Stability Analysis of Inverted Pendulum Using PID and Fuzzy Controller

Saurabh Chauhan, Rakesh Vishwakarma, Vidushi Singh, Rajat Kumar

Abstract— **An inverted pendulum, usually mounted on a cart actuated by an applied force, was designed and built for existing hydraulically driven sled or cart. Inverted Pendulum is non linear and unstable system. The aim of this study is to stabilize the Inverted Pendulum such that position of the cart on is controlled quickly and accurately so that pendulum is always maintained erected in its upright (inverted) position.**

To stable the Inverted Pendulum system a controller has to be designed. To design a controller for the system, first the transfer function of the system is obtained by mathematical modelling of the system. Since the system is non linear, it is linearized by assumption that it is operated in small region. The motion of the system is restricted to one dimension only. In analysis of uncompensated open loop system it is found that system is inherently unstable. For designing PID controller, SISO tool is used to find out the values of P, I and D. System is tested with impulse and step input.

Fuzzy system is a rule based system. IF-THEN rules are formed to design the fuzzy controller for the system. In this paper, a fuzzy controller for inverted pendulum has two inputs (angle between the pendulum and vertical position and the angular velocity) and output is force. Mamdani fuzzy inference system is used to implement the Inverted Pendulum system in the Fuzzy Logic Toolbox. System is analysed with impulse and step inputs.

Index Terms—**PID, Fuzzy Controller, SISO.**

I. INTRODUCTION

Inverted Pendulum is an inherently unstable system as its mass is above the pivoted point which is mounted on a cart. Pendulum is stable while hanging downwards but the inverted pendulum is unstable. It has one input signal (force applied to the cart) and various output signals (cart position, cart velocity, pendulum angle and pendulum angular velocity).

To balance the Inverted Pendulum some external force must be applied to the system. This external force can be applied by applying torque at the pivoted point, moving the cart horizontally or oscillating the support rapidly up and down. The Carriage Balanced Inverted Pendulum (CBIP) system as shown in Fig.1, it provides the controlling force by means of a DC servo-motor through a pulley belt system.

Saurabh Chauhan, Electrical and Electronics, College of Engineering Roorkee, Roorkee, India, 8267930516.

Rakesh Vishwakarma, Electrical and Electronics, College of Engineering Roorkee, Roorkee, India, 8439187435.

Vidushi Singh, Electrical and Electronics, College of Engineering Roorkee, Roorkee, India, 7895131886.

Rajat Kumar, Electrical and Electronics, College of Engineering Roorkee, Roorkee, India, 8430310162.

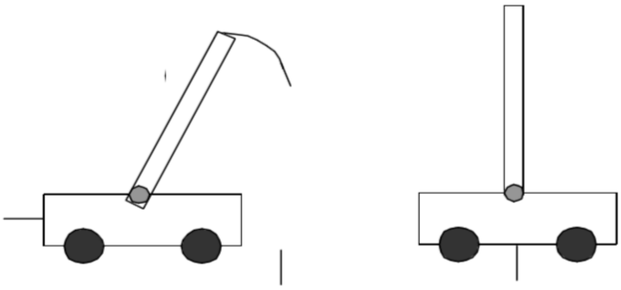

Fig.1: Carriage Balanced Inverted Pendulum

The inverted pendulum can be controlled by using the various controllers like Proportional Integral Derivative (PID), Linear Quadratic Regulator (LQR), Fuzzy Logic Controller (FLC), Artificial Neural Network Controller (ANN). This paper focus on the PID controller and Fuzzy Logic Controller technique.

Inverted Pendulum has various applications like it used in Humanoid robots, Robotic arms, Segway scooters, Robotic wheelchairs, Rockets.

II. MODELLING OF THE SYSTEM

A. System Description

The inverted pendulum is mounted on a moving cart whose motion is restricted to one dimension only. A servomotor is used for controlling the translation motion of the cart through pulley mechanism. Friction is neglected by taking the coefficient of friction equals to zero. Linearized approximated transfer function is obtained by restricting the motion of the system in small region.

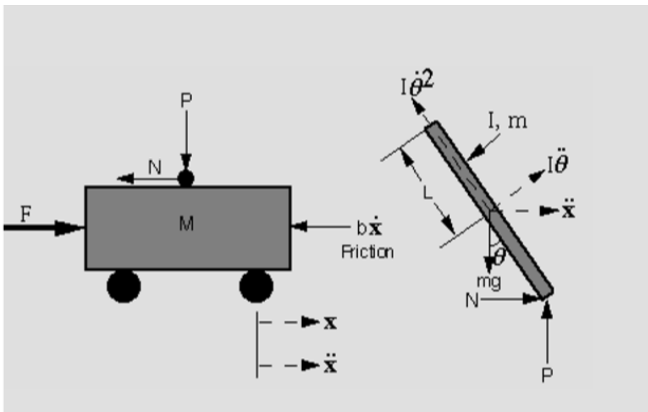

Fig.2: Free body Diagram of Inverted Pendulum

B. Transfer Function

Transfer function of the system is given by:

$$\frac{U(s)}{E(s)} = K\frac{s}{(t_m s+1)\left(\frac{s^2}{A_p^2}-1\right)}$$

Where K= $K_F K_P K_M r(M+m)$

E(s) =Error voltage

$\phi(s)$ = Angular Position of the Pendulum

And $A_p = \pm\sqrt{\frac{(M+m)mgl}{(M+m)(1+ml^2)-(ml)^2}}$

III. Analysis of Uncompensated System

Position of poles and zeros is shown in Fig.3. One pole of the system is in the right half of s-plane which makes it unstable. Thus the system is absolutely unstable.

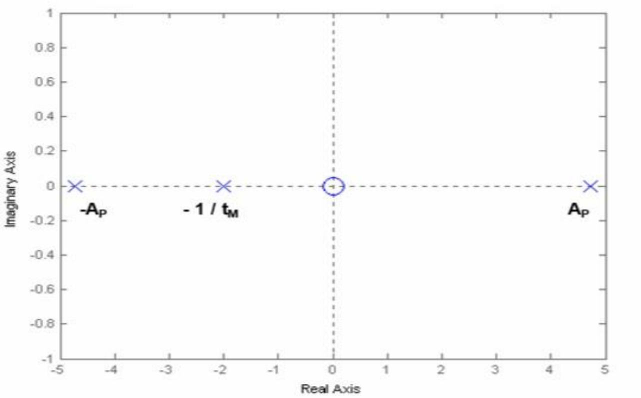

Fig.3: Poles and zero map of uncompensated system

The system is subjected to impulse and step input. Impulse represents that the force is applied for short duration of time and step represents that the force is applied continuously to the system. For both the inputs the response of the system is observed. Y-axis represents the angle in radians and X-axis time in seconds.

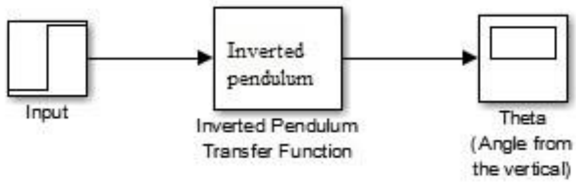

Fig.4: Uncompensated system

A. Impulse response

Impulse response of the system is shown in Fig.5. When a force is applied for a very short duration the pendulum falls downward as angle from vertical position diverges very rapidly. This indicates that the system is highly unstable.

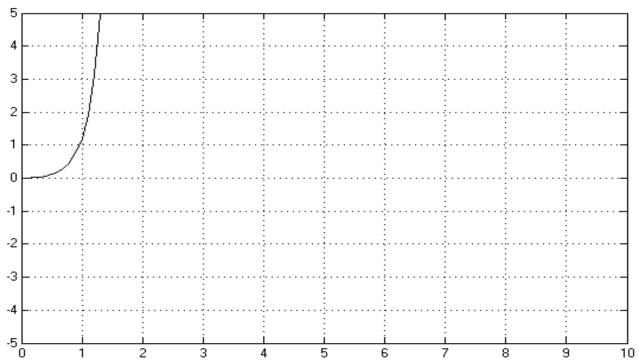

Fig.5: Impulse Response of Open Loop Uncompensated System

B. Step response

Step response of the system is shown in Fig.6. Angle increases very rapidly making it highly unstable.

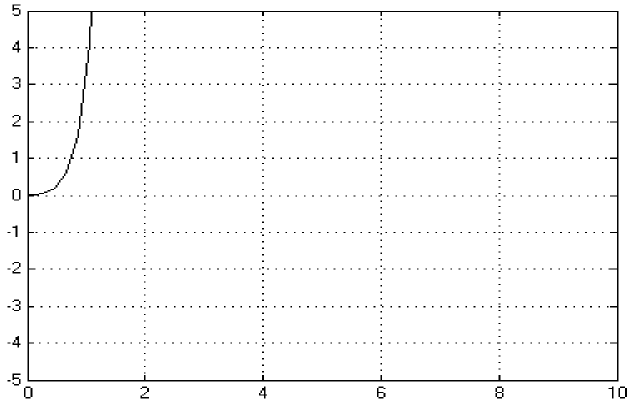

Fig.6: Step Response of Open Loop Uncompensated System

By analysing the system it is found that the system is unstable and needed to be balance. The system can be balanced using techniques discussed earlier. Here, PID and Fuzzy logic Controller techniques are presented.

IV. Compensator Design for PID controller

Compensation is to redesign the system by modifying the structure or by incorporating additional devices or components to alter the overall behaviour of the system. It is the modification of the system dynamics to satisfy the given specifications. Compensation is used when the system is absolutely unstable and compensation is required to stabilize it as well as to achieve a specified performance or when system is stable but the compensation is required to obtain the desired performance.

Compensator for the inverted pendulum system can be designed by using any of the following control analysis and design techniques:

(i) Root locus method
(ii) Bode plot
(iii) Nyquist diagram
(iv) Nichols charts

We have used Root locus technique as it permits accurate computation time domain response in addition to yielding readily available frequency response information. As shown in Fig. 5 and Fig.6 the system is unstable. The root locus of the system is shown in Fig. 7.

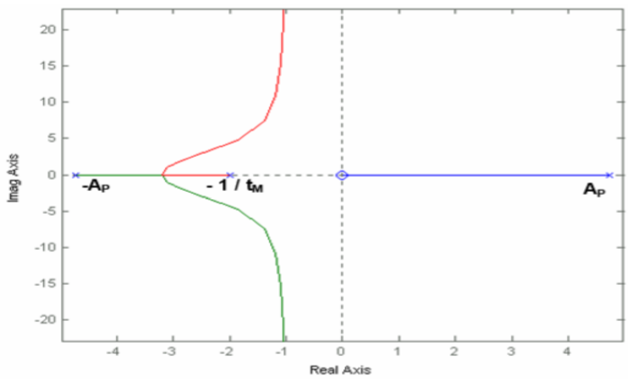

Fig.7: Root locus of uncompensated Inverted pendulum system

As shown in Fig. 7 to make the system stable reshaping of system root locus is necessary to make it stable for certain range of gain. Reshaping of the system is done by addition of

Zeros and Poles in the system which is done with the help of SISO Tool.

The Transient response of the system should have transient (settling) time of $^1/_2$ second, overshoot should be less than 20% and damping ration greater than 0.5. Steady state error must be zero.

With the help of SISO Tool we determine the values of K_P, K_I and K_D which is 20, 100 and 1 respectively. Gain K_C is 30 is found for desired performance.

V. Analysis of Inverted pendulum Compensated System with PID controller

After designing the compensator for the system, it subjected to impulse and step input.

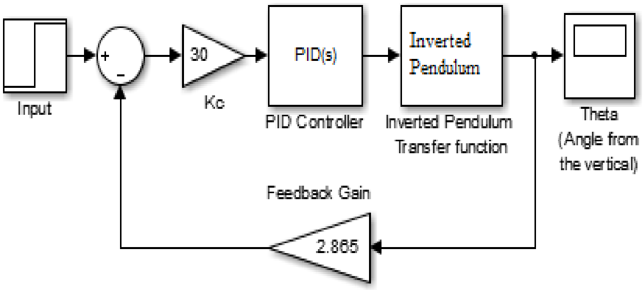

Fig.8: Compensated System

A. Impulse response with PID controller

An impulse response of the Inverted Pendulum with PID controller is shown in Fig. 9. It is a plot between angle from vertical position and time. It can be seen that the response of the system is very fast and it stabilize itself very rapidly with the time. Since the angle from vertical position is zero, the system is stable. Y-axis represents the angle in radians and X-axis time in seconds.

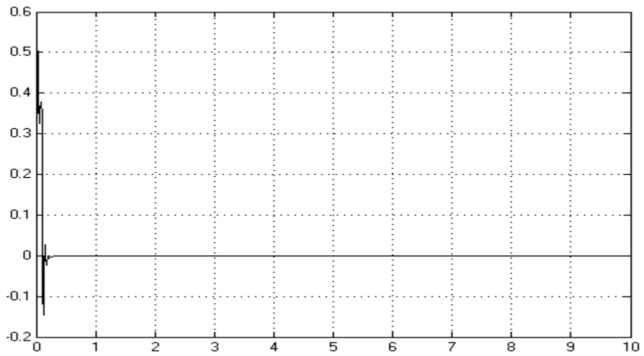

Fig.9: Impulse Response of Closed Loop Compensated System

B. Step response with PID controller

Step response of the system with compensator is shown in Fig.10. Step input represents that the force is constantly applied to inverted pendulum and after a time interval is comes in equilibrium and stabilize itself. The DC Gain of the closed loop compensated inverted pendulum is 0.35. Y-axis represents the angle in radians and X-axis time in seconds.

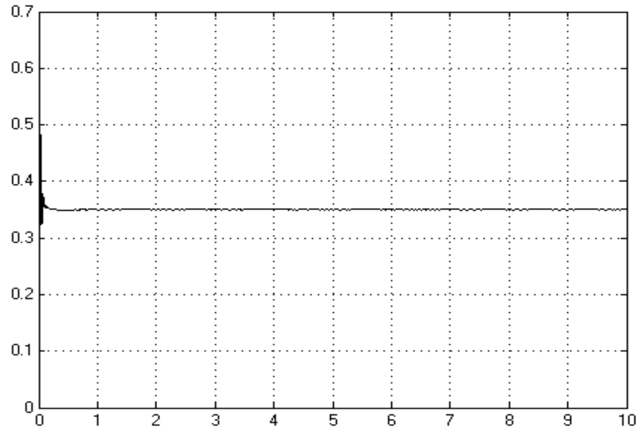

Fig.10: Step Response of Closed Loop Compensated System

VI. Fuzzy Logic Controller

Fuzzy systems are rule based system. Fuzzy system provides a systematic procedure for transforming a knowledge base into a nonlinear mapping. Fuzzy systems have been applied to various fields like signal processing, controlling, communication, medicine, expert systems to business, etc. They are multi-input single-output system. In Inverted Pendulum system, pendulum angle and pendulum angular velocity are taken as input and force on cart is taken as output of the fuzzy controller.

Force is applied at the system, with the help of sensors its angle and velocity is measured which is feedback to the fuzzy controller. On the basis of IF-THEN rules an output is generated which is applied to the inverted pendulum system to stabilise it. Fuzzy logic controller based inverted pendulum is also analyzed with impulse and step inputs. Fuzzy controller is preferred over other controllers where variation in system parameters is required. System parameters can easily be varied in fuzzy controllers.

VII. Inverted Pendulum using Fuzzy Controller

Implementation of fuzzy controller on inverted pendulum is shown in figure below.

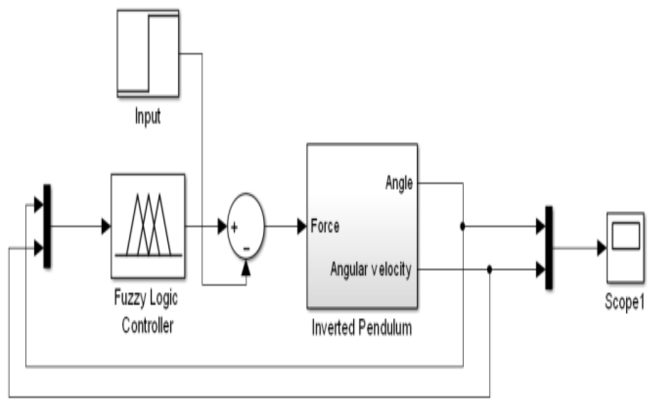

Fig.11: Inverted pendulum using fuzzy controller

A. Impulse Response

Impulse response of the system with fuzzy controller is shown in fig.12. It can be seen that both angle and angular velocity becomes zero after a short interval of time. It shows that the system has become stable.

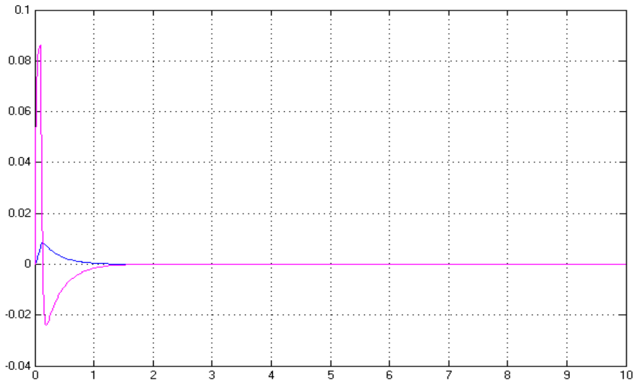

Fig.12: Impulse response of the fuzzy controlled inverted pendulum system

B.Step response

Step response of the system is shown in fig.13. Angle changes with time and attains a constant value after a time interval and at the same time angular velocity becomes zero representing that the system has achieved steady state and is stable.

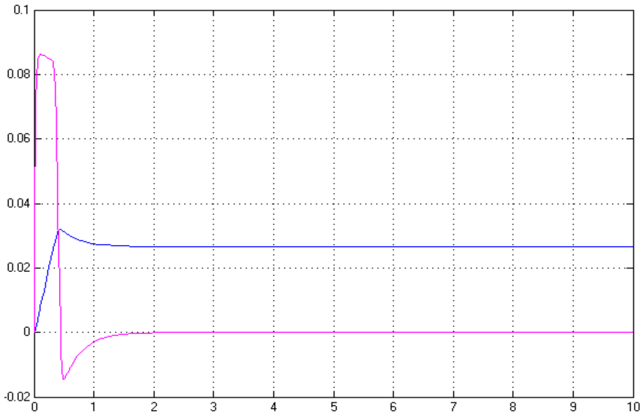

Fig.13: Step response of the fuzzy controlled inverted pendulum system

VIII. Conclusion

In PID controlled system, the angle deviation from its vertical position is very large while in Fuzzy controlled system deviation is small. PID controlled Inverted pendulum system stabilize in less time than the Fuzzy controlled Inverted Pendulum system. For impulse input, settling time for PID controlled system is 0.4 second and for Fuzzy controlled system it is 1.8 second. For step input, it is 1.125 second and 0.145 second respectively. Final steady state value by PID controlled system is 0.35 whereas in Fuzzy controlled system is 0.0265.

Fuzzy controlled system doesn't have much better characteristics in time domain than the PID controlled system, but its advantage is that it can deal with nonlinear systems.

Fuzzy controller has a lot of parameters. The most important is to make a good choice of rule base and parameters of membership functions. Once a fuzzy controller is given, whole system can actually be considered as a deterministic system. When the parameters are well chosen, the response of the system has very good time domain characteristics. The fuzzy controlled system is very sensitive to the distribution of membership functions but not to the shape of membership functions.

PID controller cannot be applied with the systems which have a fast change of parameters, because it would require the change of PID constants in the time. PID controllers are commonly used to regulate the time-domain behaviour of many different types of dynamic plants. These controllers are extremely popular because they can usually provide good closed loop response characteristics, can be tuned using relatively simple rules and are easy to construct using either analogue or digital components.

It is necessary to further study the possible combination of PID and fuzzy controller. It means that the system can be well controlled by PID which is supervised by a fuzzy system.

References

[1] Khalil Sultan, "Inverted Pendulum Analysis, Design and Implementation"

[2] Arpit Jain, Deep Tayal, Neha Sehgal, " Control of Non-Linear Inverted Pendulum using Fuzzy Logic Controller", *International Journal of Computer Applications,* (0975 – 8887), Volume 69, May 2013, pp 27.

[3] Yasar Beceriklia, B. Koray Celikb,"Fuzzy control of inverted pendulum and concept of stability using Java application", *Elsevier Mathematical and Computer Modelling* 46 (2007), pp 24–37.

[4] J. Yi, N. Yubazaki, "Stabilization fuzzy control of inverted pendulum systems", *Elsevier Artificial Intelligence in Engineering,* 14 (2000), pp 153-163.

[5] Nurfakhirah Binti Ismail , "Fuzzy logic controller design for inverted pendulum system"

[6] I.J. Nagrath, M. Gopal, "Control System Engineering" Fifth edition, *New Age International Publishers.*

Saurabh Chauhan, B.tech (Final year), College of Engineering Roorkee, Roorkee.

Rakesh Vishwakarma, B.tech (Final year), College of Engineering Roorkee, Roorkee.

Vidushi Singh, B.tech (Final year) , College of Engineering Roorkee, Roorkee.

Rajat Kumar, B.tech (Final year), College of Engineering Roorkee, Roorkee.