Moment Invariants for detection of very small objects

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Abstract—This paper focuses on the techniques used to detect small objects using moment invariants. Objects can be detected with the help of moment invariants. Different moment invariants are discussed in this paper. The pitfalls of Fourier-Mellin invariants and Legendre moment invariants are discussed. Two dimensional moments of \( N \times N \) images are sampled and proved that 2D geometrical moment invariants are far better than the other invariants in terms of the object detection.

Index Terms—Image function, algebraic invariants, polar coordinates, affine groups, information redundancy, Fourier-Mellin invariants, Legendre moment invariants, Counter-based shape descriptors, Geometric moment invariants.

I. INTRODUCTION


II. THE CONCEPT OF MOMENT INVARIANTS

Moment invariants were first introduced by the Hu. By image functions we understand any real function \( f(x, y) \in \) having a bounded support and a non-zero integral.

Translation and scale variance of a moment dimension invariants are easy to be eliminated. Let \( m_{p_1...p_n} \) represent the \( n \)-dimension \( \langle p_1 + \ldots + p_n \rangle \) th order moment of a piecewise continuous density function \( h(x_1, \ldots, x_i) \) it can be defined as

\[
m_{p_1...p_n} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_1^{p_1} \ldots x_i^{p_i} h(x_1, \ldots, x_i) dx_1 \ldots dx_i
\]

The scale variants can be eliminated based on the concept of algebraic invariants [14] as follows.

\[
\eta_{p_1...p_n} = \frac{\mu_{p_1...p_n}}{p_0 \cdot p_1 \ldots p_{n-1} \eta}
\]

A. Expression for Moment-Based Rotation invariants

Rotation invariant is achieved by using Zernike’s moment invariants, wavelet moment invariants and Li’s moment invariants.

Let \( f(x, y) \) represent a two dimensional binary image object in the \( (x, y) \) coordinate and Let \( f(r, \theta) \) be its corresponding polar coordinate. Then the relationship between \( f(x, y) \) and \( f(r, \theta) \) is specified as

\[
x = r \cos \theta, \quad y = r \sin \theta
\]

To derive rotation invariant moments, the following general expression is used

\[
f_{p_1, p_2} = \int \int f(r, \theta) g_{p_1, p_2}(r, \theta) d\theta \quad d\theta\]

WHERE IS THE ORDER MOMENT, \( g_{p_1, p_2} \) IS A FUNCTION OF RADIAL VARIABLE, AND ARE INTEGER PARAMETERS.

III FOURIER-MELLIN INVARIANTS

The optical research community introduced the Fourier-Mellin transform in 1970s. These transforms are used now-a-days in digital image and signal processing. The main concept of fourier-Mellin transforms with the study of similarity transformations. Ghorbel[15] work focused on the Fourier transform defined on 2D and 3D parameterization. Tursci[16] explained some Fourier transforms for sub groups of the affine group.

The standard fourier-Mellin transform is given by

\[
\forall (k, \nu) \in \mathbb{Z} \times \mathbb{R}, \mu_f(k, \nu) = \frac{1}{2\pi} \int_0^{2\pi} \int_0^\infty \frac{f(r, \theta) e^{-i\nu \theta} e^{-ikr}}{r} \frac{dr}{r} d\theta
\]

Is assumed to be summable over \( \mathbb{R}^+ \times \) under the measure \( d\theta \), i.e.

\[
\int_0^{\infty} \left( \int_0^{2\pi} |f(r, \theta)| e^{-i\nu \theta} d\theta \right) \frac{dr}{r} < \infty
\]
III. LEGENDRE MOMENTS

Legendre moments were introduced by Teague [17]. Tel and chain [18] stated that orthogonal Legendre moments can be used to represent an image with a minimum amount of information redundancy.

The two dimensional Legendre moments of order \((p + q)\) for image intensity function \(f(x, y)\) are defined as

\[
L_{pq} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} P_p(x) P_q(y) f(x, y) \, dx \, dy
\]

where \(P_p(x)\) is the \(p\) order Legendre polynomial. Legendre descriptors of the third and fourth order are used to test the invariance.

Third order descriptions are

\[
\begin{align*}
\phi_3 &= 25 \eta_3 \gamma_0 \\
\phi_4 &= 25 \eta_2 \gamma_1 \\
\phi_5 &= 25 \eta_1 \gamma_2 \\
\phi_6 &= 25 \eta_0 \gamma_3
\end{align*}
\]

Fourth order descriptions are

\[
\begin{align*}
\phi_7 &= \frac{3}{8} \eta_4 \gamma_0 - \frac{30}{8} \eta_2 \gamma_1 + \frac{3}{8} \eta_0 \gamma_2 \\
\phi_8 &= \frac{105}{8} \eta_3 \gamma_1 \\
\phi_9 &= \frac{3}{4} \eta_2 \gamma_2 - \frac{3}{4} \eta_0 \gamma_2 + \frac{3}{4} \eta_0 \gamma_1 \\
\phi_{10} &= \frac{105}{8} \eta_4 \gamma_2 \\
\phi_{11} &= \frac{3}{8} \eta_2 \gamma_4 - \frac{30}{8} \eta_0 \gamma_3 + \frac{3}{8} \eta_0 \gamma_1
\end{align*}
\]

IV. PROPOSED WORK

The proposed work in this paper used geometric moment invariants and found to be superior compared to Fourier-Mellin and Legendre moments.

A. Geometric invariant moments

The geometric moment invariants produce a group of features vectors that are invariants under shifting, rotation and scaling. Regular moment invariants are most popular counter-based shape descriptions derived by Hu. Geometric moment invariants were first introduced by Hu, which are derived from the theory of algebraic invariants.

Geometric moment were successfully applied on the alphabet A shown in different shapes. The alphabet image A is used to get the range of invariant. The comparison of invariants feature vectors are shown in the table.

Two dimensional moments of a digitally sampled Image has gray function \(f(x, y)\), translated by an amount \((\alpha, \beta)\), are defined as

\[
\mu_{pq} = \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} (x + \alpha)^p (y + \beta)^q f(x, y)
\]

V. RESULTS

The alphabet A in different shapes (I1, I2, I3 etc.,) has been adopted as the text image and the simulate results of average invariants \(\phi_1, \phi_2, \phi_3, \phi_4\) are compared with fourier-Mellin, Legendre and geometric invariants as shown in table.

VI. CONCLUSION

Different moment invariants are compared. The first order invariants of geometric moments are compared with Fourier and Legendre moments and are shown with bar chart.
best suitable for object detection.

REFERENCES


AUTHOR’S PROFILE

Dr V S Giridhar Akula is presently working as professor and Principal at Methodist College of Engineering and Technology, Abids, Hyderabad. He is a doctorate in Computer Science and Engineering from JNTUA. He has total 22 years of teaching experience and author of 8 text books. He has published around 63 papers in national and international journals. He has conducted 4 International Conferences. He is the editorial board member of 23 International and 12 National Journals. Dr Giridhar is the recipient of Best Promising Teacher (Engineering) of the Year National Award. He is also the recipient of Best Computer Science Teacher award (for 2010 and 2012) constituted by ISTE. He is Board of Studies Chairman for 6 Universities and also authored 3 book chapters for International Publishers.

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