \end{cases} \qquad (16)$$

ω is the angular frequency. Here subscripts 1,2,3,4 respectively denote the quantities corresponding to incident qLD, qTD, qTM and qT wave whereas the subscripts 5, 6, 7 and 8 respectively denote the corresponding reflected waves and

$$r_j = \frac{\wedge_{1j}}{\wedge_j}, s_j = \frac{\wedge_{2j}}{\wedge_j}, t_j = \frac{\wedge_{3j}}{\wedge_j}, \quad (j = 1, 2, \dots\dots 8),$$

$$\wedge_j = \begin{vmatrix} \xi^2(d_7 c_j^2 - a_2) & \xi a_5 & \xi a_7 \\ \xi a_3 & \xi^2 c_j^2 - h_1 & 0 \\ \xi a_4 & 0 & (c_j^2 \in + a_8)\xi^2 \end{vmatrix},$$

$$\wedge_{1j} = \begin{vmatrix} \xi^2 a_{11} & \xi a_5 & \xi a_7 \\ \xi a_{12} & \xi^2(c_j^2 d_{12} - a_6) - d_{11} & 0 \\ \xi a_{13} & 0 & (c_j^2 \in + a_8)\xi^2 \end{vmatrix},$$

$$\wedge_{2j} = \begin{vmatrix} \xi^2 a_{11} & \xi^2(d_7 c_j^2 - a_2) & \xi a_7 \\ \xi a_{12} & \xi a_3 & 0 \\ \xi a_{13} & \xi a_4 & (c_j^2 \in + a_8)\xi^2 \end{vmatrix},$$

$$\wedge_{3j} = \begin{vmatrix} \xi^2 a_{11} & \xi^2(d_7 c_j^2 - a_2) & \xi a_5 \\ \xi a_{12} & \xi a_3 & \xi^2(c_j^2 d_{12} - a_6) - d_{11} \\ \xi a_{13} & \xi a_4 & 0 \end{vmatrix}.$$

$h_1 = \xi^2 a_6 + d_{11}$

The expressions for $a_j, \ j = 1,2, \dots\dots, 15$ are obtained from the expressions for $a_j, \ j = 1,2, \dots\dots, 15$ are given in the equations (14) on substituting the values for p_1 and p_3.

For incident qLD-wave: $p_1 = \sin e_1, p_3 = \cos e_1,$

qTD-wave: $p_1 = \sin e_2, p_3 = \cos e_2,$

qTM-wave: $p_1 = \sin e_3, p_3 = \cos e_3,$

qT-wave: $p_1 = \sin e_4, p_3 = \cos e_4,$

For reflected qLD-wave: $p_1 = \sin e_5, p_3 = \cos e_5,$

For reflected qTD -wave: $p_1 = \sin e_6, p_3 = \cos e_6,$;

for reflected qTM-wave: $p_1 = \sin e_7, p_3 = \cos e_7, ;$
for reflected qT-wave: $p_1 = \sin e_8, p_3 = \cos e_8$.

Here $e_1 = e_5, e_2 = e_6, e_3 = e_7, e_4 = e_8$ i.e. the angle of incidence is equal to the angle of reflection, so that the velocities of reflected waves are equal to their corresponding incident wave's i.e. $c_1 = c_5, c_2 = c_6, c_3 = c_7, c_4 = c_8$.

VI. Boundary Conditions

The boundary conditions at the thermally insulated surface $x_3 = 0$ are given by

$$t_{33} = 0,\ t_{31} = 0, m_{32} = 0, \frac{\partial T}{\partial x_3} + hT = 0, \qquad (18)$$

where h is the surface heat transfer coefficient;

$h \to 0$ corresponds to thermally insulated boundaries and
$h \to \infty$ refers to isothermal boundaries.

$$\begin{aligned} t_{33} &= d_{19}\frac{\partial u_1}{\partial x_1} + \frac{1}{d_7}\frac{\partial u_3}{\partial x_3} - d_{18}T, \\ t_{31} &= d_{16}\frac{\partial u_3}{\partial x_1} + d_3\phi_2 + d_1\frac{\partial u_1}{\partial x_3}, \qquad (19) \\ m_{32} &= d_{15}\frac{\partial \phi_2}{\partial x_3}. \end{aligned}$$

The boundary conditions given by equation (18) must be satisfied for all values of x_1 and t, so we have

$$B_1(x_1, 0, t) = B_2(x_2, 0, t) = \cdots \ldots\ldots\ldots = B_8(x_8, 0, t) \qquad (20)$$

Then from equations (16) and (20), we have

$$\frac{\sin e_1}{c_1} = \frac{\sin e_2}{c_2} = \ldots\ldots\ldots\ldots = \frac{\sin e_8}{c_8} = \frac{1}{c} \qquad (21)$$

which corresponds to Snell's law in present case and

$$c_1\xi_1 = c_2\xi_2 = \ldots\ldots\ldots = c_8\xi_8 = \omega, \qquad (22)$$

Making use of equations (15) in equation (19) and substituting it into thermally insulated boundary conditions (18), we obtain

$$\sum_{j=1}^{8} A_{ij}A_j = 0,\ (i = 1,2,3,4), \qquad (23)$$

Where

$$A_{1j} = \begin{cases} \dfrac{\sin e_j}{c_j} + r_j d_5 \dfrac{\cos e_j}{c_j} - t_j d_{14}, & j = 1,2,3,4 \\ \dfrac{\sin e_j}{c_j} - r_j d_5 \dfrac{\cos e_j}{c_j} - t_j d_{14}, & j = 5,6,7,8 \end{cases},$$

$$A_{2j} = \begin{cases} d_1\dfrac{\cos e_j}{c_j} - d_{16}r_j\dfrac{\sin e_j}{c_j} + d_3 s_j, & j = 1,2,3,4, \\ -d_1\dfrac{\cos e_j}{c_j} - d_{16}r_j\dfrac{\sin e_j}{c_j} + d_3 s_j, & j = 5,6,7,8, \end{cases},$$

$$A_{3j} = \begin{cases} d_{15}s_j\dfrac{\cos e_j}{c_j}, & j = 1,2,3,4 \\ -d_{15}s_j\dfrac{\cos e_j}{c_j}, & j = 5,6,7,8, \end{cases}$$

$$A_{4j} = \begin{cases} t_j\dfrac{\cos e_j}{c_j}, & j = 1,2,3,4 \\ -t_j\dfrac{\cos e_j}{c_j}, & j = 5,6,7,8. \end{cases}$$

Incident qLD-wave:
In case of incident qLD- wave, $A_2 = A_3 = A_4 = 0$. Dividing the set of equations (23) throughout by A_1, we obtain a system of four non-homogeneous equations in four unknowns. Resulting equations can be solved by the Gauss elimination method which results in

$$Z_i = \frac{A_{i+4}}{A_1} = \frac{\Delta_i^1}{\Delta},\ (i = 1,2,3,4.) \qquad (23)$$

Incident qTD-wave:
In the case of incident qTD- wave, $A_1 = A_3 = A_4 = 0$, thus

$$Z_i = \frac{A_{i+4}}{A_2} = \frac{\Delta_i^2}{\Delta},\ (i = 1,2,3,4). \qquad (24)$$

Incident qTM-wave:
In the case of incident qTM- wave, $A_1 = A_2 = A_4 = 0$ and thus we have

$$Z_i = \frac{A_{i+4}}{A_3} = \frac{\Delta_i^3}{\Delta}, (i = 1, 2,3,4) \qquad (25)$$

Incident qT-wave:
In the case of incident qT- wave, $A_1 = A_2 = A_3 = 0$, thus

$$Z_i = \frac{A_{i+4}}{A_4} = \frac{\Delta_i^4}{\Delta},\ (i = 1,2,3,4). \qquad (26)$$

where $\Delta = \left|A_{i\,i+4}\right|_{4\times4}$ and $\Delta_i^p\ (i = 1,2,3,4\ \ p = 1,2,3,4)$ can be obtained by replacing, respectively, the 1st, 2nd, 3rd, 4th column of Δ by $\left[-A_{1p}\ -A_{2p}\ -A_{3p}\ -A_{4p}\right]^T$.

VII. Numerical Results and Discussion

In order to illustrate the theoretical results obtained in the preceding sections, we now present some numerical results. For numerical computation, we take the values for relevant parameters for transversely isotropic micropolar viscoelastic medium under the theory of thermoelasticity of type-II and type-III solid as:

$$A_{11} = 21.4\times10^9\,Nm^{-2},\ A_{77} = 5.4\times10^9\,Nm^{-2},$$
$$A_{88} = 5.2\times10^9\,Nm^{-2},\ A_{22} = 20.24\times10^9\,Nm^{-2},$$
$$A_{12} = 9.4\times10^9\,Nm^{-2}, A_{78} = 4\times10^9\,Nm^{-2},$$
$$B_{44} = 0.779\times10^5\,N, B_{66} = 0.779\times10^5\,N.$$

Also,

$$A_{11} = A'_{11}(1 + i\,\omega\,R_1^{-1}),\ A_{77} = A'_{77}(1 + i\,\omega\,R_2^{-1}),$$
$$A_{88} = A'_{88}(1 + i\,\omega\,R_3^{-1}), A_{22} = A'_{22}(1 + i\,\omega\,R_4^{-1}),$$
$$A_{12} = A'_{12}(1 + i\,\omega\,R_5^{-1}), A_{78} = A'_{78}(1 + i\,\omega\,R_6^{-1}),$$

$$B_{44} = B'_{44}(1 + i\,\omega\, R_7^{-1}), B_{66} = B'_{66}(1 + i\,\omega\, R_8^{-1}),$$

where

$$R_1^{-1} = 8.05, R_2^{-1} = 2, R_3^{-1} = 1.05, R_4^{-1} = 4.05,$$
$$R_5^{-1} = 2.05, \quad R_6^{-1} = 7.05, R_7^{-1} = 2.05, R_8^{-1} = 4.05$$

Following Gauthier [6] we take, the non-dimensional values for Aluminium epoxy like composite as

$$\rho = 2.19\times10^3\ Kg/m^3,\ \lambda = 7.59\times10^9\ N/m^2,$$
$$\mu = 1.89\times10^9\ N/m^2, K = 1.49\times10^9\ N/m^2,$$
$$j = 0.196\times10^{-4}\ m^2, \qquad K^* = 1.7\times10^2\ J/m\sec{}^oC,$$
$$C^* = 1.04\,Cal/K\ , \gamma = 2.63\times10^5\ N.$$

Graphical representation is given for the variations of amplitude ratios of reflected qLD, qTD, qTM and qT waves when four types of waves viz. qLD, qTD, qTM and qT waves are incident at the free surface to compare the results in two cases, one for the waves incident from transversely isotropic micropolar medium under the theory of thermoelasticity of type-II and type-III (MTIWED) and other from micropolar transversely isotropic viscoelastic material with energy dissipation (MVWED). In figures 1-4, the graphical representation is given for variations of amplitude ratios $|Z_1|$,$|Z_2|$,$|Z_3|$ and $|Z_4|$ for incident qLD wave. Figures 5-8, 9-12 and 13-16, respectively show the similar state in case of incident qTD, qTM, and qT waves. Here $|Z_1|$,$|Z_2|$,$|Z_3|$ and $|Z_4|$ are, respectively, the amplitude ratios of reflected qLD, qTD, qTM and qT wave.

These variations are shown for three different angle of incidence viz., $\theta = 30^o, 45^o\ and\ 90^o$. In these figures the solid lines corresponds to the case of MTIWED while the dotted lines corresponds to the case of MVWED. Also, the solid lines without center symbol, lines with center symbol $(-0-0-)$, solid lines with center symbol $(-\times-\times-)$, respectively, represent variations for $\theta = 30^o$, $\theta = 45^o$ and $\theta = 90^o$ in case on MTIWED, whereas the corresponding broken lines represent the same condition in the case of MVWED.

A. *Incident qLD wave*

It is observed from figure 1 that the amplitude ratio $|Z_1|$ of reflected qLD-wave first increases sharply to peak value and then decreases sharply to attain a constant value, when $\theta = 45^o$ for MTIWED. While, for all the other cases, its value initially increase and then oscillate to become constant ultimately.

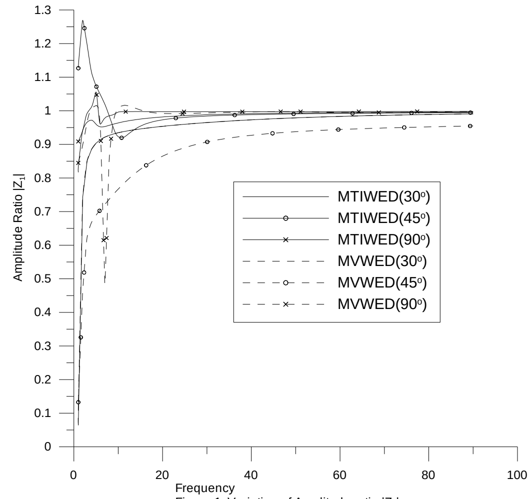

Figure 1. Variation of Amplitude ratio $|Z_1|$

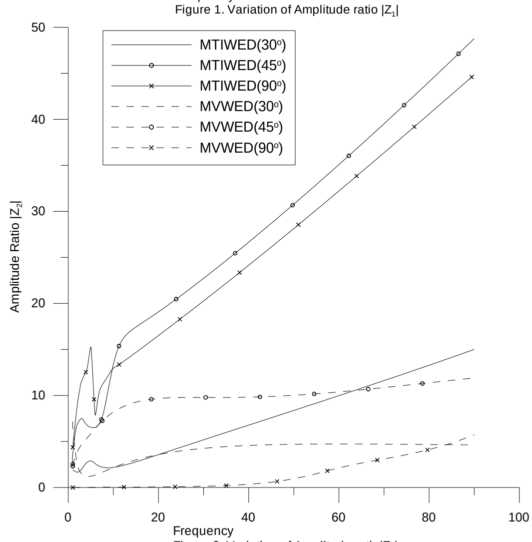

Figure 2. Variation of Amplitude ratio$|Z_2|$

Figure 2, 3 and 4 indicate the variations of amplitude ratio $|Z_2|$,$|Z_3|$ and $|Z_4|$ of reflected qTD-wave which shows that for all the cases, the value of $|Z_2|$,$|Z_3|$ and $|Z_4|$ initially oscillate with very small amplitude and then steadily increases with increase in frequency, for all the three angles of inclination.

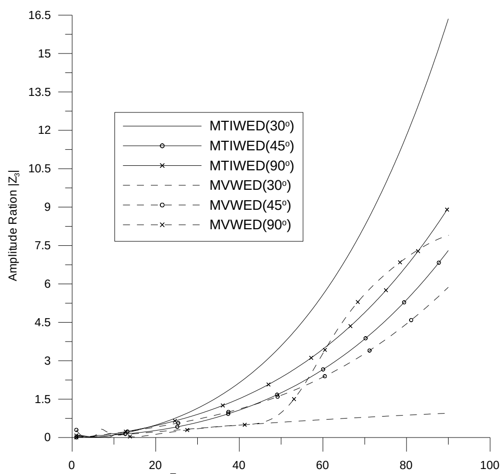

Figure 3. Variation of Amplitude ratio$|Z_3|$

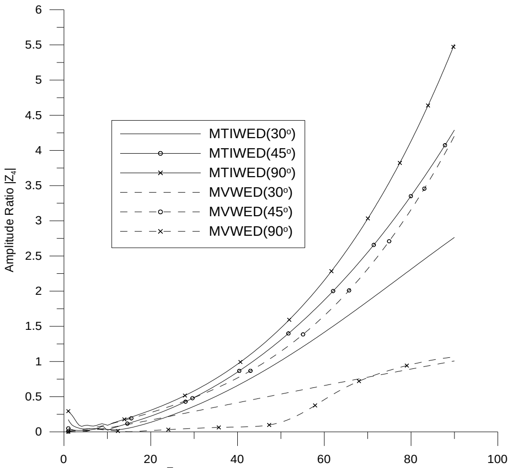

Figure 4. Variation of Amplitude ratio$|Z_4|$

B. Incident qTD wave

The variation of amplitude ratios of various reflected waves for incident qTD-wave is shown in figures 5-8. The amplitude ratio $|Z_1|$ sharply decreases to become constant for all the cases, with slight differences in their amplitudes, except for MVWED $\theta = 90^o$ where after decreasing it oscillates and then attain a constant value.

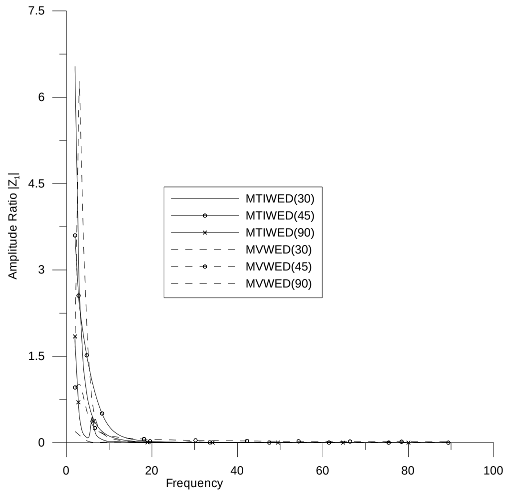

Figure 5. Variation of Amplitude ratio$|Z_1|$

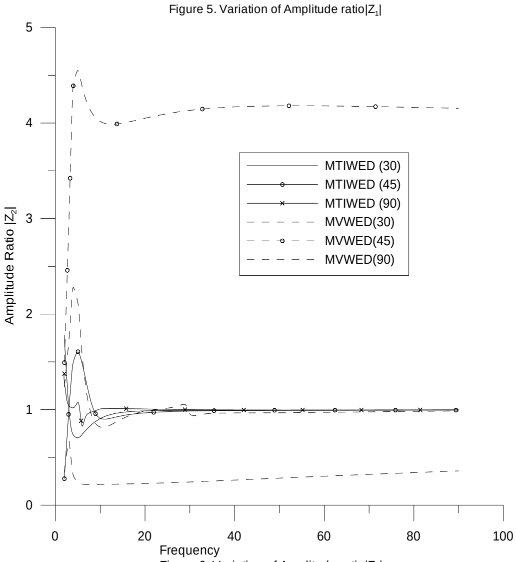

Figure 6. Variation of Amplitude ratio$|Z_2|$

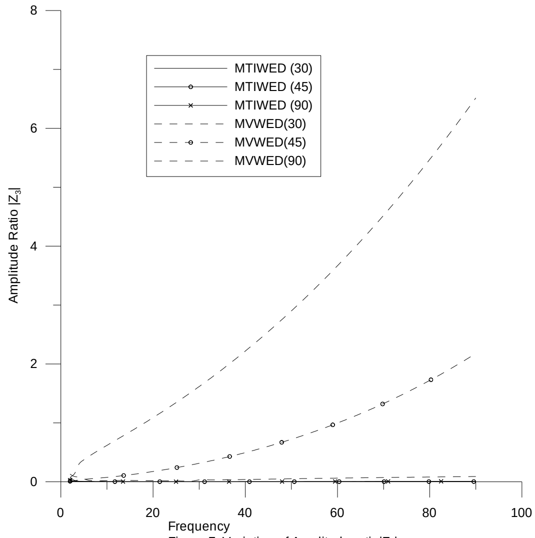

Figure 7. Variation of Amplitude ratio$|Z_3|$

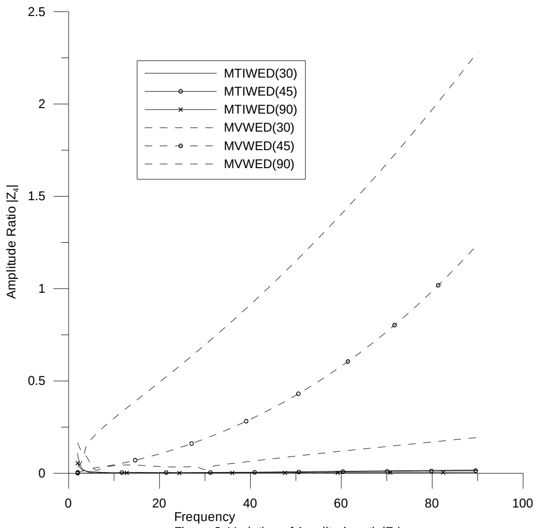

Figure 8. Variation of Amplitude ratio$|Z_4|$

It can be seen from figure 6 that the value of amplitude ratio $|Z_2|$ for MTIWED shows a hump within the interval $0 \le \omega \le 20$ and then oscillates to attain a constant value. At $= 30^o, 90^o$,its value initially decreases, then increases to become constant. However for MVWED, for all the angle of inclination, its value oscillates within the interval $0 \le \omega \le 10$ and then become constant at the end.

Figures 7 and 8 shows the variations of amplitude ratio $|Z_3|$ and $|Z_4|$ keeps on increasing with increase in frequency.

C. Incident qTM wave

The variation of amplitude ratio $|Z_1|$ initially oscillates within the interval (0, 20) for all the cases of MTIWED and then become constant with increase in frequency. Also, the values for MTIWED are higher as compared to those for MVWED. These variations are depicted in figure 9.

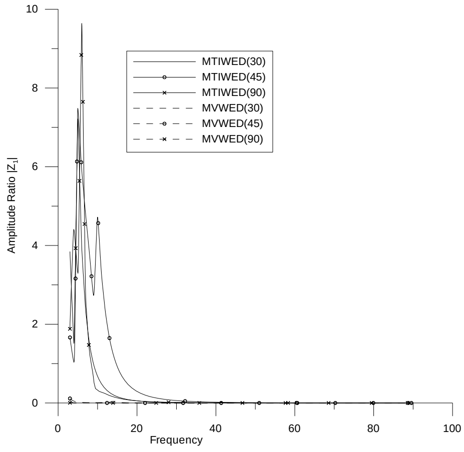

Figure 9. Variation of Amplitude ratio$|Z_1|$

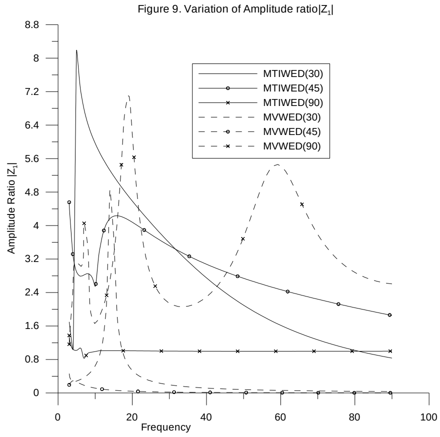

Figure 10. Variation of Amplitude ratio$|Z_2|$

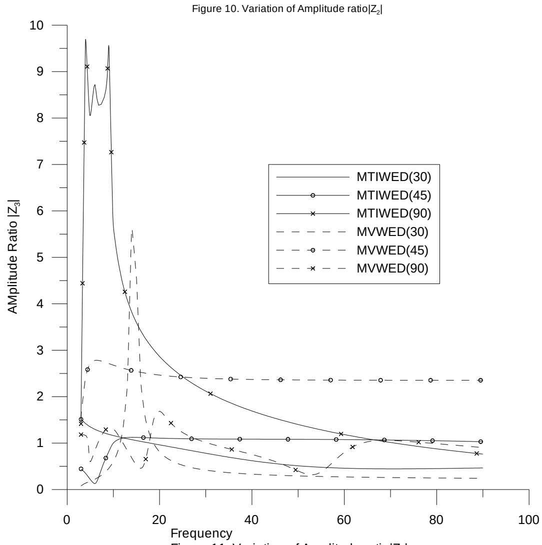

Figure 11. Variation of Amplitude ratio$|Z_3|$

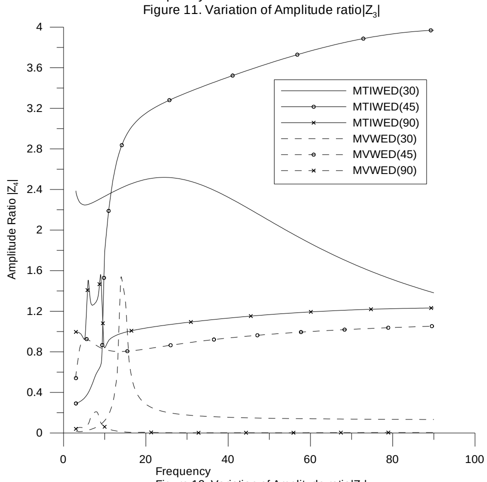

Figure 12. Variation of Amplitude ratio$|Z_4|$

For the amplitude ratio $|Z_2|$ (figure 10) of reflected qTD-wave, for MTIWED and $\theta = 30^o$, its value sharply increases to reach a maximum value and then decreases to attain a constant value at the end. While for $\theta = 45^o$and $\theta = 90^o$ its value initially oscillates to become constant at the end. However for MVWED, its value oscillates with varying amplitude.

Figure 11 shows that the curves for $|Z_3|$ start with difference in their pattern of variation but end with almost similar type of variation. It can be seen from this figure that the value of $|Z_3|$ shows the similar behavior for initial angle of incidence, while with increase in angle of incidence, its value shows the opposite behavior.

It is observed from figure 12, that $|Z_4|$ initially oscillate then show a sudden decrease at $\theta = 90^o$, goes on increasing with increase in frequency at $\theta = 45^o$, while oscillate and then decreases to become constant at $\theta = 30^o$. The variations for the case of MVWED, start with oscillating behavior showing peaks at particular value of frequency and become constant at the end. It is observed that the values get increased with increase in angle of incidence.

D. Incident qT wave

The variations of amplitude ratios of various reflected waves for incident qT-wave are shown in figures 13-16. The amplitude ratio $|Z_1|$ sharply decreases to become constant for all the cases, with slight differences in their amplitudes.

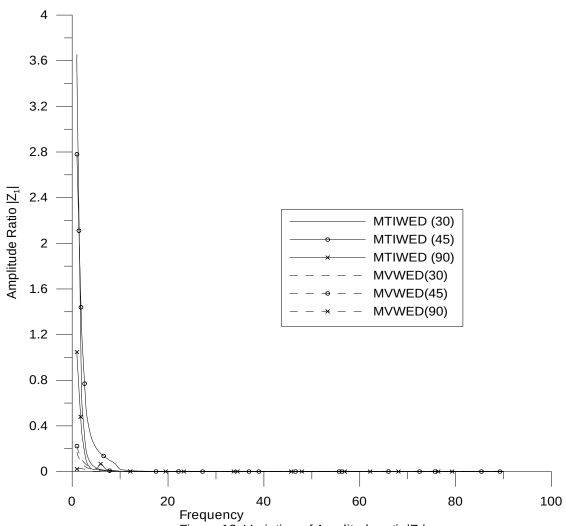

Figure 13. Variation of Amplitude ratio$|Z_1|$

The variations of $|Z_2|$and $|Z_4|$ indicated in figures 14 and 16. It can be seen from these figures that the values oscillate within the interval (0, 20), showing the peaks of different amplitudes. After this interval the values for all the cases become steady.

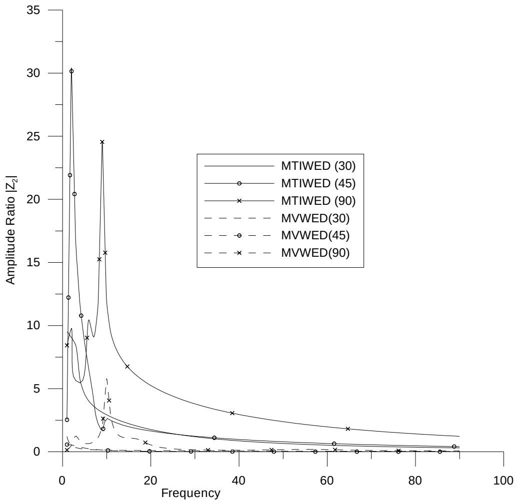

Figure 14. Variation of Amplitude ratio$|Z_2|$

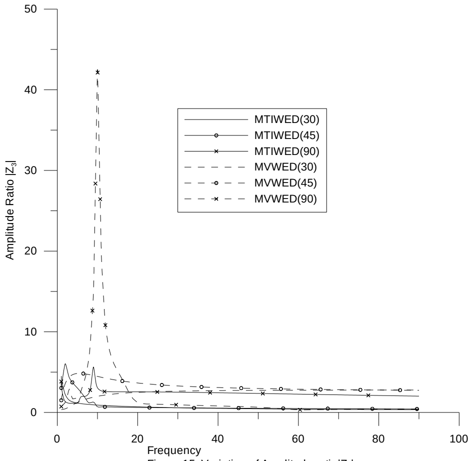

Figure 15. Variation of Amplitude ratio$|Z_3|$

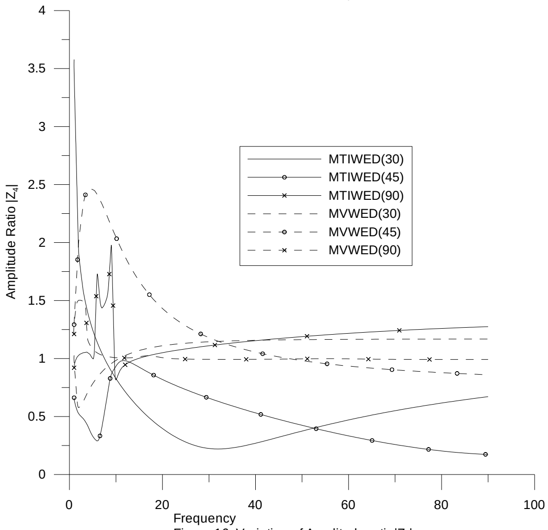

Figure 16. Variation of Amplitude ratio$|Z_4|$

Variations in the values of $|Z_3|$ indicate that viscoelasticity as well as angle of incidence show a significant impact on it throughout the whole range (figure 15). The behavior of $|Z_3|$ oscillatory, within the range $0 \leq \omega \leq 20$. The values of maximum with amplitude ratio $|Z_3|$ first increase from small values to small oscillations and ultimately decrease to become steady. The values for MVWED are higher as compared to those for MVWED at $\theta = 30^o \; and \; 45^o$, but the behavior is reversed with further increase in the angle of incidence. Viscoelasticity show a greater impact on $|Z_i|$, i=1,2,3,4 as compared to the angle of incidence.

VIII. Conclusion

The analytic expressions of amplitude ratios for various reflected waves are obtained for the transversely isotropic micropolar viscoelastic medium under the theory of thermoelasticity of type-II and type-III. It is concluded from the graphs that the values of amplitude ratios $|Z_1|,|Z_2|$ and $|Z_3|$ show sharp oscillations at initial frequencies for incident qLD and qTD waves, as compared to qTM and qT incident waves. An appreciable effect of viscoelasticity and angle of incidence is observed on amplitude ratios of various reflected waves.

References

[1] Eringen A C. "Microcontinuum fields theories I: Foundations and Solids", Springer Verlag, New York, 1999.
[2] Hu G K, Liu X N and Lu T J. "A variational method for nonlinear micropolar composite", Mechanics of Materials. 2005 37, pp. 407-425.
[3] Lakes R S. "Size effects and micromechanics of a porous solid", Journal of Material Science, 1983, 18(9), pp. 2572-2581.
[4] Green A E and Naghdi P M. "Thermoelasticity without energy dissipation", Journal of Elasticity, 1993, 31, pp. 189-208.
[5] Chandrasekharaiah D S and Srinath K S. "One-dimensional waves in a thermoelastic half-space without energy dissipation", International Journal of Engineering Science, 1996, 34(13), pp. 1447-1455.
[6] Gauthier R D. "In Experimental investigations on micropolar media, Mechanics of micropolar media (eds)" 1982, O. Brulin, RKT Hsieh. (Singapore: World Scientific) interfaces (Translation Journals style)," *IEEE Transl. J. Magn.Jpn.*, vol. 2, Aug. 1987, pp. 740–741 [*Dig. 9th Annu. Conf. Magnetics* Japan, 1982, p. 301].

Rajani Rani Gupta earned her Bachelors and master's degree from University of Pune, India by achieving third and first positions respectively. She completed her Ph. D. from Kurukshtera University, India in 2012. She has 28 research papers published in the Journals of internal repute and also served as a reviewer in many international journals. Her field of research is based on the theory of Micropolar Elasticity involving the effects of thermoelasticity, Viscoelasticity, Initial stress, rotation. She attended various national and international level workshops and conferences. Presently she's working in the field of teaching experiences using technology.

Raj Rani Gupta earned her master's degree from University of Pune by achieving overall first position in 2003, M. Phill degree from Kurukshetra University. She qualified JRF (NET) conducted by U. G. C. She has more than 16 research papers published in the journals of international repute. Her area of research work is Continuum Mechanics (Thermoelasticity, Viscoelasticity, Initial stress, rotation). She also managed various activities like Member of Grievance Cell, Women Cell, Convocation, Exam center etc..