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BPI Observer-based Fault Tolerant Control Design for a Class of Fuzzy Bilinear Systems

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Abstract— **In this paper, a new methodology of Bilinear Proportional Integral (BPI) observer-based Fault Tolerant Control (FTC) for a class of nonlinear systems with bilinear terms is proposed. Nonlinear systems are represented by the Takagi-Sugeno (T-S) Fuzzy Bilinear Model (FBM). The objective is to design a stable and robust fault-tolerant controller based on a T-S fuzzy BPI observer. The proposed BPI observer estimates simultaneously the states of the system and the occurring time varying faults with the final goal to adapt control law strategies in order that the system remains stable even in fault case. Further, the stability conditions of the designed BPI observer and the state feedback control are analysed with Lyapunov theory to ease the design of FTC. The gains of the BPI observer and the state feedback control law are obtained by solving linear matrix inequalities (LMI) under equality constraints. Moreover, we proposed some sufficient conditions for robust BPI observer-based fault tolerant control of the FBM with parametric uncertainties. Finally, an academic system is presented to illustrate the robustness of the proposed controller with respect to uncertainties.**

Index Terms—**T-S fuzzy bilinear model, Bilinear proportional integral observer, Observer-based control, Fault tolerant control, Lyapunov function, Parametric uncertainty, LMI.**

I. Introduction

The Bilinear systems are a special class of nonlinear systems, where the dynamics are jointly linear in the state and input variables, that attract the interest of researchers [1], [2]. Moreover, there are many engineering applications as well as models in biology, ecology, nuclear engineering are naturally described by bilinear systems. Furthermore, many nonlinear systems can be adequately approximated bilinear systems.

As a special extension, fuzzy bilinear systems based on the T-S fuzzy model with bilinear rule consequence have attracted the interest of researchers [3]-[5]. It is proved in these papers that often nonlinear behaviors can be approximated by T-S bilinear multimodel description. Some topics of control are extended to T-S FBM, such as stability and stabilization in [6], [7], observers and state estimation in [8]-[10], faults detection and fault tolerant control [9]-[11].

Fault Tolerant Control have become challenging problems in the area of modern control theory. FTC allows having a control loop that fulfils its objectives when faults appear. Thus great interest has been accorded to the fault tolerant controllers for nonlinear systems in the literature [12]-[14]. Moreover, it is noted that all of the aforementioned assume that the system states are measured, which is not true in many control systems and real applications. For this reason, observer-based fuzzy controllers were considered in many researches. In [15], [16], the authors have studied the controller designs based on fuzzy observers for T-S fuzzy systems. Nonetheless, in [17], [18], the authors have proposed sufficient design conditions for robust stabilization of T-S fuzzy models with parametric uncertainties based on state estimation. However, to the best of our knowledge, no previous study has investigated the problem of an BPI observer-based FTC for T-S fuzzy bilinear models and a robust stabilization for uncertain T-S fuzzy models.

In this paper, we consider the fault-tolerant control problem for the fuzzy bilinear systems with parametric uncertainties. Fuzzy BPI observer is designed to estimate simultaneously the states and the occurring time varying faults. Based on the fault estimation, the fault-tolerant controller is designed to ensure that the closed-loop system with faults is asymptotically stable.

This paper is organized as follows. In section II, the considered structure of the T-S fuzzy bilinear system is presented. In Section III, the problem statement is described. In Section IV, the problem of observer-based control for the FBM is developed. In section V, the robust control of the FBM with parametric uncertainties is proposed. An example is provided to show the effectiveness of the proposed design in the subject of section VI.

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II. T-S Fuzzy Bilinear Systems modeling

The T–S fuzzy bilinear models based on the T-S fuzzy model with bilinear rule consequence are defined by extending the T-S fuzzy ordinary model. Similar to [19], the fuzzy bilinear model can be represented by the following fuzzy *if-then* rules:

R^i : if $\xi_1(t)$ is F_{i1} and$\cdots$and $\xi_g(t)$ is F_{ig}

$$\text{then } \begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + N_i x(t)\,\mathrm{u(t)} \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where R_i denotes the i^{th} fuzzy rule $\forall i = \{1, \cdots, r\}$, r is the number of *if-then* rules, $\xi_i(t)$ are the premise variables which can be measurable or not measurable, and $F_{ij}\left(\xi_j(t)\right)$ are fuzzy set. $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re$ is the control input, and $y(t) \in \Re^p$ is the system output. The matrices A_i , B_i , N_i , C are known matrices. Then, the overall FBM can be described as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(\xi(t))\left(A_i x(t) + B_i u(t) + N_i x(t)\,\mathrm{u(t)}\right) \\ y(t) = Cx(t) \end{cases} \tag{2}$$

where $h_i(.)$ verify the following properties:

$$\begin{cases} \sum_{i=1}^{r} h_i(\xi(t)) = 1 & \forall i \in \{1, 2, \cdots, r\} \\ 0 \le h_i(\xi(t)) \le 1 \end{cases} \tag{3}$$

In the remainder of the paper, the following lemmas are used.

Lemma 1: *For any matrices A and B with appropriate dimensions, the following property holds for any positive scalar λ :*

$$A^T B + B^T A \le \lambda A^T A + \lambda^{-1} B^T B \tag{4}$$

Lemma 2: *Given a scalar $\mu > 0$ and a symmetric positive definite matrix P, the following inequality holds:*

$$2x^T y \le \mu^{-1} x^T P x + \mu y^T P^{-1} y \,, \quad x, y \in \Re^n \tag{5}$$

Lemma 3: *Let us consider P a positive definite matrix and Q a full column rank matrix. It follows that the matrix QPQ^T is a positive definite matrix.*

III. PROBLEM STATEMENT

Let us consider the T-S fuzzy bilinear models containing the actuator faults which can be rewritten in the following form:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(\xi(t)) \begin{pmatrix} A_i x(t) + B_i u(t) \\ + N_i x(t) u(t) + G_i f(t) \end{pmatrix} \\ y(t) = Cx(t) \end{cases} \tag{6}$$

where $f(t)$ is an actuator fault.

Assumption 1: *The fault $f(t)$ satisfies $\|f(t)\| \le \alpha_1$ and the first time derivative of $f(t)$ with respect to time is norm bounded i.e. $\|\dot{f}(t)\| \le \alpha_2$ and $0 \le \alpha_1 \le \infty$, $0 \le \alpha_2 \le \infty$.*

The main objective of FTC design is to find a control law $u(t)$ such that the closed-loop system remains stable despite the presence of actuator faults. Then, it is necessary to estimate the states and faults. In this paper, a BPI observer is utilized for estimate both the states and the actuator faults with the final goal to adapt control law strategies such that the system remains stable even in actuator fault case. Following the design concept in [1], the fuzzy control law for FBM is formulated as follows:

$$\begin{aligned} u(t) &= \sum_{i=1}^{r} h_i(\xi(t)) \frac{\rho D_i \hat{x}(t)}{\sqrt{1 + \hat{x}^T(t) D_i^T D_i \hat{x}(t)}} \\ &= \sum_{i=1}^{r} h_i(\xi(t)) \rho \sin \hat{\theta}_i(t) \\ &= \sum_{i=1}^{r} h_i(\xi(t)) \rho D_i \hat{x}(t) \cos \hat{\theta}_i(t) \end{aligned} \tag{7}$$

where:

$$\sin \hat{\theta}_i(t) = \frac{D_i \hat{x}(t)}{\sqrt{1 + \hat{x}^T(t) D_i^T D_i \hat{x}(t)}}$$

$$\cos \hat{\theta}_i(t) = \frac{1}{\sqrt{1 + \hat{x}^T(t) D_i^T D_i \hat{x}(t)}}$$

$$\hat{\theta}_i \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$$

ρ is a given scalar, and $D_i \in \Re^{1 \times n}$, $i = 1, \cdots, r$ are vectors to be determined.

In the following section, we will be developing the BPI observer-based control problem for the T-S FBM.

IV. STABILIZING FAULT TOLERANT CONTROL FOR FBM

In order to estimate both state and faults of FBM, we propose a BPI observer and make an analysis of the error system, from which we provides a design method of a fuzzy observer for the fuzzy bilinear system (2). The considered BPI observer is described by the following equation:

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^{r} h_i(\xi(t)) \begin{pmatrix} H_i z(t) + L_i y(t) + J_i u(t) \\ + M_i u(t) y(t) + \varphi_i \hat{f}(t) \end{pmatrix} \\ \hat{x}(t) = z(t) - Ey(t) \\ \hat{y}(t) = C\hat{x}(t) \\ \dot{\hat{f}}(t) = \sum_{i=1}^{r} h_i(\xi(t)) \Psi_i (\hat{y}(t) - y(t)) \end{cases} \tag{8}$$

where $z(t) \in \Re^n$, $\hat{x}(t) \in \Re^n$, $\hat{y}(t) \in \Re^p$ and $\hat{f}(t) \in \Re^f$ are the observer state vector, the estimated state vector, the estimated output vector and the estimated faults vector respectively. H_i , M_i , L_i , J_i , φ_i , Ψ_i and E are unknown matrices of the BPI observer of appropriate dimensions to be determined.

Note that the premise variables do not depend on the state variables estimated by a BPI observer.

Let us consider the state and fault estimation errors defined respectively by:

$$e_x(t) = x(t) - \hat{x}(t) \tag{9}$$

$$e_f(t) = f(t) - \hat{f}(t) \tag{10}$$

From (8) and (9), the state estimation error given as follows:

$$e_x(t) = Tx(t) - z(t) \tag{11}$$

where $T = I_n + EC$.

By taking into account (2) and (8), the dynamics of the state estimation error and the closed-loop system with the state feedback control law defined in (7) becomes after some calculations:

$$\dot{e}_x(t) = \sum_{i=1}^{r} h_i(\xi(t)) \begin{pmatrix} H_i e_x(t) + (TA_i - L_i C - H_i T) x(t) \\ + (TN_i - M_i C) x(t) u(t) + (TB_i - J_i) u(t) \\ + (TG_i - \varphi_i) f(t) + \varphi_i e_f(t) \end{pmatrix} \tag{12}$$

$$\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t)) h_j(\xi(t)) \begin{pmatrix} \Phi_{ij} x(t) - \rho B_i D_j \cos \hat{\theta}_j e_x(t) \\ + G_i f(t) \end{pmatrix}$$

(13)

where:

$$\Phi_{ij} = A_i + \rho N_i \sin\hat{\theta}_j + \rho B_i D_j \cos\hat{\theta}_j \tag{14}$$

Hence, if the following conditions hold true $\forall i \in \{1,2,\cdots,r\}$

$$TA_i - L_iC - H_iT = 0 \tag{15}$$

$$TN_i - M_iC = 0 \tag{16}$$

$$TB_i - J_i = 0 \tag{17}$$

Then, the equation of the observing error becomes

$$\dot{e}_x(t) = \sum_{i=1}^{r} h_i(\xi(t))\left(H_ie_x(t) + \Xi_i f(t) + \varphi_i e_f(t)\right) \tag{18}$$

with $\Xi_i = TG_i - \varphi_i$

From (15) and using $T = I_n + EC$, we get

$$H_i = TA_i - K_iC \tag{19}$$

with $K_i = H_iE + L_i$ (20)

The goal is to design a feedback control law such that the system remains stable even if a fault occurs. A result is summarized in the following theorem.

Theorem 1: If there exist symmetric positive definite matrices Q, Γ_1, Γ_2, Γ_3, P_1 *and* P_2, matrices W_i, V_i Y_i, Z_i, R_i, U_i, S and positive scalars ε, β, $\rho > 0$ such that LMIs (21) subject to linear equality constraints (22), (23) are satisfied $\forall i = 1\cdots r$, $\forall j = 1\cdots r$.

$$\begin{bmatrix} LM1 & * & * & * & * & * & * & * \\ N_iQ & -I & * & * & * & * & * & * \\ B_iV_j & 0 & -I & * & * & * & * & * \\ Q & 0 & 0 & -\varepsilon\Gamma_3^{-1} & * & * & * & * \\ \rho(B_iV_j)^T & 0 & 0 & 0 & -2\beta Q & * & * & * \\ 0 & 0 & 0 & 0 & 0 & -2\beta I & * & * \\ 0 & 0 & 0 & 0 & \beta I & 0 & LM2 & * \\ 0 & 0 & 0 & 0 & 0 & \beta I & LM3 & \varepsilon^{-1}\Gamma_2 \end{bmatrix} < 0 \tag{21}$$

$$R_i = (P_2 + SC)B_i \tag{22}$$

$$U_iC = (P_2 + SC)N_i \tag{23}$$

$$\begin{aligned} LM1 &= A_iQ + QA_i^T + \rho^2 I \\ LM2 &= (P_2+SC)A_i + A_i^T(P_2+SC)^T - W_iC - (W_iC)^T + \varepsilon^{-1}\Gamma_1 \\ LM3 &= Y_i^T + Z_iC \end{aligned} \tag{24}$$

Then, the state $x(t)$ of the system, the state estimation error $e_x(t)$ and the fault estimation error $e_f(t)$ are bounded. The gains of the observer and the state feedback control law are given by:

$$\begin{aligned} &E = P_2^{-1}S\ ,\ J_i = P_2^{-1}R_i\ ,\ M_i = P_2^{-1}U_i\ , \\ &H_i = (I_n + EC)A_i - P_2^{-1}W_iC\ ,\ \varphi_i = P_2^{-1}Y_i\ , \\ &L_i = P_2^{-1}W_i - H_iE\ ,\ \Psi_i = P_3^{-1}Z_i\ ,\ D_i = V_iQ^{-1} \end{aligned} \tag{25}$$

Proof: In the framework of a FTC by state-feedback, a new procedure is proposed such that the closed-loop system and the convergence of the state and fault estimation errors becomes asymptotically robust stable by using a Lyapunov function-based design approach. Let us consider a Lyapunov function $V(t)$ depending on $x(t)$, $e_x(t)$ and $e_f(t)$ defined by:

$$V(t) = x^T(t)P_1x(t) + e_x^T(t)P_2e_x(t) + e_f^T(t)P_3e_f(t) \tag{26}$$

where P_1, P_2 and P_3 are symmetric and positive definite matrices with appropriate dimensions. Stability condition for the estimation error yields that the time derivative of (26) should be negative definite. The derivative of $V(t)$ with respect to time yields:

$$\dot{V}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))\begin{Bmatrix} \dot{x}^T(t)P_1x(t) + x^T(t)P_1\dot{x}(t) \\ +\dot{e}_x^T(t)P_2e_x(t) + e_x^T(t)P_2\dot{e}_x(t) \\ +\dot{e}_f^T(t)P_3e_f(t) + e_f^T(t)P_3\dot{e}_f(t) \end{Bmatrix} \tag{27}$$

that becomes by using equations (18) and (13):

$$\dot{V}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))\begin{Bmatrix} x^T\Pi_{ij}x + e_x^T\Omega_ie_x + 2x^TP_1G_if \\ -2\rho x^TP_1B_iD_j\cos\hat{\theta}_je_x \\ +2e_x^TP_2\Xi_if + 2e_x^TP_2\varphi_ie_f \\ +2e_f^TP_3\dot{f} + 2e_f^TP_3\Psi_iCe_x \end{Bmatrix} \tag{28}$$

where:

$$\Pi_{ij} = \Phi_{ij}^TP_1 + P_1\Phi_{ij} \tag{29}$$

$$\Omega_i = H_i^TP_2 + P_2H_i \tag{30}$$

By using the Assumption (1) and applying Lemma (2) for three terms of the above inequality (28), it follows that:

$$
\begin{aligned}
2e_x^T P_2 \Xi_i f &\le \varepsilon^{-1} e_x^T \Gamma_1 e_x + \varepsilon f^T \left(\Xi_i^T P_2 \Gamma_1^{-1} P_2 \Xi_i\right) f \\
&\le \varepsilon^{-1} e_x^T \Gamma_1 e_x + \eta^1 \\
2e_f^T P_3 \dot{f} &\le \varepsilon^{-1} e_f^T \Gamma_2 e_f + \varepsilon \dot{f}^T \left(P_3 \Gamma_2^{-1} P_3\right) \dot{f} \\
&\le \varepsilon^{-1} e_f^T \Gamma_2 e_f + \eta^2 \\
2x^T P_1 G_i f &\le \varepsilon^{-1} x^T \Gamma_3 x + \varepsilon f^T \left(G_i^T P_1 \Gamma_3^{-1} P_1 G_i\right) f \\
&\le \varepsilon^{-1} x^T \Gamma_3 x + \eta^3
\end{aligned}
\tag{31}
$$

with:

$$
\begin{aligned}
\eta^1 &= \varepsilon \alpha_1^2 \lambda_{\max}\left(\Xi_i^T P_2 \Gamma_1^{-1} P_2 \Xi_i\right) \\
\eta^2 &= \varepsilon \alpha_2^2 \lambda_{\max}\left(P_3 \Gamma_2^{-1} P_3\right) \\
\eta^3 &= \varepsilon \alpha_1^2 \lambda_{\max}\left(G_i^T P_1 \Gamma_3^{-1} P_1 G_i\right)
\end{aligned}
\tag{32}
$$

Let us define the scalar δ the maximum value of the sum of all constant coefficients obtained in the equalities (32):

$$\delta = \max\left(\eta^1 + \eta^2 + \eta^3\right) \tag{33}$$

Then, the time derivative of the Lyapunov function (28) is bounded as follows:

$$
\dot{V}(t) \le \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(\xi(t)\right) h_j\left(\xi(t)\right)
\left\{
\begin{array}{l}
x^T \Pi_{ij} x + e_x^T \Omega_i e_x \\
-2\rho x^T P_1 B_i D_j \cos\hat{\theta}_j e_x \\
+2e_x^T P_2 \varphi_i e_f + 2e_f^T P_3 \Psi_i C e_x \\
+\varepsilon^{-1} e_x^T \Gamma_1 e_x + \varepsilon^{-1} e_f^T \Gamma_2 e_f \\
+\varepsilon^{-1} x^T \Gamma_3 x + \delta
\end{array}
\right\}
\tag{34}
$$

The above inequality (34) can be reformulated as follows:

$$\dot{V}(t) \le X_a^T(t) \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(\xi(t)\right) h_j\left(\xi(t)\right) \Delta_{ij} X_a(t) + \delta \tag{35}$$

where the augmented vector X_a and the matrix Δ_{ij} are respectively given by:

$$X_a(t) = \begin{bmatrix} x(t) \\ e_x(t) \\ e_f(t) \end{bmatrix} \tag{36}$$

$$
\Delta_{ij} = \begin{bmatrix}
\Pi_{ij} + \varepsilon^{-1}\Gamma_3 & -\rho P_1 B_i D_j \cos\hat{\theta}_j & 0 \\
* & \Omega_i + \varepsilon^{-1}\Gamma_1 & P_2 \varphi_i + \left(\Psi_i C\right)^T P_3 \\
* & * & \varepsilon^{-1}\Gamma_2
\end{bmatrix}
\tag{37}
$$

Let us define the positive scalar τ

$$\tau = \min_{t>0} \lambda_{\min}\left(-\sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(\xi(t)\right) h_j\left(\xi(t)\right) \Delta_{ij}\right) \tag{38}$$

which can be also bounded by:

$$\tau \le \min_{i,j} \lambda_{\min}\left(-\Delta_{ij}\right) \tag{39}$$

If the following inequality holds:

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(\xi(t)\right) h_j\left(\xi(t)\right) \Delta_{ij} < 0 \tag{40}$$

Then, we can obtain that

$$\dot{V}(t) \le -\tau \left\| X_a(t) \right\|^2 + \delta \tag{41}$$

It follows that $\dot{V}(t) < 0$ if $\tau \left\| X_a(t) \right\|^2 > \delta$, $\forall t > 0$. Then, we can deduce that the state $x(t)$, the state estimation error $e_x(t)$ and the fault estimation error $e_f(t)$ converge to a small set according to Lyapunov stability theory and lie in it. This set is smaller as the constant δ converges to zero.

Let us introduce the following notations for the sake of simplicity:

$$Y_\xi = \sum_{i=1}^{r} h_i\left(\xi(t)\right) Y_i \tag{42}$$

$$Y_{\xi\xi} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(\xi(t)\right) h_j\left(\xi(t)\right) Y_{ij} \tag{43}$$

where Y_i and Y_{ij} are given matrices. By using these notations, the inequality (40) becomes:

$$\Delta_{\xi\xi} = \begin{bmatrix} \Theta_{\xi\xi}^1 & \Theta_{\xi\xi}^2 \\ * & \Theta_{\xi\xi}^4 \end{bmatrix} < 0 \tag{44}$$

with:

$$\Theta_{\xi\xi}^1 = \Pi_{ij} + \varepsilon^{-1}\Gamma_3 \tag{45}$$

$$\Theta_{\xi\xi}^2 = \begin{bmatrix} -\rho P_1 B_i D_j \cos\hat{\theta}_j & 0 \end{bmatrix} \tag{46}$$

$$\Theta_{\xi\xi}^4 = \begin{bmatrix} \Omega_i + \varepsilon^{-1}\Gamma_1 & P_2 \varphi_i + \left(\Psi_i C\right)^T P_3 \\ * & \varepsilon^{-1}\Gamma_2 \end{bmatrix} \tag{47}$$

Let us consider a symmetric matrix χ defined as:

$$\chi = \begin{bmatrix} P_1^{-1} & 0 \\ 0 & \chi_1 \end{bmatrix}, \quad \chi_1 = \begin{bmatrix} P_1^{-1} & 0 \\ 0 & I \end{bmatrix} \tag{48}$$

By using Lemma (3), and post and pre-multiplying the inequality (44) by χ, we can obtain that:

$$\begin{bmatrix} P_1^{-1}\Theta_{\xi\xi}^1 P_1^{-1} & P_1^{-1}\Theta_{\xi\xi}^2\chi_1 \\ * & \chi_1\Theta_{\xi\xi}^4\chi_1 \end{bmatrix} < 0 \tag{49}$$

The term $\chi_1\Theta_{\xi\xi}^4\chi_1$ can be replaced by considering the following inequality which holds for any scalar β such that

$$\left(\chi_1 + \beta\left(\Theta_{\xi\xi}^4\right)^{-1}\right)^T \Theta_{\xi\xi}^4 \left(\chi_1 + \beta\left(\Theta_{\xi\xi}^4\right)^{-1}\right) \le 0 \tag{50}$$

$$\Leftrightarrow \chi_1\Theta_{\xi\xi}^4\chi_1 \le -2\beta\chi_1 - \beta^2\left(\Theta_{\xi\xi}^4\right)^{-1} \tag{51}$$

Then from the above inequality (51) and with the Schur Complement, it follows that the inequality (49) holds if:

$$\begin{bmatrix} P_1^{-1}\Theta_{\xi\xi}^1 P_1^{-1} & P_1^{-1}\Theta_{\xi\xi}^2\chi_1 & 0 \\ * & -2\beta\chi_1 & \beta I \\ * & * & \Theta_{\xi\xi}^4 \end{bmatrix} < 0 \tag{52}$$

Now, the term $P_1^{-1}\Theta_{\xi\xi}^1 P_1^{-1}$ of (52) can be rewritten as:

$$\begin{aligned} P_1^{-1}\Theta_{\xi\xi}^1 P_1^{-1} &= P_1^{-1}A_i^T + A_i P_1^{-1} + \rho N_i P_1^{-1}\sin\hat{\theta}_j + \rho P_1^{-1}N_i^T\sin\hat{\theta}_j \\ &+ \rho B_i D_j P_1^{-1}\cos\hat{\theta}_j + \rho P_1^{-1}\left(B_i D_j\right)^T\cos\hat{\theta}_j + \varepsilon^{-1}P_1^{-1}\Gamma_3 P_1^{-1} \end{aligned} \tag{53}$$

Applying lemma (1), the previous equation can be rewritten as:

$$\begin{aligned} P_1^{-1}\Theta_{\xi\xi}^1 P_1^{-1} &\le P_1^{-1}A_i^T + A_i P_1^{-1} + P_1^{-1}N_i^T N_i P_1^{-1} + \rho^2 I \\ &+ P_1^{-1}\left(B_i D_j\right)^T B_i D_j P_1^{-1} + \varepsilon^{-1}P_1^{-1}\Gamma_3 P_1^{-1} \end{aligned} \tag{54}$$

From the inequality (54) and the matrices (46), (47), using the notations (42), (43), replacing H_i by (19) and $T = I_n + EC$ and by introducing the following variable changes:

$$\begin{aligned} &Q = P_1^{-1},\ D_j = V_j Q,\ W_i = P_2 K_i,\ Y_i = P_2\varphi_i \\ &R_i = P_2 J_i,\ U_i = P_2 M_i,\ Z_i = P_3\Psi_i,\ S = P_2 E \end{aligned} \tag{55}$$

we can obtain the inequalities given in theorem (1) by applying the Schur's complement [20] to (52) under equality constraints (22), (23). This completes the proof of the theorem.

This section is devoted to the design of a BPI observer-based control for T-S FBM. The system parameters used are known but in reality the system parameters may either be uncertain or time-dependent. Moreover, the problem of stabilization remains a key problem in the study of uncertain T-S fuzzy control systems and their controller designs. The study of BPI observer-based control for T-S FBM with parametric uncertainties will be subject the next section.

V. Robust Control of the Fuzzy Bilinear Model with Uncertainties

In this section, we shall consider a uncertain nonlinear system which can be described by the following T-S fuzzy bilinear model with actuator faults:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(\xi(t))\begin{pmatrix} \left(A_i + \Delta A_i\right)x(t) + \left(B_i + \Delta B_i\right)u(t) \\ +\left(N_i + \Delta N_i\right)x(t)u(t) + G_i f(t) \end{pmatrix} \\ y(t) = Cx(t) \end{cases} \tag{56}$$

The parameter uncertainties considered here are norm-bounded and presented by the form

$$\begin{bmatrix} \Delta A_i & \Delta B_i & \Delta N_i \end{bmatrix} = \hbar_i F_i(t)\begin{bmatrix} E_{1i} & E_{2i} & E_{3i} \end{bmatrix} \tag{57}$$

where $\hbar_i$, E_{1i}, E_{2i}, E_{3i} are known real constant matrices of appropriate dimensions, and $F_i(t)$ is an unknown matrix function satisfying $F_i^T(t)F_i(t) \le I$, in which I is the identity matrix of appropriate dimension.

The feedback controller and the state of BPI observer for T-S FBM with parametric uncertainties (56) is the same as that for (6).

In order to describe the dynamic of BPI observer (8), the state and fault estimation errors is defined by (9), (10). Then, from (56) and (8), we have:

$$\dot{e}_x(t) = \sum_{i=1}^{r} h_i(\xi(t))\begin{pmatrix} H_i e_x(t) + \left(TA_i - L_i C - H_i T\right)x(t) \\ +\left(TN_i - M_i C\right)x(t)u(t) + \left(TB_i - J_i\right)u(t) \\ +T\Delta A_i x(t) + T\Delta B_i u(t) + T\Delta N_i x(t)u(t) \\ +\left(TG_i - \varphi_i\right)f(t) + \varphi_i e_f(t) \end{pmatrix} \tag{58}$$

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))\begin{pmatrix} \left(\Phi_{ij} + \Delta\Phi_{ij}\right)x(t) + G_i f(t) \\ -\rho\left(B_i + \Delta B_i\right)D_j\cos\hat{\theta}_j e_x(t) \end{pmatrix} \tag{59}$$

where:

$$\Phi_{ij} = A_i + \rho N_i\sin\hat{\theta}_j + \rho B_i D_j\cos\hat{\theta}_j \tag{60}$$

$$\Delta\Phi_{ij} = \Delta A_i + \rho\Delta N_i\sin\hat{\theta}_j + \rho\Delta B_i D_j\cos\hat{\theta}_j \tag{61}$$

If the conditions (15), (16), (17) are satisfied $\forall i \in \{1, 2, \cdots, r\}$, we get:

$$\dot{e}_x(t) = \sum_{i=1}^{r} h_i(\xi(t))\begin{pmatrix} H_i e_x(t) + T\Delta A_i x(t) + T\Delta B_i u(t) \\ +T\Delta N_i x(t)u(t) \\ +\left(TG_i - \varphi_i\right)f(t) + \varphi_i e_f(t) \end{pmatrix} \tag{62}$$

By using (7), we obtain:

$$\dot{e}_x(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t)) h_j(\xi(t)) \begin{pmatrix} \Xi_i^1 e_x(t) + \Xi_i^2 f(t) \\ +\Xi_i^3 x(t) + \varphi_i e_f(t) \end{pmatrix} \tag{63}$$

where :

$$\Xi_i^1 = H_i - \rho T \Delta B_i D_j \cos\hat{\theta}_j \tag{64}$$

$$\Xi_i^2 = TG_i - \varphi_i \tag{65}$$

$$\Xi_i^3 = T\Delta A_i + \rho T \Delta B_i D_j \cos\hat{\theta}_j + \rho T \Delta N_i \sin\hat{\theta}_j \tag{66}$$

Hereafter, we give the following theorem which presents sufficient conditions such that the closed-loop fuzzy system (59) is globally asymptotically stable in the presence of actuator faults and which gives the gains of the observer and the state feedback control law.

Theorem 2: If there exist symmetric positive definite matrices Q , Γ_1 , Γ_2 , Γ_3 , P_1 *and* P_2, matrices W_i , V_i Y_i , Z_i , R_i , U_i , S and positive scalars ε , β , ρ , $\mu > 0$ such that LMIs (67) subject to linear equality constraints (68), (69) are satisfied $\forall i = 1\cdots r$, $\forall j = 1\cdots r$.

$$\begin{bmatrix} LM1 & * & * & * & * & * & ** & * & * & * & * & * \\ N_iQ & -I & * & * & * & * & * & * & * & * & * & * \\ B_iV_j & 0 & -I & * & * & * & * & * & * & * & * & * \\ Q & 0 & 0 & -\varepsilon\Gamma_3^{-1} & * & * & * & * & * & * & * & * \\ E_{1i}Q & 0 & 0 & 0 & -\mu I & * & * & * & * & * & * & * \\ E_{2i}V_j & 0 & 0 & 0 & 0 & -\mu I & * & * & * & * & * & * \\ E_{3i}Q & 0 & 0 & 0 & 0 & 0 & -\mu I & * & * & * & * & * \\ LM4 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu I & * & * & * & * \\ E_{2i}V_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu^{-1}I & * & * & * \\ LM5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu^{-1}I & * & * \\ \rho(B_iV_j) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & LM6 & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & LM7 \end{bmatrix} < 0 \tag{67}$$

$$R_i = (P_2 + SC) B_i \tag{68}$$

$$U_i C = (P_2 + SC) N_i \tag{69}$$

where:

$$LM1 = A_i Q + Q A_i^T + \rho^2 I$$
$$LM2 = (P_2 + SC) A_i + A_i^T (P_2 + SC)^T - W_i C - (W_i C)^T + \varepsilon^{-1}\Gamma_1$$
$$LM3 = Y_i^T + Z_i C$$
$$LM4 = \rho \hbar_i^T (I + EC)^T$$
$$LM5 = \sqrt{(1+2\rho^2)}\,\hbar_i^T (I + EC)^T$$

$$LM6 = \begin{bmatrix} \mu^{-1}E_{1i}^T E_{1i} & * & * & * & * & * \\ E_{2i}D_j & -\mu I & * & * & * & * \\ E_{3i} & 0 & -\mu I & * & * & * \\ \rho\hbar_i^T & 0 & 0 & -\mu I & * & * \\ E_{2i}D_j & 0 & 0 & 0 & -\mu^{-1}I & * \\ \sqrt{(1+2\rho^2)}\,\hbar_i^T & 0 & 0 & 0 & 0 & -\mu^{-1}I \end{bmatrix}$$

$$LM7 = \begin{bmatrix} -2\beta\chi_1 & * \\ LM3 & \wedge \end{bmatrix}$$

$$\wedge = \begin{bmatrix} LM2 & * \\ LM3 & \varepsilon^{-1}\Gamma_2 \end{bmatrix}$$

Then, the state $x(t)$ of the system, the state estimation error $e_x(t)$ and the fault estimation error $e_f(t)$ are bounded. The gains of the observer and the state feedback control law are given by:

$$\begin{aligned} &E = P_2^{-1}S \ , J_i = P_2^{-1}R_i \ , M_i = P_2^{-1}U_i \ , \\ &H_i = (I_n + EC)A_i - P_2^{-1}W_i C \ , \varphi_i = P_2^{-1}Y_i \ , \\ &L_i = P_2^{-1}W_i - H_i E \ , \Psi_i = P_3^{-1}Z_i \ , D_i = V_i Q^{-1} \end{aligned} \tag{71}$$

Proof: In order to prove the stability of the closed-loop system and the convergence of the state and fault estimation errors, sufficient conditions are derived using Lyapunov function (26). Then, the derivative of $V(t)$ with respect to time yields:

$$\dot{V}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t)) h_j(\xi(t)) \left\{ \begin{aligned} &x^T(\Pi_{ij} + \Delta\Pi_{ij})x + e_x^T \bar{\Omega}_{ij} e_x \\ &+2x^T P_1 G_i f + 2e_x^T P_2 \varphi_i e_f \\ &-2\rho x^T P_1 (B_i + \Delta B_i) D_j \cos\hat{\theta}_j e_x \\ &+2e_x^T P_2 \Xi_i^2 f + 2e_x^T P_2 \Xi_i^3 x \\ &+2e_f^T P_3 \dot{f} + 2e_f^T P_3 \Psi_i C e_x \end{aligned} \right\} \tag{72}$$

where:

$$\Pi_{ij} = P_1 \Phi_{ij} + \Phi_{ij}^T P_1 \tag{73}$$

$$\Delta\Pi_{ij} = P_1 \Delta\Phi_{ij} + \Delta\Phi_{ij}^T P_1$$
(74)

$$\begin{aligned} \bar{\Omega}_{ij} = &\left(H_i^T - \rho (T\Delta B_i D_j)^T \cos\hat{\theta}_j\right) P_2 \\ &+ P_2 \left(H_i - \rho T \Delta B_i D_j \cos\hat{\theta}_j\right) \end{aligned} \tag{75}$$

By using (31), the time derivative of $V(t)$ (72) is bounded as follows:

$$\dot{V}(t) \le \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(\xi(t)\right)h_j\left(\xi(t)\right)\begin{Bmatrix} x^T\left(\Pi_{ij}+\Delta\Pi_{ij}\right)x + e_x^T\bar{\Omega}_{ij}e_x \\ -2\rho x^T P_1\left(B_i+\Delta B_i\right)D_j\cos\hat{\theta}_j e_x \\ +2e_x^T P_2\varphi_i e_f + 2e_x^T P_2 \Xi_i^3 x \\ +2e_f^T P_3 \Psi_i C e_x + \varepsilon^{-1}e_x^T\Gamma_1 e_x \\ +\varepsilon^{-1}e_f^T\Gamma_2 e_f + \varepsilon^{-1}x^T\Gamma_3 x + \delta \end{Bmatrix}$$
(76)

where δ is defined by (33).

The above inequality (76) can be reformulated as follows:

$$\dot{V}(t) \le X_a^T(t)\sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(\xi(t)\right)h_j\left(\xi(t)\right)\bar{\Delta}_{ij}X_a(t) + \delta \tag{77}$$

where the augmented vector X_a is defined by (36) and the matrix $\bar{\Delta}_{ij}$ is given by:

$$\bar{\Delta}_{ij} = \begin{bmatrix} \bar{\Delta}_{ij}^1 & \bar{\Delta}_{ij}^2 & 0 \\ * & \bar{\Omega}_{ij}+\varepsilon^{-1}\Gamma_1 & \bar{\Delta}_{ij}^3 \\ * & * & \varepsilon^{-1}\Gamma_2 \end{bmatrix} \tag{78}$$

with

$$\bar{\Delta}_{ij}^1 = \Pi_{ij} + \Delta\Pi_{ij} + \varepsilon^{-1}\Gamma_3 \tag{79}$$

$$\bar{\Delta}_{ij}^2 = -\rho P_1\left(B_i+\Delta B_i\right)D_j\cos\hat{\theta}_j + \left(\Xi_i^3\right)^T P_2$$
(80)

$$\bar{\Delta}_{ij}^3 = P_2\varphi_i + \left(\Psi_i C\right)^T P_3 \tag{81}$$

From (91), we have:

$$\bar{\Delta}_{ij} = \Delta_{ij} + \tilde{\Delta}_{ij} \tag{82}$$

where Δ_{ij} is defined by (37) and :

$$\tilde{\Delta}_{ij} = \begin{bmatrix} \tilde{\Delta}_{1ij} & \tilde{\Delta}_{3ij} & 0 \\ * & \tilde{\Delta}_{2ij} & 0 \\ * & * & 0 \end{bmatrix}$$
(83)

with:

$$\begin{aligned} \tilde{\Delta}_{1ij} &= \Delta\Pi_{ij} \\ \tilde{\Delta}_{2ij} &= \rho P_2 T\Delta B_i D_j\cos\hat{\theta}_j + \rho\left(T\Delta B_i D_j\right)^T P_2\cos\hat{\theta}_j \\ \tilde{\Delta}_{3ij} &= -\rho P_1\Delta B_i D_j\cos\hat{\theta}_j + \left(\Xi_i^3\right)^T P_2 \end{aligned} \tag{84}$$

Multiplying the previous equations (84) on the left and right by P_1^{-1} and P_2^{-1}, we can rewrite (84) as:

$$\begin{aligned} \tilde{\Delta}_{1ij} &= \Delta A_i P_1^{-1} + \rho\Delta N_i P_1^{-1}\sin\hat{\theta}_j + \rho\Delta B_i D_j P_1^{-1}\cos\hat{\theta}_j \\ &+ P_1^{-1}\Delta A_i^T + \rho P_1^{-1}\Delta N_i^T\sin\hat{\theta}_j + \rho P_1^{-1}\left(\Delta B_i D_j\right)^T\cos\hat{\theta}_j \\ \tilde{\Delta}_{2ij} &= \rho T\Delta B_i D_j P_2^{-1}\cos\hat{\theta}_j + \rho P_2^{-1}\left(T\Delta B_i D_j\right)^T\cos\hat{\theta}_j \\ \tilde{\Delta}_{3ij} &= -\rho\Delta B_i D_j P_1^{-1}\cos\hat{\theta}_j + P_2^{-1}\left(T\Delta A_i\right)^T \\ &+ \rho P_2^{-1}\left(T\Delta B_i D_j\right)^T\cos\hat{\theta}_j + \rho P_2^{-1}\left(T\Delta N_i\right)^T\sin\hat{\theta}_j \end{aligned} \tag{85}$$

The equation $\tilde{\Delta}_{ij}$ can be reformulated as follows:

$$\tilde{\Delta}_{ij} = \tilde{\tilde{\Delta}}_{ij}^1 + \tilde{\tilde{\Delta}}_{ij}^2 + \tilde{\tilde{\Delta}}_{ij}^3 \tag{86}$$

where:

$$\tilde{\tilde{\Delta}}_{ij}^1 = \begin{bmatrix} 0 & \tilde{\tilde{\Delta}}_{ij}^{11} & 0 \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix}, \tilde{\tilde{\Delta}}_{ij}^2 = \begin{bmatrix} 0 & \tilde{\tilde{\Delta}}_{ij}^{21} & 0 \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix}, \quad \tilde{\tilde{\Delta}}_{ij}^3 = \begin{bmatrix} \tilde{\Delta}_{1ij} & 0 & 0 \\ * & \tilde{\Delta}_{2ij} & 0 \\ * & * & 0 \end{bmatrix} \tag{87}$$

$$\begin{aligned} \tilde{\tilde{\Delta}}_{ij}^{11} &= P_2^{-1}\left(T\Delta A_i\right)^T + \rho P_2^{-1}\left(T\Delta B_i D_j\right)^T\cos\hat{\theta}_j \\ &\quad + \rho P_2^{-1}\left(T\Delta N_i\right)^T\sin\hat{\theta}_j \\ \tilde{\tilde{\Delta}}_{ij}^{21} &= -\rho\Delta B_i D_j P_1^{-1}\cos\hat{\theta}_j \end{aligned} \tag{88}$$

Using the uncertainties structure and by using the separation lemma 1 [21], we obtain

$$\tilde{\Delta}_{ij} = \begin{bmatrix} \tilde{T}_{ij}^1 & 0 & 0 \\ * & \tilde{T}_{ij}^2 & 0 \\ * & * & 0 \end{bmatrix} \tag{89}$$

where:

$$\begin{aligned} \tilde{T}_{ij}^1 &= \left(1+2\rho^2\right)\mu T\hbar_i F_i F_i^T\hbar_i^T T^T + \mu P_1^{-1}D_j^T E_{2i}^T E_{2i}D_j P_1^{-1} \\ &\quad + \mu^{-1}P_1^{-1}E_{1i}^T E_{1i}P_1^{-1} + \mu^{-1}P_1^{-1}D_j^T E_{2i}^T E_{2i}D_j P_1^{-1} \\ &\quad + \mu^{-1}P_1^{-1}E_{3i}^T E_{3i}P_1^{-1} + \rho^2\mu^{-1}T\hbar_i F_i F_i^T\hbar_i^T T^T \\ \tilde{T}_{ij}^2 &= \mu^{-1}P_2^{-1}E_{1i}^T E_{1i}P_2^{-1} + \mu^{-1}P_2^{-1}E_{3i}^T E_{3i}P_2^{-1} \\ &\quad + \mu^{-1}P_2^{-1}D_j^T E_{2i}^T E_{2i}D_j P_2^{-1} + \mu^{-1}\rho^2\hbar_i F_i F_i^T\hbar_i^T \\ &\quad + \left(1+2\rho^2\right)\mu\hbar_i F_i F_i^T\hbar_i^T + \mu P_2^{-1}D_j^T E_{2i}^T E_{2i}D_j P_2^{-1} \end{aligned} \tag{90}$$

From (82) and (89), we have:

$$\begin{bmatrix} \Theta_{\xi\xi}^1 & \Theta_{\xi\xi}^2 \\ * & \Theta_{\xi\xi}^4 \end{bmatrix} + \begin{bmatrix} \tilde{\Theta}_{\xi\xi}^1 & \tilde{\Theta}_{\xi\xi}^2 \\ * & \tilde{\Theta}_{\xi\xi}^4 \end{bmatrix} < 0 \tag{91}$$

where Θ_{ij}^1 , Θ_{ij}^2 , Θ_{ij}^4 are defined by (45)-(47) respectively and $\tilde{\Theta}_{ij}^1$, $\tilde{\Theta}_{ij}^2$, $\tilde{\Theta}_{ij}^4$ are given by:

$$\begin{aligned}&\tilde{\Theta}_{ij}^{1}=\tilde{T}_{ij}^{1}\\&\tilde{\Theta}_{ij}^{2}=\begin{bmatrix}0 & 0\end{bmatrix}\\&\tilde{\Theta}_{ij}^{4}=\begin{bmatrix}\tilde{T}_{ij}^{2} & 0\\ * & 0\end{bmatrix}\end{aligned} \tag{92}$$

Using (49), (51) and with the Schur Complement, it follows that the inequality (91) holds if:

$$\begin{bmatrix}P_1^{-1}\Theta_{\xi\xi}^{1}P_1^{-1}+\tilde{\Theta}_{\xi\xi}^{1} & P_1^{-1}\Theta_{\xi\xi}^{2}\chi_1+\tilde{\Theta}_{\xi\xi}^{2} & 0 & 0\\ * & \tilde{\Theta}_{\xi\xi}^{4} & 0 & 0\\ * & * & -2\beta\chi_1 & \beta I\\ * & * & * & \Theta_{\xi\xi}^{4}\end{bmatrix}<0 \tag{93}$$

We calculate each terms of inequality (93) and by considering the variable changes (55), we can obtain the inequalities given in theorem (2) by applying the Schur's complement [20] to (93) under equality constraints (68), (69). This completes the proof of the theorem.

VI. Numerical Example

To illustrate the effectiveness of the proposed method, an example is considered. The dynamical FBM is given by the following data:

$$A_1=\begin{bmatrix}-2 & 1 & 0\\ 1 & -3 & 0\\ 2 & 1 & -8\end{bmatrix},\ B_1=\begin{bmatrix}5\\ 1.2\\ 2.5\end{bmatrix},\ N_1=\begin{bmatrix}-2 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{bmatrix},\ G_1=\begin{bmatrix}1\\ 0\\ 1\end{bmatrix}$$

$$A_2=\begin{bmatrix}-2 & 0 & 1\\ 0.5 & -2 & 0\\ 2 & 1 & -1\end{bmatrix},\ B_2=\begin{bmatrix}4\\ 2\\ 0\end{bmatrix},\ N_2=\begin{bmatrix}-2 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{bmatrix},\ G_1=\begin{bmatrix}1\\ 0\\ 1\end{bmatrix}$$

$$C=\begin{bmatrix}1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$$

Uncertainties are also defined by the following matrices:

$$\hbar_1=\begin{bmatrix}0.1 & 0 & 0\\ 0 & 0 & 0.1\\ 0 & 0.1 & 0\end{bmatrix},\ E_{11}=\begin{bmatrix}0 & 0.5 & 0\\ 0 & 0 & 0.3\\ 0 & 0 & 0\end{bmatrix},\ E_{21}=\begin{bmatrix}1\\ 0.5\\ 0\end{bmatrix},\ E_{31}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$\hbar_2=\begin{bmatrix}0.1 & 0 & 0\\ 0 & 0 & 0.1\\ 0 & 0.1 & 0\end{bmatrix},\ E_{12}=\begin{bmatrix}0 & 0.5 & 0\\ 0 & 0 & 0.6\\ 0 & 0 & 0\end{bmatrix},\ E_{22}=\begin{bmatrix}1\\ 0.5\\ 0\end{bmatrix},\ E_{32}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

The weighting functions are defined by:

$$h_1(x(t))=\exp\left(\frac{1}{2}\left(\frac{x_1+5}{2}\right)^2\right)\ ,\ h_2(x(t))=1-h_1(x(t))$$

The additive actuator fault signal is defined as follows :

$$f(t)=\begin{cases}0 & t<1s\\ 0.5\sin(0.5\pi) & 1s\le t<7s\\ 0.3 & 7s\le t<9s\\ 0 & t>9s\end{cases}$$

Thus, the resolution of the conditions of theorem 2 leads to the following controller and observer gain matrices respectively with $\rho=0.05$

$$D_1=\begin{bmatrix}3.343 & -0.098 & 1.254\end{bmatrix},\ D_2=\begin{bmatrix}-0.041 & -1.008 & -0.254\end{bmatrix}$$

$$E=\begin{bmatrix}9.022 & -4.533\\ 5.976 & -8.965\\ 2.046 & -3.569\end{bmatrix},\ M_1=M_2=\begin{bmatrix}9.022 & 0\\ 5.976 & 0\\ 3.046 & 0\end{bmatrix}$$

$$L_1=10^3*\begin{bmatrix}0.688 & -1.056\\ -0.399 & 0.584\\ -1.567 & 2.344\end{bmatrix},\ L_2=\begin{bmatrix}-504.851 & 752.266\\ 656.529 & -987.282\\ 165.730 & -249.618\end{bmatrix}$$

$$J_1=\begin{bmatrix}-18.166 & -9.956 & -4.461\end{bmatrix}^T,\ J_2=\begin{bmatrix}-47.110 & -27.882 & -13.229\end{bmatrix}^T$$

$$H_1=\begin{bmatrix}-51.288 & -10.218 & -77.669\\ 29.304 & 31.204 & -32.793\\ 112.873 & 80.082 & 15.084\end{bmatrix},\ H_2=\begin{bmatrix}23.644 & 46.592 & -0.148\\ -53.635 & -17.602 & -25.086\\ -14.954 & -1.058 & -9.543\end{bmatrix}$$

$$\varphi_1=\varphi_2=\begin{bmatrix}4.511 & 2.988 & 1.523\end{bmatrix}^T$$

$$\Psi_1=\Psi_2=\begin{bmatrix}-1.523 & -2.988\end{bmatrix}$$

The simulation results are shown in figures (1)-(4). The time-varying actuator fault and its estimate are depicted on Figure 1. It appears clearly the good estimation of this actuator fault. Furthermore, the BPI observer provides the fault estimation which errors are illustrated in the figures 2. One can shows the fault estimation error is zero-mean except at time 1s, 7s and 9s that suits abrupt changes of the fault.

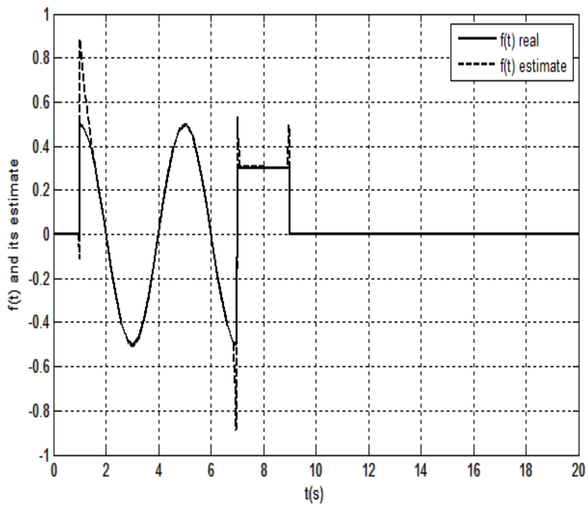

Fig 1. Evolution of the fault and its estimate

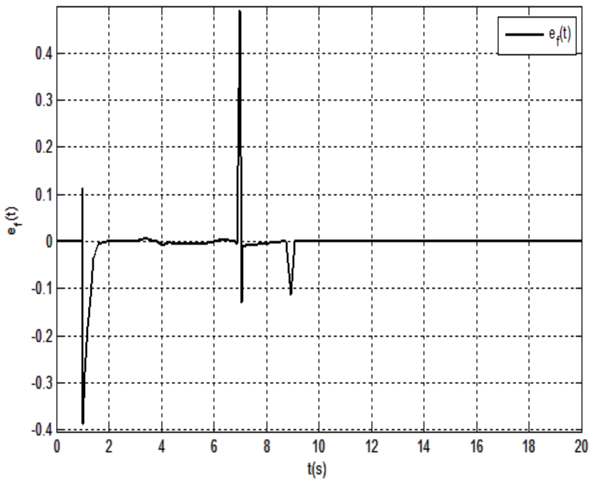

Fig 2. Fault estimation error

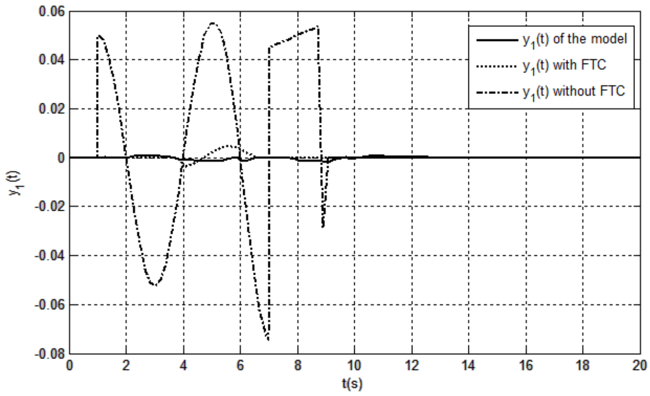

Fig 3. The output $y_1(t)$ of the system: nominal output, output without FTC and output with FTC

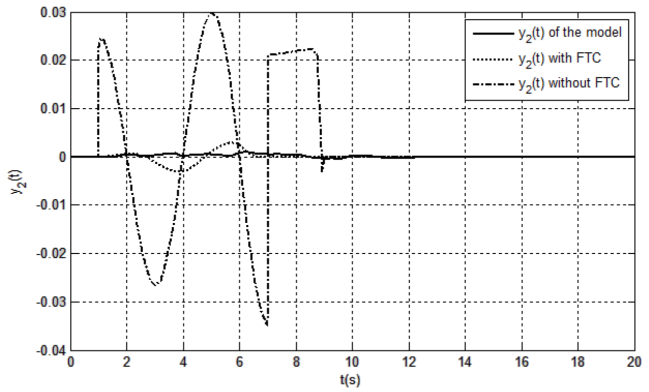

Fig 4. The output $y_2(t)$ of the system: nominal output, output without FTC and output with FTC

The figures (3-4) compares the outputs of the nominal model, the outputs of the faulty system without FTC and the outputs with FTC. One can see that the trajectory of the system follows the trajectory of the reference model, even in fault case. It is clear that the proposed strategy is robust with respect to the actuator additive fault f(t) and with parametric uncertainties. Moreover, the BPI observer-based FTC scheme provides good results in the presence of norm-bounded parametric uncertainties, as shown in the figures.

VII. Conclusion

This paper proposes a state feedback fault tolerant control law methodology for T-S fuzzy bilinear systems. A T-S fuzzy BPI observer is designed for the proposed FTC strategy in order to simultaneously estimate time varying faults and state variables. This method aims to adapt the control law in order to compensate the effect of the time-varying faults by maintaining the stability of the system. The stability conditions of the designed observer and the state-feedback control has been provided and solved through a set of Linear Matrix Inequalities under equality constraints. Moreover, sufficient conditions for controller design based on state estimation for robust stabilization of T-S FBM with parametric uncertainties has been proposed. Finally, the performance of the proposed BPI observer based FTC approach has been given to an example in order to illustrate the validity of this method.

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