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Performance Evaluation of Queuing System in Mega Petroleum Stations A Case of Nigerian National Petroleum Corporation (NNPC) Mega Petroleum Station Enugu

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***Abstract*— The operations managers of Mega Petroleum Stations were not able to determine the best number of servers that can serve arriving customers at various demand periods which affects their queue performance. This study was conducted at NNPC Mega Petroleum Station in Enugu, Nigeria with the aim of addressing the identified problem. Experimental observations were conducted simultaneously on the referenced service facilities in order to collect the daily arrival rates of customers. Arrival rates and combined service rates were however, collected at every 15 minutes interval and at peak demand periods of the day. From the results of the queuing evaluations, it was discovered that at an average of 6 servers being used, with combined service rate ($\bar{\mu}_c$) of 1.5320 cars/minutes and average customer arrival rate $(\bar{\lambda})$ of 1.5153 cars/minutes, for NNPC mega petroleum station Enugu gave a system utilization (P) of 0.9892 which gave a percentage system utilization of 98.9%. It was discovered that the service systems were being over utilized at almost 100% which resulted to the longer waiting time of customers at the service facilities. The result from the Queue Evaluation Environment showed that 8 servers gave the best system utilization value of 74.2% which reduced the customers waiting times (Ws) by 92.7%. The expected probability of system idleness for the case study is negligible at 8 server utilization. The Queue Evaluation Environment was later adopted in developing a Decision Support System for the referenced service facilities. The Decision Support System was finally recommended to guide the operations manager in determining the best number of servers to engage at various demand periods for PMS refill only.**

***Index Terms*— Queuing, NNPC Mega Station, Server, PMS, Decision support system, waiting time, system utilization, Arrival rate and Service rate.**

I. INTRODUCTION

Queuing theory is the mathematical study of waiting lines [1]. The theory permits the derivation and calculation of several performance measures which includes the average waiting time in the queue or the system, the expected number waiting or receiving service, the probability of encountering the system empty, having an available server or having to wait a certain time to be served and most importantly the system utilization [2]. As a result of its applications in industries, technology, telecommunications networks, information technology and management sciences, it has been an interesting research area for many researchers active in the field.

The theory of queues was initiated by the Danish mathematician Erlang, who in 1909 published “The theory of Probabilities and Telephone Conversation”. He observed that a telephone system was generally characterized by either (1) Poisson input (the number of calls), exponential holding (service) time, and multiple channels (servers), or (2) Poisson input, constant holding time and a single channel. Erlang was also responsible in his later works for the notion of stationary equilibrium and for the first consideration of the optimization of a queuing system.

In 1927, Molina published “Application of the Theory of Probability to Telephone Trunking Problems”, and one year later Thornton Fry printed “Probability and its Engineering Uses” which expanded much of Erlang’s earlier work. Kendall was the pioneer who viewed and developed queuing theory from the perspective of stochastic processes [3]. Kleinrock also did some extensive work on the theory of queuing systems and their computer applications [4]. The work in queuing theory picked up momentum rather slowly in its early days, but in 1960’s started to accelerate and there have been a great deal of work in the area and its applications since then [5].

II. REVIEW OF RELATED WORK

In recent times, queuing theory and the diverse areas of its applications has grown tremendously. Takagi considered queuing phenomena with regard to its applications and performance evaluation in computer and communication systems [6]. Obamiro Applied Queuing Model in Determining the Optimum number of Service Facility needed in Nigerian Hospitals. He however achieved this by determining some queuing parameters which enabled him to improve the performance of the system [7]. Azmat also applied queuing theory to determine the sales checkout operation in ICA supermarket using a multiple queue multiple server model. This was used to obtain efficiency of the models in terms of utilization and waiting length, hence increasing the number of queues so customers will not have to wait longer when servers are too busy. The model contains five (5) servers which are checkout sales counters and it helps to reduce queue [8].

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Yankovic N. and Green L. developed a queuing model to help identify nurse staffing levels in hospital clinical units based on providing timely responses to patient needs. The model represents the crucial interaction between the nurse and bed systems and therefore includes the nursing workload due to admissions, discharges and transfers, as well as the observed impact of nursing availability on bed occupancy levels [9].

Mgbemena was able to model the queuing system of some banks in Nigeria using regression analysis. In her work, she created queuing management software in MATLAB that shows at a glance, the behavior of the queuing system and the unit that needs attention at any time. The essence was to improve the customer service system in Nigerian banks [10]. Vasumathi and Dhanavanthan applied Simulation Technique in Queuing Model for ATM Facility. The main purpose of their study was to develop an efficient procedure for ATM queuing problem, which can be daily used by banks to reduce the waiting time of customers in the system. In their work, they formulated a suitable simulation technique which will reduce idle time of servers and waiting time of customers for any bank having ATM facility [11].

Ahmed S.A and Huda K.T focused on banks lines system, the different queuing algorithms that are used in banks to serve customers, and the average waiting time. The aim of their paper was to build an automatic queuing system for organizing and analyzing queue status and take decision of which customer to serve in banks. The new queuing architecture model can switch between different scheduling algorithms according to the testing results and the factor of the average waiting time. The main innovation of their work concerns the modeling of the average waiting time taken into processing, in addition with the process of switching to the scheduling algorithm that gives the best average waiting time [12].

Chinwuko and Nwosu adopted the single line multi-server queuing existing model to analyze the queuing system of First Bank Nigeria PLC. In their work, they suggested the need to increase the number of servers in order to serve customers better in the case study organization [13]. Tabari et al used queuing theory to reorganize the optimal number of required human resources in an educational institution carried out in Iran. Multi-queuing analysis was used to estimate the average waiting time, queue lengths, number of servers and service rates. The analysis was performed for different numbers of staff members. Finally, the result shows that the staff members in this department should be reduced [14].

Ohaneme et al, proposed the single line multi-server queuing system which they simulated using c-programming to be adopted at NNPC Mega petroleum station in Awka, Anambra State in order to avoid congestion and delay of customers [1]. Akpan N.P et al, studied queuing theory and its application in waste management authority in LAWMA Igando dump site, Lagos state by adopting the M/M/S queuing model. He used the Queuing performance measures to estimate the inter-arrival, service and waiting time of the queue. His study showed that both the service and the inter-arrival time made a good fit to Exponential distribution.

However, this work goes further in evaluating the performance of the queuing system, creating a Queue Evaluation Environment that gives expected queue performance and developing a Decision Support System that recommends the best number of servers to use at various demand periods [15].

The objective of the study is to address the queuing problem at NNPC Mega Petroleum Stations by developing a Decision Support System that recommends the best number of servers needed to be engaged at various demand periods.

The structure of the studied system is shown in figure 1. The structures can be approximated as a single-line multi-server queuing systems. . At the NNPC mega petroleum station, there are six dispensers i.e. fuel metering pumps (S_1 *to* S_6) in the system. Each of the fuel dispensers has two nozzles. This means that at full capacity of operation the service facility should be considered as a twelve-server system.

III. Methodology

The research method used in this work was the quantitative research approach. . The single line multi-server queuing model was adopted for developing the results of the queue performance. This model was adopted because it showed a good representation of the model structure of both case studies of queuing systems.

3.1 Method of Data Analysis

The data generated was first organized and descriptive statistics was used to compute the total average arrival rates and total average combined service rates for the year. The service rates per server of both facilities were established and the single line multi server queuing model was coded in Microsoft Excel using 2 – 12 servers (i.e. when M = 2 – 12 servers) in creating the Queue Evaluation Environment that generates the expected queue performance results at the respective average arrival rates of customers in the referenced service facilities. The flow chat developed of the Queue Evaluation Environment is presented in figure 2. The Queue Evaluation Environment was later adopted in developing the decision support system using the application of Microsoft Excel.

3.2. Models Applied for the Queuing Analysis

Based on the assumptions of the single line multi-server queuing model, the expressions for the performance measures which are derived from the analysis of the birth-and-death models, [16, 17and18] are;

i. The average utilization of the system:

When m = 6 is

$$P = \frac{\lambda}{\overline{\mu}_c} \tag{1}$$

When m = 2 – 12 is

$$P = \frac{\lambda}{M(\overline{\mu})} \tag{2}$$

ii. The probability that there are no customers in the system is

$$P_0 = \left[\sum_{n=0}^{M-1} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} + \frac{\left(\frac{\lambda}{\mu}\right)^M}{M!\left(1-\frac{\lambda}{M\mu}\right)} \right]^{-1} \tag{3}$$

iii. The average number of customers waiting for service.

$$L_q = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^M}{(M-1)!\,(M\mu-\lambda)^2} P_0 \tag{4}$$

iv. The average number of customers in the system.

$$L_s = L_q + (\bar{\lambda}/\bar{\mu}) \quad (5)$$

v. The average time a customer spends in line waiting for service

$$W_q = \frac{L_Q}{\bar{\lambda}} \quad (6)$$

vi. The average time a customer spends in the system.

$$W_s = \frac{L_s}{\bar{\lambda}} \quad (7)$$

vii. The average waiting time of a customer on arrival not immediately served.

$$W_a = \frac{1}{M\mu - \bar{\lambda}} \quad (8)$$

viii. Probability that an arriving customer must wait

$$P_w = \frac{W_Q}{W_a} \quad (9)$$

It is seen that these performance measures depend on two basic queue parameters, namely; $\bar{\lambda}$ and $\bar{\mu}$. Given $\bar{\lambda}$ and $\bar{\mu}$, the values computed for these measures gives an indication of how well the referenced service facilities handle the volume of arriving customers.

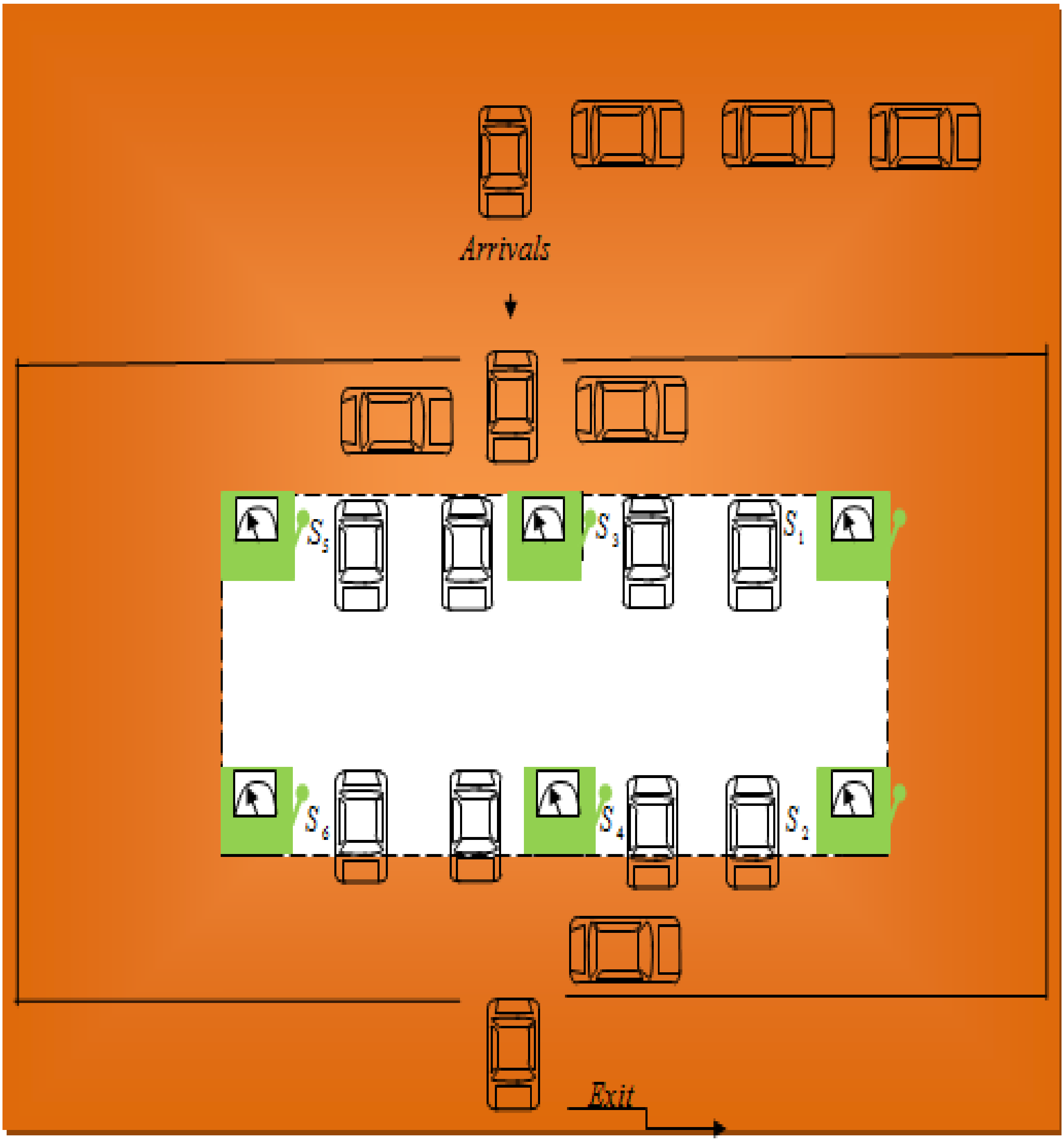


Figure 1: Structure of the PMS Dispensary pump system of the studied NNPC mega petroleum station Enugu

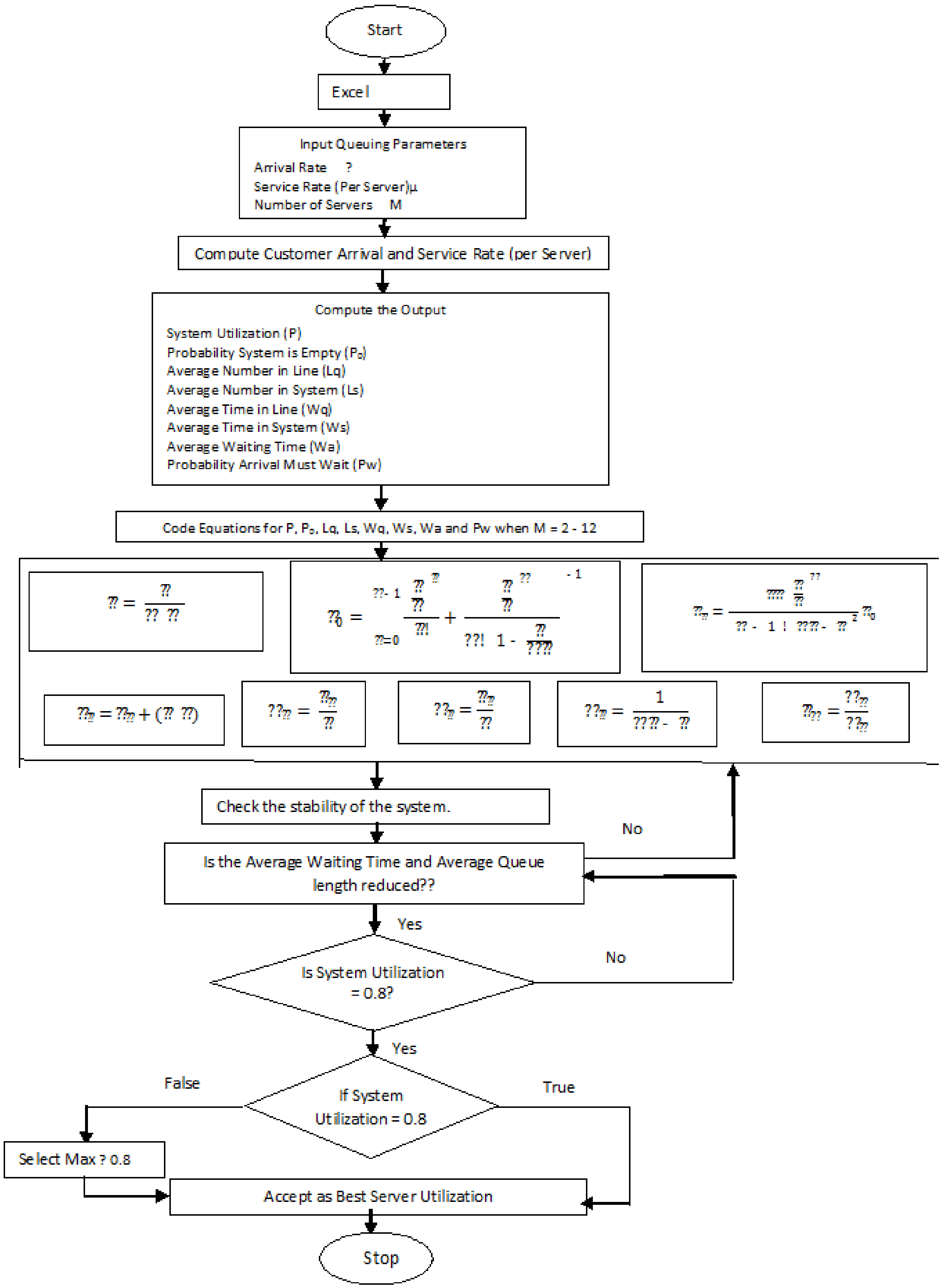


Figure 2: Development Flow Chart of the Queue Evaluation Environment and Best Server Utilization.

IV. Data Analysis (Case Study: NNPC Mega Petroleum Station Enugu)

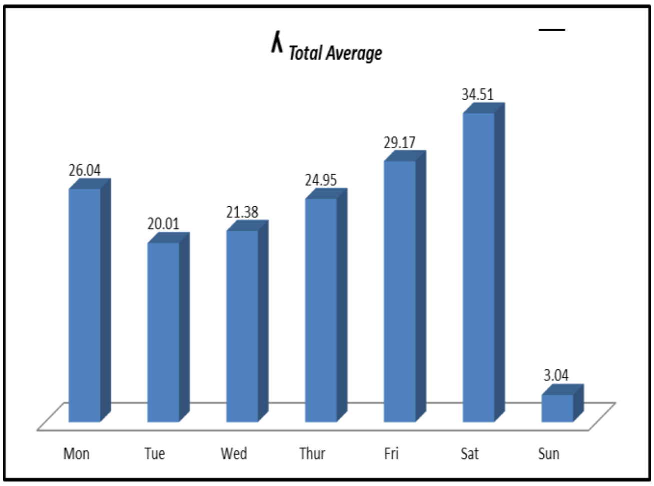


Figure 3: Total Daily Average Arrival Rate / 15 Minutes

In figure 3, the bar chart shows the total daily average arrival rate of customers Per 15 Minutes for the year from Monday - Sunday at NNPC mega petroleum station Enugu. From the chart, it is observed that Saturday was with the highest arrivals which shows that the mega station is being patronage more by customers on Saturdays than the rest of the days being the fact that Saturday is a work free day for civil servants and most public servants so customers buy large quantity of PMS to last them for the week day activities as well as weekend travels. While Sunday was with the lowest arrivals which shows less patronage of customers being the fact that Sunday is a worship day for Christians and the Mega Station don't always open for service on that day and also customers most especially civil and public servants must have bought large quantity of PMS on Saturday and Friday to last them for the week. The week days (i.e. Mondays – Fridays) were mostly patronage more by commercial transporters

Table 1: Weekly Mean Arrival Rate of Customers, Weekly Mean Combined Service Rate of Customers and Mean Number of Servers Engaged at NNPC Mega Petroleum Station Enugu for the year.

		Weekly Mean Arrival Rate Per Mins	Weekly Mean Combined Service Rate Per Mins	Mean Number of Servers Being Used (M)
DEC	1st Week	1.5488	1.5571	6
	Last Week	0.3893	0.4226	4
JAN	1st Week	0.9036	0.9554	5
	Last Week	1.5982	1.6089	5
FEB	1st Week	1.6619	1.6786	8
	Last Week	1.5631	1.5804	6
MAR	1st Week	1.5601	1.5738	6
	Last Week	1.6857	1.7077	6
APR	1st Week	1.3911	1.4083	7
	Last Week	1.6405	1.6536	7
MAY	1st Week	1.6542	1.6714	8
	Last Week	1.603	1.6202	7
JUN	1st Week	1.5863	1.5976	6
	Last Week	1.5512	1.5661	6
JUL	1st Week	1.5804	1.5952	8
	Last Week	1.5899	1.6006	7
AUG	1st Week	1.6083	1.6268	6
	Last Week	1.5679	1.581	6
SEP	1st Week	1.644	1.6589	7
	Last Week	1.6351	1.6494	7
OCT	1st Week	1.6226	1.6375	6
	Last Week	1.5964	1.6036	6
NOV	1st Week	1.572	1.5881	7
	Last Week	1.6131	1.6244	7
	Total	36.3667	36.7672	154
	Average	1.5153	1.532	6

Total average arrival rate for the year

$= \frac{\sum\bar{\lambda}_{(Weekly)}}{24} = 1.5153$ cars/minutes

Total average number of servers being used for the year $= \frac{\sum\bar{M}_{(Weekly)}}{24} = 6$

Total average combined service rate for the year = $\frac{\sum\bar{\mu}_{c(Weekly)}}{24} = 1.5320$ cars/minutes.

(Note) It is assumed that each server contributes an average service rate of $\frac{\bar{\mu}_{c(Year)}}{\bar{M}}$ cars/minutes. Where $\bar{M}$ = 6, and $\bar{\mu}_{c\,(Year)}$ = 1.5320 cars/minutes. This implies that each server contributes an average service rate of 0.2553 cars/minutes in the service facility

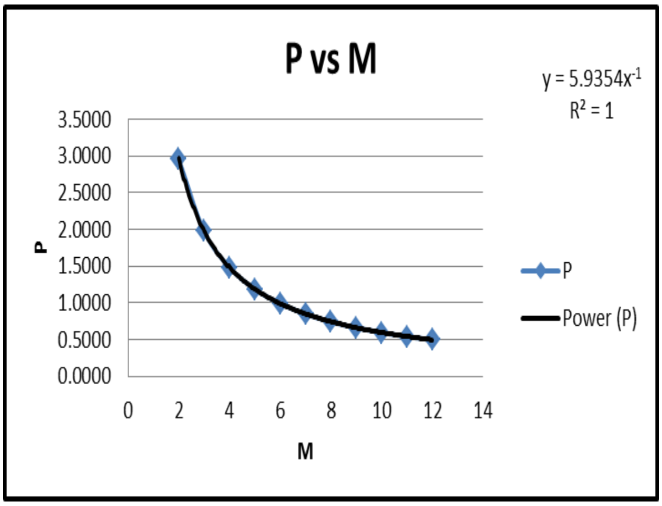
V. Presentation of Queue Performance Evaluation Results

The results of the performance measures of the queuing system are presented below.

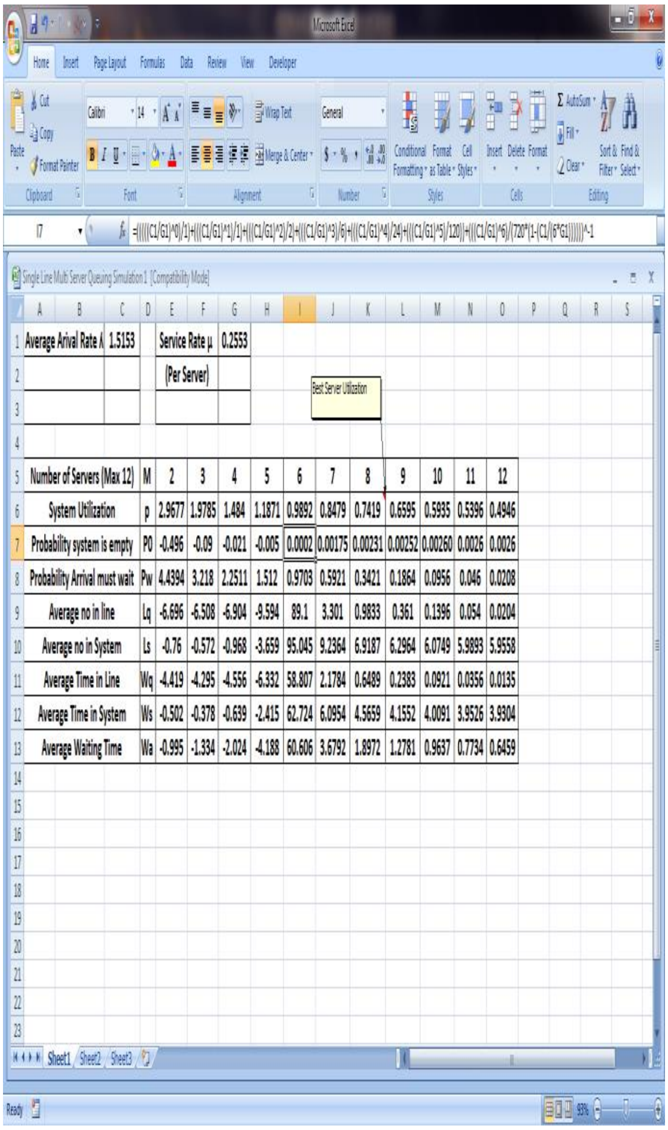
Table 2: Results of the performance evaluation of the queuing system with parameters $\bar{\lambda}$ = 1.5153 cars/minutes and $\bar{\mu}_c$ = 1.5320cars/minuteswhen (M= 6)

Average Arival Rate ʎ	1.5153		Average Combined Service Rate μc	1.5320

System Utilization	p	0.9892
Probability system is empty	P0	2E-04
Probability Arrival must wait	Pw	0.9703
Average no in line	Lq	89.1
Average no in System	Ls	95.045
Average Time in Line	Wq	58.807
Average Time in System	Ws	62.724
Average Waiting Time	Wa	60.606



From figure 4, the charts of the queue output results were developed using the application of Microsoft Excel and trend line was used to test for the best goodness fit in developing the relationship that exists best between the queue output results.



Average Arival Rate ʎ	1.5153	Service Rate μ (Per Server)	0.2553									
Number of Servers (Max 12)	M	2	3	4	5	6	7	8	9	10	11	12
System Utilization	p	2.9677	1.9785	1.484	1.1871	0.9892	0.8479	0.7419	0.6595	0.5935	0.5396	0.4946
Probability system is empty	P0	-0.496	-0.09	-0.021	-0.005	0.0002	0.00175	0.00231	0.00252	0.00260	0.0026	0.0026
Probability Arrival must wait	Pw	4.4394	3.218	2.2511	1.512	0.9703	0.5921	0.3421	0.1864	0.0956	0.046	0.0208
Average no in line	Lq	-6.696	-6.508	-6.904	-9.594	89.1	3.301	0.9833	0.361	0.1396	0.054	0.0204
Average no in System	Ls	-0.76	-0.572	-0.968	-3.659	95.045	9.2364	6.9187	6.2964	6.0749	5.9893	5.9558
Average Time in Line	Wq	-4.419	-4.295	-4.556	-6.332	58.807	2.1784	0.6489	0.2383	0.0921	0.0356	0.0135
Average Time in System	Ws	-0.502	-0.378	-0.639	-2.415	62.724	6.0954	4.5659	4.1552	4.0091	3.9526	3.9304
Average Waiting Time	Wa	-0.995	-1.334	-2.024	-4.188	60.606	3.6792	1.8972	1.2781	0.9637	0.7734	0.6459

Figure 4: Queue Evaluation Environment displaying the results of the queue performance when $\bar{\lambda}$ is fixed, $\bar{\mu}$= 0.2553 cars/minutes (per server) and *M* = 2 – 12 servers

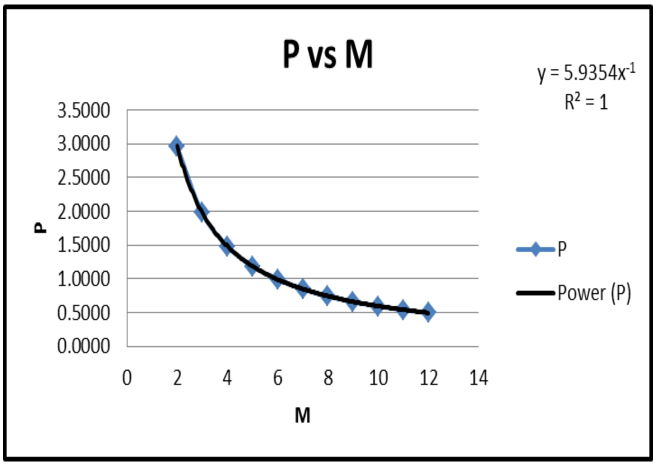


Figure 5: Scatter Plot of System Utilization vs. Number of Servers

In figure 5, the scatter plot shows that the best fit between the two variables i.e. (P and M) from 2 to 12 servers is a trend line power.

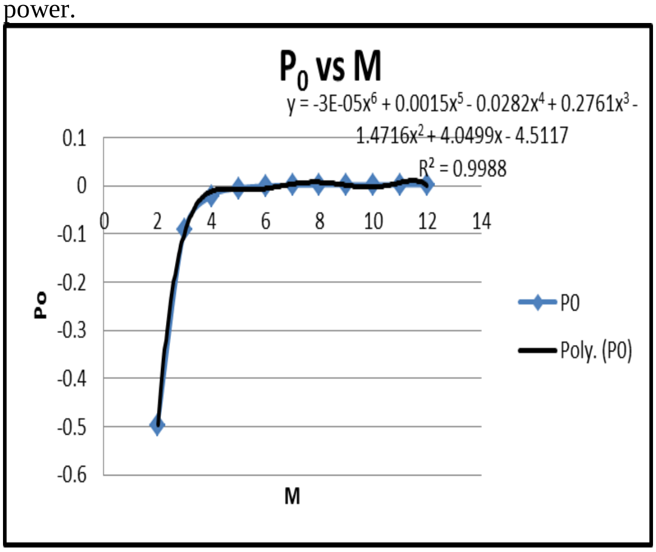


Figure 5: Scatter Plot of Probability System is Empty vs. Number of Servers

In figure 5,the scatter plot shows that the best fit between the two variables i.e. (P_0 and M) from 2 to 12 servers is a nonlinear polynomial function in the sixth order.

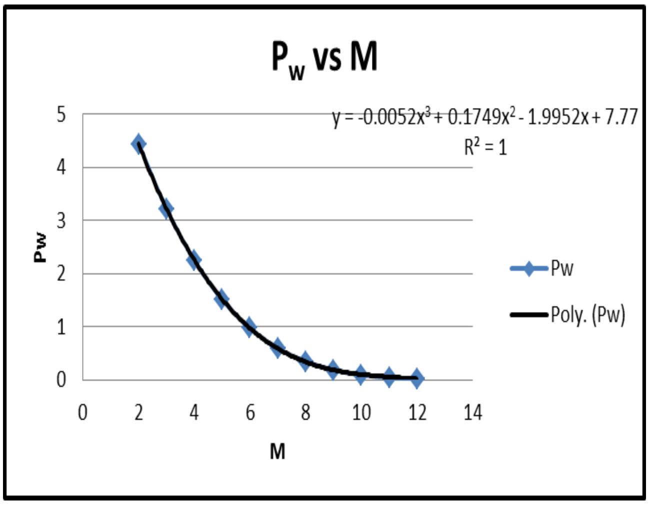


Figure 6: Scatter Plot of Probability Arrival Must Wait vs. Number of Servers

In figure 6,the scatter plot shows that the best fit between the two variables i.e. (P_w and M) from 2 to 12 servers is a nonlinear polynomial function in the third order.

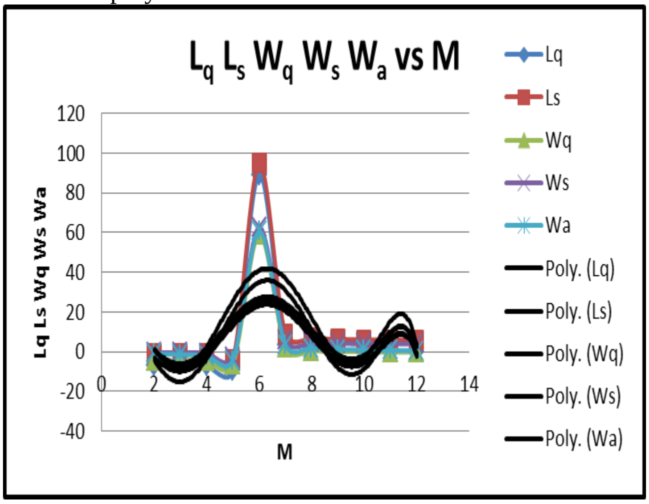


Figure 7:Scatter Plot of Average Number in Line, Average Number in System, Average Time in Line, Average Time in System and Average Waiting Time vs. Number of Servers.

In figure 7, the scatter plot shows the relationship that exists between the queue output variables i.e. (L_q, L_s, W_q, W_s and W_a plotted against M) from 2 to 12 servers. The chart can hardly be interpreted and the best fit between the variables is a nonlinear polynomial function in the sixth order which produced poor R^2 values at 0.360 for L_q, L_s, W_q, W_s and 0.356 for W_a. (See equation 10 – 15). The R^2 values are poor and thus, the relationship that exists between each of the dependent variables i.e. (L_q, L_s, W_q, W_s and W_a) and the independent variable (M) were poor. This resulted from the negative values of L_q, L_s, W_q, W_s, and W_a from 2 to 5 servers (see figure 1). The resultant of these negative values shows that system utilization is greater than 1. The scatter plots of the variables were later plotted separately from the positive result outputs (i.e. from 6 to 12 servers) and the results produced excellent R^2 values. (See figure 8 – 12).

$$L_q = -0.012m^6 + 0.471m^5 - 6.701x^4 + 44.44m^3 - 137.6m^2 + 177.5m - 71.61 \quad (10)$$
$R^2 = 0.360$

$$L_s = -0.012m^6 + 0.471m^5 - 6.701m^4 + 44.44m^3 - 137.6m^2 + 177.5m - 65.67 \quad (12)$$
$R^2 = 0.360$

$$W_q = -0.008m^6 + 0.311m^5 - 4.422m^4 + 29.33m^3 - 90.84m^2 + 117.1m - 47.26 \quad (13)$$
$R^2 = 0.360$

$$W_s = -0.008m^6 + 0.311m^5 - 4.422m^4 + 29.33m^3 - 90.84m^2 + 117.1m - 43.34 \quad (14)$$
$R^2 = 0.360$

$$W_a = -0.008m^6 + 0.311m^5 - 4.422m^4 + 29.33m^3 - 90.82m^2 + 116.6m - 42.81 \quad (15)$$
$R^2 = 0.356$

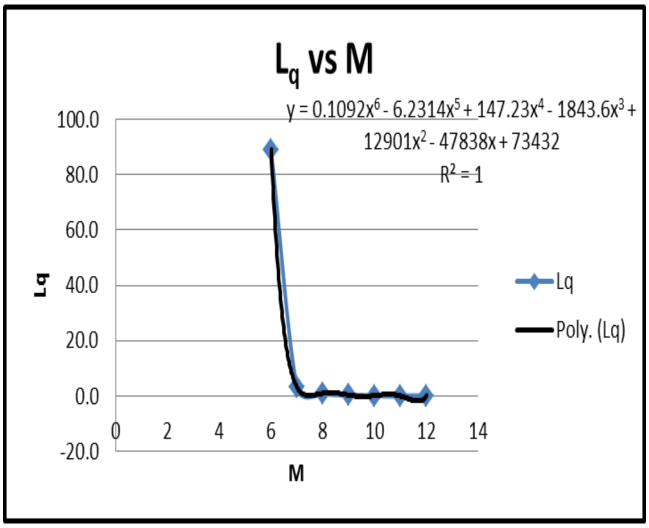


Figure 8: Scatter Plot of Average Number in Line vs. Number of Servers

In figure 8, the scatter plot shows that the best fit between the two variables i.e. (L_q and M) from 6 to 12 servers is a nonlinear polynomial function in the sixth order.

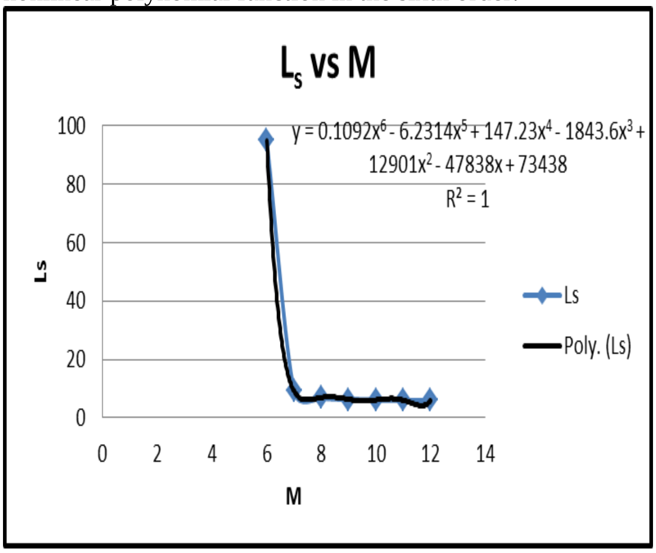


Figure 9: Scatter Plot of Average Number in System vs. Number of Servers

In figure 9, the scatter plot shows that the best fit between the two variables i.e. (L_s and M) from 6 to 12 servers is a nonlinear polynomial function in the sixth order.

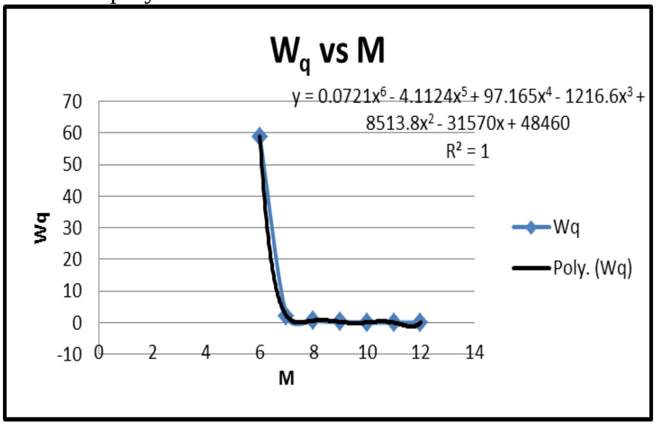


Figure 10: Scatter Plot of Average Time in Line vs. Number of Servers

In figure 10, the scatter plot shows that the best fit between the two variables i.e. (W_q and M) from 6 to 12 servers is a nonlinear polynomial function in the sixth order.

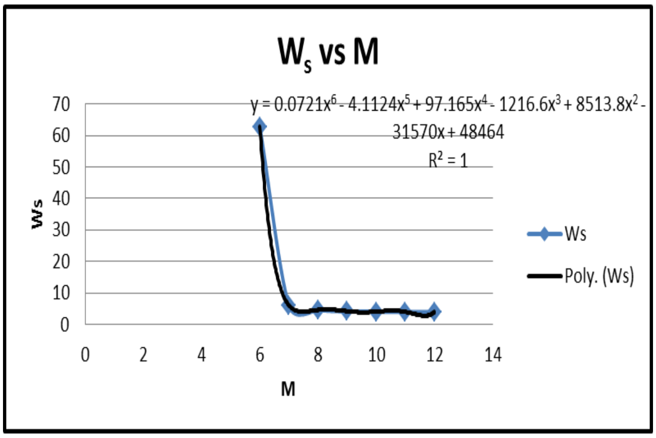


Figure 11: Scatter Plot of Average Time in System vs. Number of Servers

In figure 11, the scatter plot shows that the best fit between the two variables i.e. (W_s and M) from 6 to 12 servers is a nonlinear polynomial function in the sixth order.

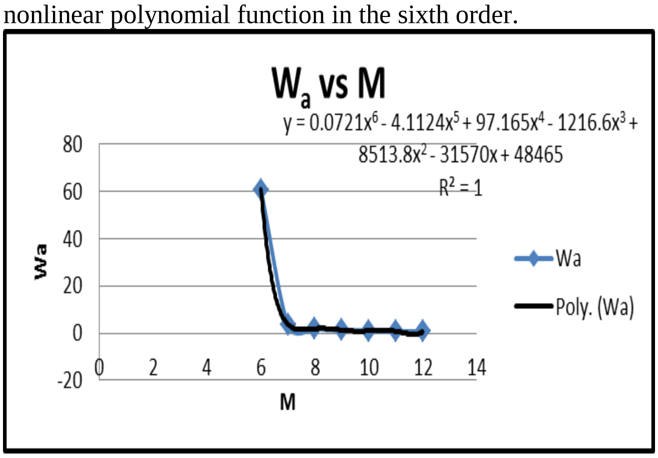


Figure 12: Scatter Plot of Average Waiting Time vs. Number of Servers

In figure 12, the scatter plot shows that the best fit between the two variables i.e. (W_a and M) from 6 to 12 servers is a nonlinear polynomial function in the sixth order.

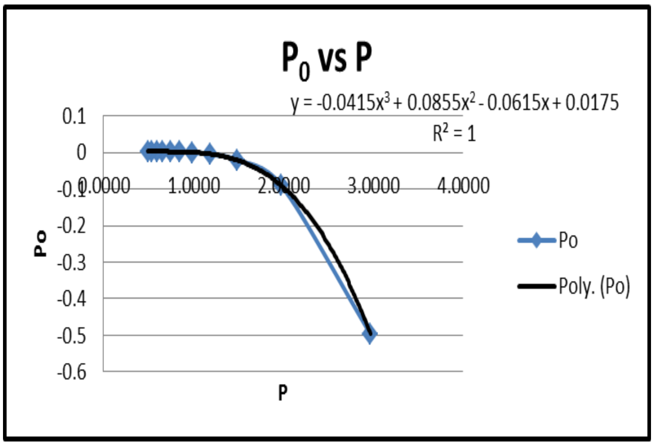


Figure 13: Scatter Plot of Probability System is Empty vs. System Utilization

In figure 13, the scatter plot shows that the best fit between the two variables i.e. (P_0 and P) from 2 server utilization to 12 server utilization is a nonlinear polynomial function in the third order.

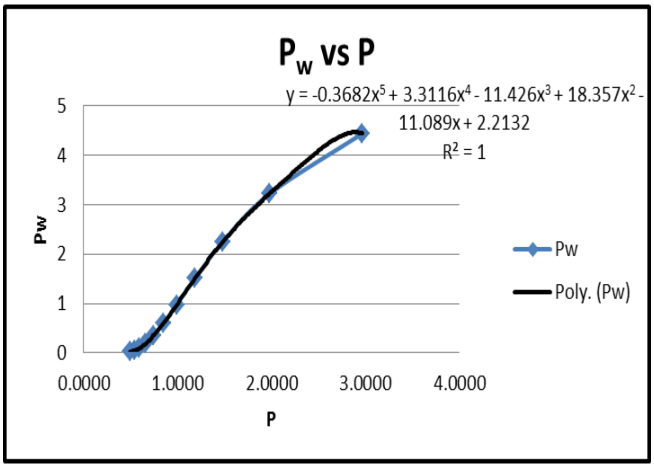


Figure 14: Scatter Plot of Probability Arrival Must Wait vs. System Utilization

In figure 14, the scatter plot shows that the best fit between the two variables i.e. (P_w and P) from 2 server utilization to 12 server utilization is a nonlinear polynomial function in the fifth order.

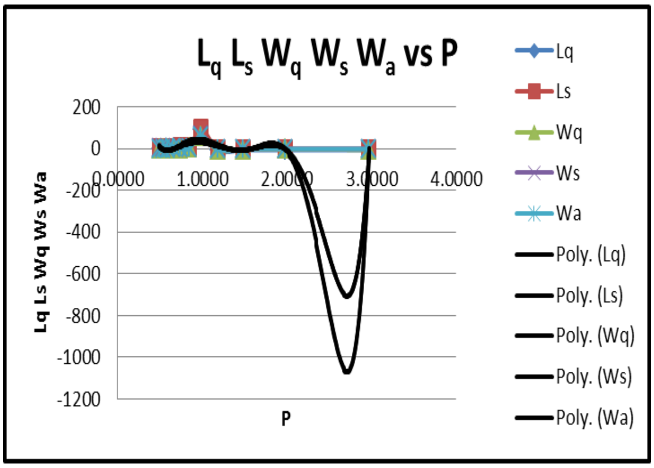


Figure 15:Scatter Plot of Average Number in Line, Average Number in System, Average Time in Line, Average Time in System and Average Waiting Time vs. System Utilization

In figure 15, the scatter plot shows the relationship that exists between the queue output variables i.e. (L_q, L_s, W_q, W_s and W_a plotted against P) from 2 Server Utilization to 12 Server Utilization. The chart can hardly be interpreted and the best fit between the variables is a nonlinear polynomial function in the sixth order which produced poor R^2 values at 0.430 for L_q, L_s, W_q, W_s and 0.426 for W_a (See equations 13 – 17). The R^2 values are poor and thus, the relationship that exists between each of the dependent variables i.e. (L_q, L_s, W_q, W_s and W_a) and the independent variable (P) were poor. This resulted from the negative values of L_q, L_s, W_q, W_s, and W_a from 2 to 5 Server Utilization (see figure 3). The resultant of these negative values shows that system utilization is greater than 1. The scatter plots of the variables were later plotted separately from the positive result outputs (i.e. from 6 servers' utilization to 12 servers' utilization) and the result produced excellent R^2 values. (See figure 16 – 20)

$L_q = 795.7p^6 - 7176.p^5 + 25352p^4 – 44779p^3 + 41488p^2 – 19002p + 3360.$ (16)

$R^2 = 0.430$

$L_s = 795.7p^6 - 7176.p^5 + 25352p^4 – 44779p^3 + 41488p^2 – 19002p + 3366.$ (17)

$R^2 = 0.430$

$W_q = 525.1p^6 - 4736.p^5 + 16731p^4 – 29551p^3 + 27379p^2 – 12540p + 2217.$ (18)

$R^2 = 0.430$

$W_s = 525.1p^6 - 4736.p^5 + 16731p^4 – 29551p^3 + 27379p^2 – 12540p + 2221.$ (19)

$R^2 = 0.430$

$W_a = 525.4p^6 - 4739.p^5 + 16743p^4 – 29574p^3 + 27401p^2 – 12547p + 2218.$ (20)

$R^2 = 0.426$

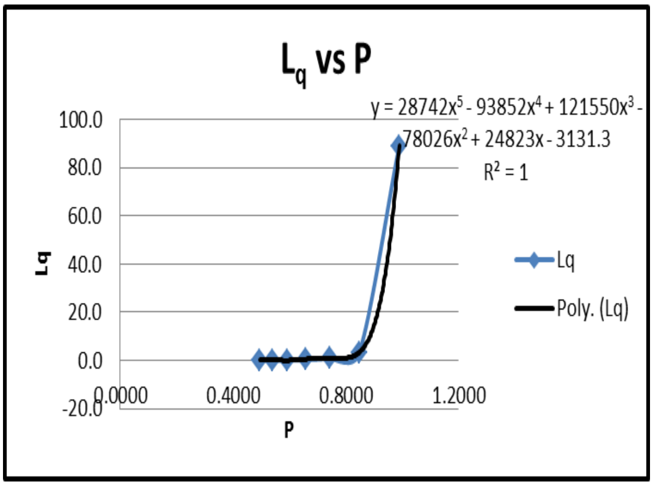


Figure 16: Scatter Plot of Average Number in Line vs. System Utilization

In figure 16, the scatter plot shows that the best fit between the two variables i.e. (L_q and P) from 6 server utilization to 12 server utilization is a nonlinear polynomial function in the sixth order.

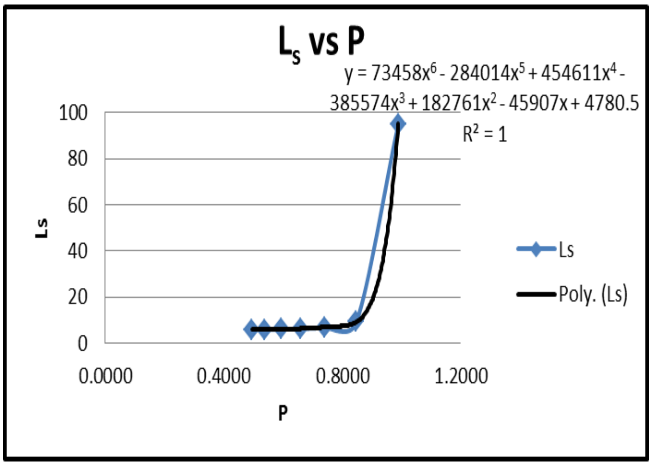


Figure 17: Scatter Plot of Average Number in System vs. System Utilization

In figure 17, the scatter plot shows that the best fit between the two variables i.e. (L_s and P) from 6 server utilization to 12 server utilization is a nonlinear polynomial function in the sixth order.

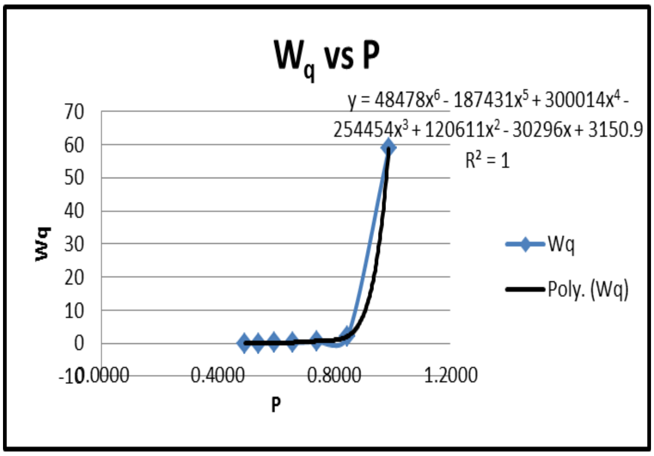


Figure 18: Scatter Plot of Average Time in Line vs. System Utilization

In figure 18, the scatter plot shows that the best fit between the two variables i.e. (W_q and P) from 6 server utilization to 12 server utilization is a nonlinear polynomial function in the sixth order.

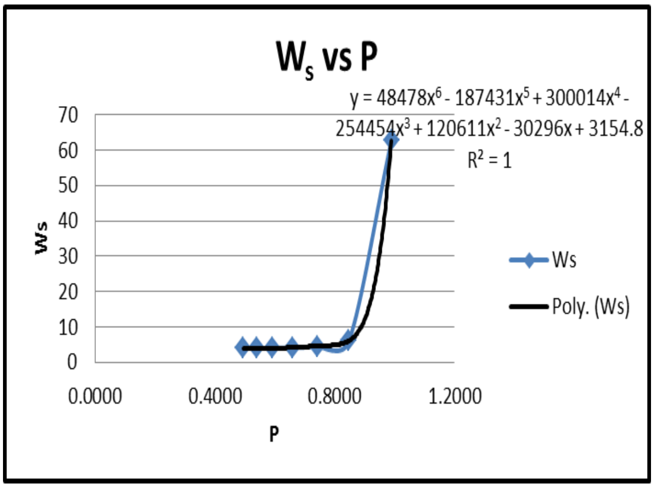


Figure 19: Scatter Plot of Average Time in System vs. System Utilization

In figure 19, the scatter plot shows that the best fit between the two variables i.e. (W_s and P) from 6 server utilization to 12 server utilization is a nonlinear polynomial function in the sixth order.

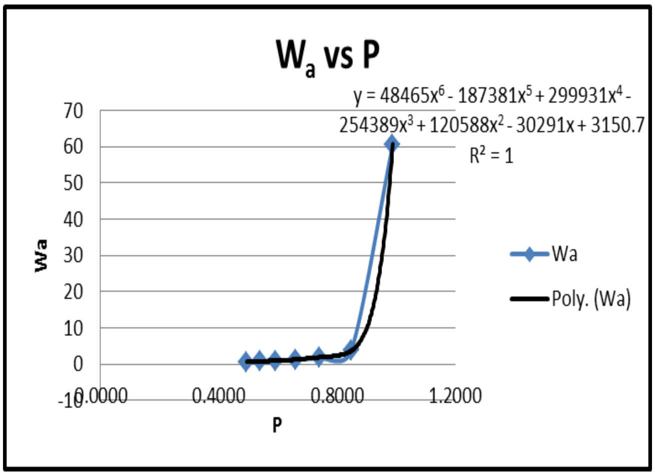


Figure 20: Scatter Plot of Average Waiting Time vs. System Utilization

In figure 20, the scatter plot shows that the best fit between the two variables i.e. (W_a and P) from 6 server utilization to 12 server utilization is a nonlinear polynomial function in the sixth order.

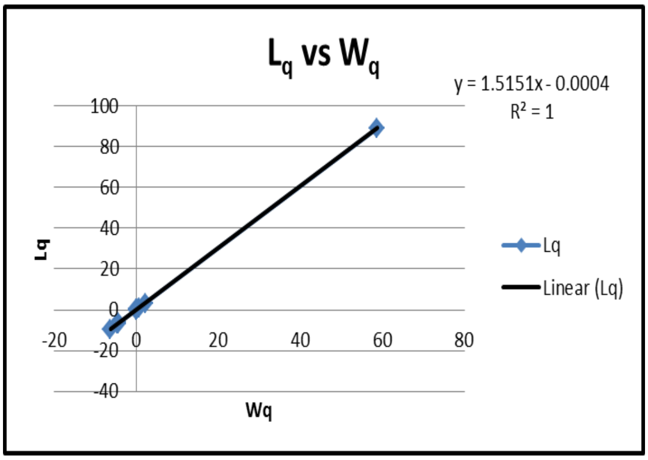


Figure 21: Scatter Plot of Average Number in Line vs. Average Time in Line

In figure 21, the scatter plot shows that the best fit between the two variables i.e. (L_q and W_q) from 2 to 12 servers is a linear function.

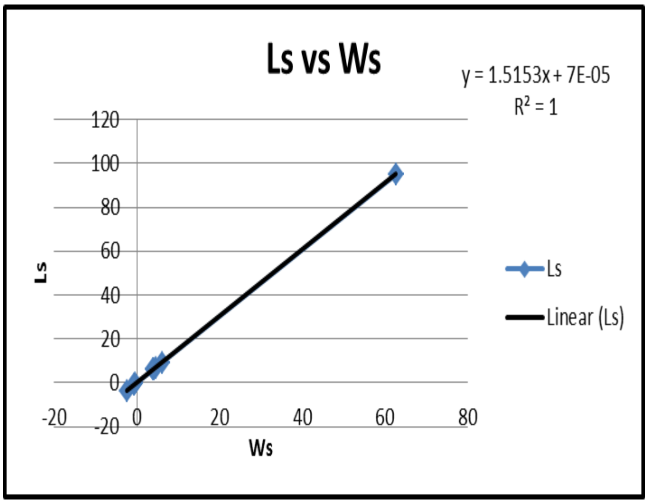


Figure 22: Scatter Plot of Average Number in System vs. Average Time in System

In figure 22, the scatter plot shows that the best fit between the two variables i.e. (L_s and W_s) from 2 to 12 servers is a linear function.

VI. Development of the Decision Support System for Case Study A and B

From the Queue Evaluation Environment created, the service rates (per server) of each of the referenced facilities were fixed and arrival rates were simulated using 2 – 12 servers to see the expected queue performance and to determine the best number of servers that gives the best system utilization value at various arrival rates of customers. The summary result outputs were plotted on a chart using the application of Microsoft Excel and trend line was used to test for the best goodness fit between the dependent variable i.e. Number of Servers (M) and the independent variable i.e. Average Arrival Rates/Minutes (λ). See summary result output and charts below.

Table 3: Summary result output of simulated arrival rates of customers/minutes (NNPC Mega Petroleum Station Enugu)

$\lambda/Mins$	Best No. Servers
0.3	2
0.4	2
0.5	3
0.6	3
0.7	4
0.8	4
0.9	5
1.0	5
1.1	6
1.2	6
1.3	7
1.4	7
1.5	8
1.6	8
1.7	9
1.8	9
1.9	10

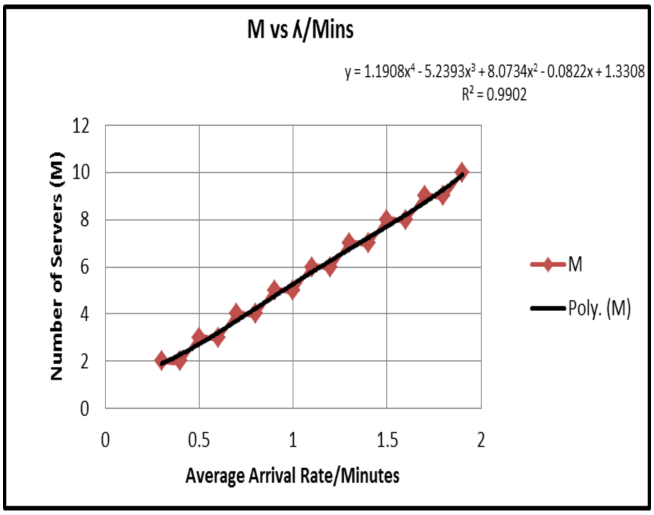


Figure 23: Scatter Plot of Number of Servers (M) vs. Average Arrival Rate/Minutes

From figure 23, the scatter plot shows the number of servers plotted against average arrival rate of customers/minutes in

NNPC Mega Petroleum Station Enugu. From the chart, it is observed that number of servers is expected to be stepping up as average arrival rate increases and the best fit between the two variables i.e. Number of Servers (M) and Average Arrival Rate (λ) is a nonlinear polynomial function in the fourth order as depicted in the chart.

VII. Discussion of Results

From the analysis, table 2 showed the results of the performance measures of the queuing system as seen at the NNPC mega petroleum station Enugu. From the results, it was discovered at an average number of 6 servers being used, with average combined service rate($\bar{\mu}_c$) of 1.5320 cars/minutes and average customer arrival rate$(\bar{\lambda})$ of 1.5153 cars/minutes, gave a system utilization (P) of 0.9892 which gives a percentage system utilization of 98.92%, while the probability of the system being empty and the probability of waiting gave 0.0002 and 0.9703 respectively, this means that when service commences, the system is never idle and a customer must wait before receiving service with a 97.03% probability. However, the average number of customers in line and the average number of customers in system including any being served gave 89.1 and 95.045 respectively. Furthermore, the average waiting time of customers in line, the average waiting time of customers in the system including service and the average waiting time of customers on arrival not immediately served gave 58.807, 62.724 and 60.606 minutes respectively.

The results of table 2 studies showed that the systems were heavily utilized at an average of 6 servers because system utilization was almost 100%. This resulted to the longer waiting time of customers experienced at the service facilities. However in respect of this, the service rate per server was determined for the studies and a Queue Evaluation Environment was created using 2 – 12 servers to see the expected queue performance and to determine the best number of servers that gives a good tradeoff between system utilization and waiting time at the collected average arrival rates of customers in the referenced service facilities.

The results from the Queue Evaluation Environment showed that 8 servers gave the best system utilization values of 0.7419 which is expected to reduce the respective customers waiting times (Ws) by 92.72% for the study establishments. This is based on the statement of Egolum, which says that system utilization should be greater than 0 but less than 0.8 [19]. From the charts of system utilization versus waiting time plotted for both case studies, it is observed that there’s no significant decrease in waiting time anymore from system utilization value of 0.8, which shows that waiting time has reached its optimum at the respective best server utilization values of 0.7419 of the referenced service facilities. See figures (18, 19 and 20). This shows that there will be no need of making use of more than 8 servers at the respective average arrival rates of customers in the referenced service facilities. Also the expected probabilities of system idleness for the study is negligible at 8 server utilization because at that point, probabilities of system idleness has also reached its optimum and it no longer has any effect on the service systems. (See figure 13).

From the study, it was revealed that the probability of system being empty increased to optimum as system utilization drops; the probability of an arrival waiting drops as system utilization reduces; the average number in line and average number in system drops to optimum as system utilization reduced; the average time in line, average time in system and average waiting drops to optimum as system utilization reduced; the average number in line increases as average time in line increases; the average number in system increases as average time in system increases (See figures (13 – 14), (16 – 20) and (21 – 22).

It was also revealed that system utilization drops as number of server’s increases; the probability of system being empty increased to optimum as number of server’s increases; the probability of an arrival waiting reduces as number of server’s increases; the average number in line and average number in system drops to optimum as number of server’s increased; the average time in line, average time in system and average waiting time drops to optimum as number of server’s increased (See figure 8 – 12).

Finally, from figure 23, the models for the decision support system were developed using trend line analysis. For NNPC Mega Petroleum Station Enugu, the recommended best number of server (M) for PMS refill is given by:

$$\mathbf{M = 1.190\lambda^4 - 5.239\lambda^3 + 8.073\lambda^2 - 0.082\lambda + 1.330} \quad (21)$$

VIII. Conclusion

The evaluation of queuing system in an establishment is very essential for the betterment of the establishment. Most establishments are not aware on the significance of evaluating their queue performance. The implication of this is that operations managers are not able to determine the best number of servers to engage for service at various demand periods which affects their queue performance. As it concerns the case study establishments, the analysis and evaluation of their queuing system showed that both service systems were being over utilized which resulted to customers spending longer time than necessary before receiving service. However, the need of creating a Queue Evaluation Environment to find out the number of servers that gives the best server utilization at the collected average arrival rates became very essential. From the Queue Evaluation Environment, using 8 servers at the collected average arrival rates of customers in the referenced service facilities gave a good tradeoff between system utilization and waiting time which is expected to reduce the waiting time of customers’ inthe system while server idleness is neglected. In conclusion, the Queue Evaluation Environment created and the decision support system developed for the case study establishments will go every long way in addressing their queuing problems.

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