Adaptive Group Consensus of Multi-agent in Switching Networks

Yujuan Cheng, Hui Yu

***Abstract*—In this paper, the group consensus problem of multi-agent systems is studied in networks with switching topologies, in which the nonlinear dynamics of all agents are unknown and non-identical. We assume that the unknown dynamics are linearly parameterized, and take adaptive control method in the algorithm design. A novel adaptive algorithm is proposed for the multi-agent systems in networks with switching topologies to reach group consensus by using pinning control strategies, which only rely on the relative position measurements between each agent and its neighbours. Based on Lyapunov theory, Barbalat's lemma, adaptive control theory and algebraic graph theory, the stability and parameter convergence analysis are derived. Finally, a simulation example is given to validate our theoretical results.**

***Index Terms*—Group consensus, Multi-agent systems, Switching topologies, Adaptive control, Parameter convergence**

I. Introduction

With the high development of communication technology and the increase of people's demands, consensus coordination control of multi-agent systems is indispensable in our life, and is taken attention by a mass of researchers [1-5]. The consensus problem is an important topic in cooperative control of multi-agent systems. We say consensus is reached if all agents can approach to a common value.

Recently, the group consensus problem has drawn greater attention of researchers [6-18]. The so-called group consensus problem is to force groups of agents to reach different group decision values. In [7], the authors study group consensus problem in networks with undirected communication graphs, present a novel consensus protocol and establish some criteria. In [10], the authors study two cases of interconnection topologies: one is the case of switching topologies without communication delay; another is the case of switching topologies with communication delays. The dynamics of agents in [10] are transformed into reduced-order systems by double-tree-form transformations, and they obtain some analysis results for the two topologies. In [13], the second-order nonlinear multi-agent systems in networks with directed and weakly connected topologies are considered to investigate the group consensus problem via pinning leader-following approach. Based on the property of all agents, the authors propose a pinning control protocol and some consensus criteria. Under the laws, all agents asymptotically follow the virtual leader in each group, and each group is independent. In [14], the authors consider group consensus problem of the first-order and the second-order multi-agent systems in linearly coupled networks, respectively. Two novel linear protocols are proposed by using the interactive influence between the agents in the multi-agent networks. Moreover, the authors also study the group consensus problem for multi-agent networks with generally connected topologies.

In this paper, the group consensus problem of uncertain multi-agent systems, in which unknown and identical nonlinear dynamics exist, is considered. A novel group consensus algorithm is proposed such that groups of agents can follow different groups' leaders only using the relative position information. The unknown nonlinear dynamics of agents are assumed to be linearly parameterized, and adaptive control design method is adopted to tune the unknown parameters. The analysis of stability and parameter convergence are based on Lyapunov theory, adaptive control theory and PE (persistently exciting) condition.

The contributions of this paper are mainly in two aspects. First, we present a novel group consensus algorithm for uncertain multi-agent systems. To the best of our knowledge, few researches address this issue. Second, a sufficient condition is obtained for multi-agent systems to reach group consensus. The PE condition plays a key role to guarantee the parameter convergence, and the consensus stability is ensured by Lyapunov theory.

The paper is constituted as follows. In section 2, some descriptions, lemmas about multi-agent networks and some preliminaries on graph theory are given. The problems are stated in section 3. In section 4, we present our main results. The simulation results are shown in section 5 to verify our theoretical results. In section 6, we conclude our paper.

II. Preliminaries

In a multi-agent system, each agent may receive or sense information from other agents. Graph theory is a good choice to model the communication relationships among these agents.

A graph $G=(V,E)$ consists of a set of node V and a set of edge $E\subset V\times V$, where $V=\{1,2,\cdots,N\}$ corresponding to the N agents in the multi-agent system. If agent i can receive information from agent j, then an edge $(i,j)\in E$ and agent j is called the neighbor of agent i. The set of neighbors of agent i is denoted by $N_i=\{j\in V|(i,j)\in E, j\neq i\}$. The graph is undirected if agent i and agent j can receive information from each other. Let the adjacency matrix of graph G be $A=[a_{ij}]_{N\times N}$, in which $a_{ij}=1$ if there exists an edge (i,j) between agent i and j, otherwise

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$a_{ij}=0$. The Laplacian matrix of graph G is defined as.

$$l_{ij}=\begin{cases}-a_{ij}, & i\neq j\\ \sum_{j=1}^{N}a_{ij}, & i=j,\end{cases} \quad (1)$$

which is symmetric for an undirected graph. From algebraic graph theory [19], the following lemma is well known.

Lemma 1. The Laplacian L of graph G has at least one zero eigenvalue with $1_N=(1,1,\cdots,1)^T\in R^N$ as its eigenvector, and all the non-zero eigenvalues of L are positive. The Laplacian L has a simple zero eigenvalue if and only if graph G is connected.

In this paper, we consider the case that the interconnected graphs of the systems are switching over time. The set of all possible switching graphs is denoted by $\{G_p \mid p\in P\}$, where P is an index set. For describing the time dependence of graphs, a piecewise constant switching signal $\sigma(t)$: $[0,\infty)\to P$ is defined. We use $G_{\sigma(t)}$ to denote the underlying graphs at time t on N nodes. Since the Laplacian L of the graph, the neighbors set N_i of agent i, and the (i,j) th entry a_{ij} of A are all vary with time, their time varying versions are denoted by $L_{\sigma(t)}$, $a_{ij}(t)$ and $N_i(t)$, respectively.

Assumption 1. The switching times of the switching signal $\sigma(t)$ is finite in any bounded time intervals.

We assume that N agents in multi-agent systems are divided into m groups, and denote the set of the i th group of agents by $V^i_{\sigma(t)}, i=1,2,\cdots,m$. Then $V_{\sigma(t)}=V^1_{\sigma(t)}\cup V^2_{\sigma(t)}\cup\cdots\cup V^m_{\sigma(t)}=\{1,2,\cdots,N\}$ and $V^k_{\sigma(t)}\cap V^s_{\sigma(t)}=\varnothing(k\neq s)$. Let $\eta(i)$: $V_{\sigma(t)}\to\{i=1,2,\cdots,m\}$ be a switching function and $V^{\eta(i)}_{\sigma(t)}$ be the agents set of group $\eta(i)$. Then $\eta(i)=k\in\{1,2,\cdots,m\}$ means that agent i belongs to group k.

Since the communication relationships also exist in different groups, a group in multi-agent system consists of the set of inter-agent and the set of intra-agent. Let $\tilde{V}^{\eta(i)}_{\sigma(t)}$ be the set of inter-agent with $\tilde{V}^{\eta(i)}_{\sigma(t)}\subseteq V^{\eta(i)}_{\sigma(t)}$. Agent i is called inter-agent if agent $i\in\tilde{V}^{\eta(i)}_{\sigma(t)}$, and agent i is called intra-agent if agent $i\in V^{\eta(i)}_{\sigma(t)}\setminus\tilde{V}^{\eta(i)}_{\sigma(t)}$. From [13], we have the following lemma:

Lemma 2. If node i is the intra-act agent, namely, $i\in V^{\eta(i)}_{\sigma(t)}\setminus\tilde{V}^{\eta(i)}_{\sigma(t)}$, then the following results hold $c\sum_{j=1}^{N}l_{ij}x_{0\eta(i)}(t)=0$.

Lemma 3. (Barbalat's lemma) If $V(x,t)$ satisfies the following conditions:

(1) $V(x,t)$ is lower bounded;

(2) $\dot{V}(x,t)$ is negative semi-definite;

(3) $\dot{V}(x,t)$ is uniformly continuous in time or $\ddot{V}(x,t)$ is bounded, then $\dot{V}(x,t)\to 0$ as $t\to\infty$.

III. Problem Statement

Consider a multi-agent system consists of N agents divided into m groups, the dynamics of N agents are given by

$$\dot{x}_i(t)=f_i(x_i,t)+u_i(t),\quad i=1,2,\cdots,N, \quad (2)$$

where $x_i(t)\in R$ is the position of agent i, $u_i(t)\in R$ is the control input, and $f_i(x_i,t)\in R$ is the unknown dynamics of agent i which is nonlinear. To guarantee the existence of unique solution of equation (2), we suppose that $f_i(x_i,t)$ is continuous in t and Lipschitz in $x_i(t)$.

We assume that there are m virtual leaders in m groups respectively, and the dynamics of these leaders are described by

$$\dot{x}_{0\eta(i)}(t)=v_{0\eta(i)}(t),\quad i=1,2,\cdots,N, \quad (3)$$

where $x_{0\eta(i)}(t)\in R$ is the virtual leader's position of group $\eta(i)$ and $v_{0\eta(i)}(t)\in R$ the velocity which is also unknown.

Remark 1. The states of all agents are assumed to be in R. It is trivial to extend this case to R^n by introducing Kronecker product.

Definition 1. For the N agents in the multi-agent system (2), we say group consensus is achieved if, there exist controllers $u_i(t), i=1,2,\cdots,N$, such that

$$\lim_{t\to\infty}|x_i(t)-x_{0\eta(i)}(t)|=0, \quad (4)$$

for any initial condition $x_i(0), i=0,1,\cdots,N$.

IV. Main Results

Similar to [21], we assume that the unknown nonlinear dynamics in the system can be linearly parameterized. Then, the unknown nonlinear functions $f_i(x_i,t)$, $i=1,\cdots,N$ and the unknown velocity $v_{0\eta(i)}(t)$ of the leader can be parameterized as

$$f_i(x_i,t)=\varphi_i^T(x_i,t)\theta_i,\quad i=1,2,\cdots,N, \quad (5)$$

and

$$v_{0\eta(i)}(t)=\varphi^T_{0\eta(i)}(t)\theta_{0\eta(i)}, \quad (6)$$

where $\varphi_{0\eta(i)}(t)$, $\varphi_i(x_i,t)\in R^m$ are some basis function vectors and $\theta_{0\eta(i)}$, $\theta_i\in R^m$ are unknown constant parameter vectors which will be estimated later.

Let the estimate of $\theta_{0\eta(i)}$ be $\hat{\theta}_{0\eta(i)}$ and $v_{0\eta(i)}(t)$ be $\hat{v}_{0\eta(i)}(t)$. Then $\hat{v}_{0\eta(i)}(t)$ can be expressed as

$$\hat{v}_{0\eta(i)}(t)=\varphi^T_{0\eta(i)}(t)\hat{\theta}_{0\eta(i)},\quad i=1,2,\cdots,N. \quad (7)$$

Similarly, let the estimates of θ_i and $f_i(x_i,t)$ be $\hat{\theta}_i$ and $\hat{f}_i(x_i,t)$, respectively. Then $\hat{f}_i(x_i,t)$ can be expressed as

$$\hat{f}_i(x_i,t)=\varphi_i^T(x_i,t)\hat{\theta}_i,\quad i=1,2,\cdots,N. \quad (8)$$

A. Adaptive group consensus algorithm design

For multi-agent systems (1), let $[t_k,t_{k+1}), k=0,1,2,\cdots$ be an infinite time interval sequence, where $t_0=0$ is the initial time instant, $T_0 \le t_{k+1} - t_k \le T$, and T_0, T are some positive constants.

Suppose that each time interval $[t_k, t_{k+1}), k = 0,1,2,\cdots,$ is divided into a finite time interval sequence

$$[t_k^0, t_k^1), \cdots, [t_k^l, t_k^{l+1}), \cdots, [t_k^{l_k-1}, t_k^{l_k}),$$

where $t_k = t_k^0, t_{k+1} = t_k^{l_k}$ for some integer $l_k \ge 0$, $t_k^{l+1} - t_k^l \ge \tau, 0 \le l \le l_k - 1$, $\tau > 0$ is a positive constant, the so-called dwell time. $t_k^0, t_k^1, \cdots, t_k^{l_k-1}$ are the time instants at which the time-varying graph switches, in other words, in each such time subinterval $[t_k^l, t_k^{l+1})$ the switching graph $G_{\sigma(t)}$ is time invariant and connected.

Denote the stack column vector $(x_1, x_2, \cdots, x_N)^T$ of $x_i(t), i = 1,2,\cdots,N,$ by $\mathrm{col}(x_i)$. Let $x = \mathrm{col}(x_i)$, $u = \mathrm{col}(u_i)$, $\Theta_0 = \mathrm{col}(\theta_{0\eta(i)})$, $\Theta_f = \mathrm{col}(\theta_i)$, $\hat{\Theta}_0 = \mathrm{col}(\hat{\theta}_{0\eta(i)})$, $\hat{\Theta}_f = \mathrm{col}(\hat{\theta}_i)$, $i = 1,2,\cdots,N$. The adaptive group consensus algorithm is proposed as follows.

Control Laws:

$$\begin{cases} u_i(t) = c\sum_{j\in N_i(t)} a_{ij}(t)[x_j(t) - x_i(t)] \\ \quad - cb_i(t)[x_i(t) - x_{0\eta(i)}(t)] + c\sum_{j=1}^{N} l_{ij}x_{0\eta(j)}(t) \\ \quad + \varphi_{0\eta(i)}^T(t)\hat{\theta}_{0\eta(i)} - \varphi_i(x_i,t)^T\hat{\theta}_i, \quad i \in \tilde{V}_{\sigma(t)}^{\eta(i)} \\ u_i(t) = c\sum_{j\in N_i(t)} a_{ij}(t)[x_j(t) - x_i(t)] \\ \quad - cb_i(t)[x_i(t) - x_{0\eta(i)}(t)] + \varphi_{0\eta(i)}^T(t)\hat{\theta}_{0\eta(i)} \\ \quad - \varphi_i(x_i,t)^T\hat{\theta}_i, \quad i \in V_{\sigma(t)}^{\eta(i)} \backslash \tilde{V}_{\sigma(t)}^{\eta(i)}, \end{cases} \tag{9}$$

where $c > 0$ is a constant number, $b_i(t) > 0$ and let $B_{\sigma(t)}$ be a diagonal matrix with diagonal elements $\{b_1(t), b_2(t), \cdots, b_N(t)\}$.

Parameter adaptive laws:

$$\begin{cases} \dot{\hat{\theta}}_{0\eta(i)} = -\frac{\gamma}{c}\varphi_{0\eta(i)}(t)\{\sum_{j\in N_i(t)} a_{ij}(t)[x_j(t) - x_i(t)] \\ \quad - b_i(t)[x_i(t) - x_{0\eta(i)}(t)] + \sum_{j=1}^{N} l_{ij}x_{0\eta(j)}(t)\}, \\ \dot{\hat{\theta}}_i = \frac{\gamma}{c}\varphi_i(x_i,t)\{\sum_{j\in N_i(t)} a_{ij}(t)[x_j(t) - x_i(t)] \\ \quad - b_i(t)[x_i(t) - x_{0\eta(i)}(t)] + \sum_{j=1}^{N} l_{ij}x_{0\eta(j)}(t)\} \\ i \in \tilde{V}_{\sigma(t)}^{\eta(i)}, \end{cases} \tag{10}$$

$$\begin{cases} \dot{\hat{\theta}}_{0\eta(i)} = -\frac{\gamma}{c}\varphi_{0\eta(i)}(t)\{\sum_{j\in N_i(t)} a_{ij}(t)[x_j(t) - x_i(t)] \\ \quad - b_i(t)[x_i(t) - x_{0\eta(i)}(t)]\}, \\ \dot{\hat{\theta}}_i = \frac{\gamma}{c}\varphi_i(x_i,t)\{\sum_{j\in N_i(t)} a_{ij}(t)[x_j(t) - x_i(t)] \\ \quad - b_i(t)[x_i(t) - x_{0\eta(i)}(t)]\}, \\ i \in V_{\sigma(t)}^{\eta(i)} \backslash \tilde{V}_{\sigma(t)}^{\eta(i)} \end{cases}$$

where $\gamma > 0$ is a constant number.

Remark 2. The controllers and adaptive law proposed in (9) and (10) depend on the relative position information between neighboring agents and the state information of the group leaders. When agent i is inter-agent, the state information of some group leaders is needed.

B. Stability analysis

Let $e_i(t) = x_i(t) - x_{0\eta(i)}(t)$, from **Lemma 2**, we have

$$\begin{aligned} \dot{e}(t) &= \dot{x} - \dot{x}_{0\eta} \\ &= f + u - v_\eta \\ &= -c(L_{\sigma(t)} + B_{\sigma(t)})e + \Phi^T\bar{\Theta}. \end{aligned} \tag{11}$$

where $f = (f_1, f_2, \cdots, f_N)^T$, $\Phi = \begin{pmatrix} \Phi_0 \\ -\Phi_f \end{pmatrix}$ is a regressor matrix, $\bar{\Theta} = \begin{pmatrix} \bar{\Theta}_0 \\ \bar{\Theta}_f \end{pmatrix}$, $\Phi_0 = \mathrm{diag}\{\varphi_{0\eta(1)}, \varphi_{0\eta(2)}, \cdots, \varphi_{0\eta(N)}\}$, $\Phi_f = \mathrm{diag}\{\varphi_1, \varphi_2, \cdots, \varphi_N\}$, $\bar{\Theta}_0 = \hat{\Theta}_0 - \Theta_0 = \mathrm{col}(\bar{\theta}_{0\eta(i)})$ and $\bar{\Theta}_f = \hat{\Theta}_f - \Theta_f = \mathrm{col}(\bar{\theta}_i)$, in which $\bar{\theta}_{0\eta(i)} = \hat{\theta}_{0\eta(i)} - \theta_{0\eta(i)}$ and $\bar{\theta}_i = \hat{\theta}_i - \theta_i$. We have the error dynamics of system as follows:

$$\begin{cases} \dot{e} = -cH_{\sigma(t)}e + \Phi^T\bar{\Theta}, \\ \dot{\bar{\Theta}} = -\frac{\gamma}{c}\Phi H_{\sigma(t)}e, \end{cases} \tag{12}$$

where $H_{\sigma(t)} = L_{\sigma(t)} + B_{\sigma(t)}$, and the matrix $H_{\sigma(t)}$ of graph $G_{\sigma(t)}$ has the well-known properties [22] as follows:

Lemma 4. (1) Matrix $H_{\sigma(t)}$ has nonnegative eigenvalues; (2) Matrix $H_{\sigma(t)}$ is positive definite if and only if graph $G_{\sigma(t)}$ is connected.

Let $\mu_p = \lambda_{\min}(H_p)$, $p \in P$ be the smallest non-zero eigenvalue of positive semi-definite matrix H_p. Then we have $\varpi_{\min} = \min\{\mu_p \,|\, p \in P\}$ is positive.

We assume that the regression matrix Φ is persistently exciting (PE) [23], i.e., there exist two positive reals δ_0 and α such that

$$\int_t^{t+\delta_0} \Phi\Phi^T d\tau \ge \alpha I > 0, \quad \forall t \ge 0, \tag{13}$$

where I is an appropriate dimension identity matrix. The PE condition has another explanation, that is

$$\int_t^{t+\delta_0} \omega^T\Phi\Phi^T\omega d\tau \ge \alpha, \quad \forall t \ge 0, \forall \omega : \|\omega\| = 1. \tag{14}$$

By introducing the PE condition, the parameter convergence, that is,

$$\begin{aligned} &\lim_{t\to\infty} \|\hat{\theta}_{0\eta(i)} - \theta_{0\eta(i)}\| = 0, \\ &\lim_{t\to\infty} \|\hat{\theta}_i - \theta_i\| = 0, \quad i = 1,2,\cdots,N. \end{aligned} \tag{15}$$

for any initial condition $\hat{\theta}_{0\eta(i)}(0), \hat{\theta}_i(0), i = 1,2,\cdots,N$, can be guaranteed. A detailed analysis will be given later.

Theorem 1. Consider the multi-agent systems (2)-(3) in networks with undirected topologies. Suppose that $\varphi_{0\eta(i)}$, $\dot{\varphi}_{0\eta(i)}$, φ_i, and $\dot{\varphi}_i, i = 1,2,\cdots,N$ are uniformly bounded and the PE condition is satisfied. Under the control law (9) and the adaptive law (10), group consensus of the multi-agent system is reached globally, uniformly and asymptotically, in the meanwhile, the global, uniform, and asymptotical parameter convergence is also guaranteed.

Proof. Consider a Lyapunov function candidate

$$V(t)=\frac{1}{2c}e^{T}H_{p}e+\frac{1}{2\gamma}\bar{\Theta}^{T}\bar{\Theta}. \quad (16)$$

for systems (15). Except for the switching instants, $V(t)$ is continuous and differentiable .

Consider a time interval $[t_k^l, t_k^{l+1}]$ with switching graph $G_p, p\in P$, included in the time interval $[t_k, t_{k+1}]$. The time derivative of this Lyapunov function candidate along the trajectory of the system (12) is

$$\dot{V}(t)=-e^{T}H_{p}^{2}e. \quad (17)$$

Due to the fact that H_p is positive definite from **Lemma 4**, there exists an orthogonal matrix U_p such that

$$U_pH_pU_p^T=\Lambda_p=\text{diag}\{\lambda_p^1,\lambda_p^2,\cdots,\lambda_p^N\}, \quad (18)$$

where $\lambda_p^1,\lambda_p^2,\cdots,\lambda_p^N$ are the N positive eigenvalues of H_p.

Let $\tilde{e}=U_pe$, we have

$$\begin{aligned}\dot{V}(t)&=-\tilde{e}^T\Lambda_p{}^2\tilde{e}\\&\le-\varpi_{\min}^2\tilde{e}^T\tilde{e}\\&\le 0.\end{aligned} \quad (19)$$

Therefore, $\lim_{t\to\infty}V(t)=V(\infty)$ exists.

Consider the infinite sequence $\{V(t_k), k=0,1,\cdots\}$. Based on the Cauchy's convergence criteria, for any $\varepsilon>0$, there exists $\tau>0$ such that $|V(t_{k+1})-V(t_k)|<\varepsilon$, i.e., $\left|\int_t^{t+\tau}\dot{V}(t)dt\right|<\varepsilon$.

From (19), we have

$$-\varepsilon<\int_{t_k}^{t_{k+1}}\dot{V}(t)dt\le-\varpi_{\min}^2\int_{t_k}^{t_{k+1}}\tilde{e}^T(t)\tilde{e}(t)dt, \quad (20)$$

and due to $t_k^{l+1}-t_k^l\ge\tau$, then

$$\int_{t_k^l}^{t_k^l+\tau}\tilde{e}^T(t)\tilde{e}(t)dt\le\int_{t_k^l}^{t_k^{l+1}}\tilde{e}^T(t)\tilde{e}(t)dt\le\frac{\varepsilon}{\varpi_{min}^2} \quad (21)$$

Therefore,

$$\lim_{t\to\infty}\int_t^{t+\tau}\tilde{e}^T(s)\tilde{e}(s)ds=0. \quad (22)$$

From (12), (16), (19), it follows that $e(t)$, $\bar{\Theta}_0(t)$, $\bar{\Theta}_f(t)$ and $\dot{e}$ are all uniformly bounded, and from the assumption of the theorem, $\varphi_{0\eta(i)}$, $\dot{\varphi}_{0\eta(i)}$, φ_i and $\dot{\varphi}_i$ are also uniformly bounded. Therefore the function $\tilde{e}^T(t)\tilde{e}(t)$ is uniformly continuous. From **Lemma 3**, we have $\lim_{t\to\infty}e(s)^Te(s)=0$, then

$$\lim_{t\to\infty}e(t)=0. \quad (23)$$

Thus, $\lim_{t\to\infty}|x_i(t)-x_{0\eta(i)}(t)|=0, i=1,2,\cdots,N$.

In the following, we prove that

$$\lim_{t\to\infty}\|\bar{\Theta}(t)\|=0, \quad (24)$$

for any initial condition $\bar{\Theta}(0)$, i.e, for any $\varepsilon>0$, there exist $T_\varepsilon>0$ such that

$$\|\bar{\Theta}(t)\|<\varepsilon,\quad t\ge T_\varepsilon, \quad (25)$$

for any initial condition $(e(0),\bar{\Theta}(0))$.

To prove (25), we first prove that, given any $\varepsilon>0$ and $T>0$, for any $e(0)$ and $\bar{\Theta}(0)$, there exists $t>T$ such that

$$\|\bar{\Theta}(t)\|<\varepsilon. \quad (26)$$

To prove (26), we equivalently show by contradiction that, for any $\varepsilon>0$, a time $T_1>0$ such that

$$\|\bar{\Theta}(t)\|\ge\varepsilon,\ \forall t\ge T_1, \quad (27)$$

does not exist.

Consider a time intervals $[t_k,t_{k+1})=[t_k,t_k+T_k)$ with $T_0\le T_k\le T$ and construct the following function

$$\Psi(\bar{\Theta}(t),t)=\frac{1}{2}[\bar{\Theta}^T(t+T_k)\bar{\Theta}(t+T_k)-\bar{\Theta}^T(t)\bar{\Theta}(t)]. \quad (28)$$

From the definition of the Lyapunov function candidate (16), $\lim_{t\to\infty}|e(t)|=0$ and $\lim_{t\to\infty}V(t)=V(\infty)$, we have $\lim_{t\to\infty}\bar{\Theta}^T(t)\bar{\Theta}(t)=\rho V(\infty)$, where ρ is a positive constant. Thus, $\lim_{t\to\infty}\Psi(\bar{\Theta}(t),t)=0$. Therefore, for $\forall\varepsilon_1>0$, there exists $t_2>0$ such that

$$\|\Psi(\bar{\Theta}(t),t)-\Psi(\bar{\Theta}(t'),t')\|<\varepsilon_1,\quad\forall t,t'>T_2. \quad (29)$$

Calculating the time derivative of the function $\Psi(\bar{\Theta}(t),t)$ define in (34) at the time instant $t\in[t_k,t_{k+1})$, we have

$$\begin{aligned}&\dot{\Psi}(\bar{\Theta}(t),t)\\&=\frac{1}{2}[\dot{\bar{\Theta}}^T(t+T_k)\bar{\Theta}(t+T_k)+\bar{\Theta}^T(t+T_k)\dot{\bar{\Theta}}(t+T_k)]\\&\quad-\frac{1}{2}[\dot{\bar{\Theta}}^T(t)\bar{\Theta}(t)+\bar{\Theta}^T(t)\dot{\bar{\Theta}}(t)]\\&=\bar{\Theta}^T(t+T_k)\dot{\bar{\Theta}}(t+T_k)-\bar{\Theta}^T(t)\dot{\bar{\Theta}}(t)\\&=\int_t^{t+T_k}\frac{d}{d\tau}[\bar{\Theta}^T(\tau)\dot{\bar{\Theta}}(\tau)]d\tau\\&=\int_t^{t+T_k}\frac{d}{d\tau}(-\frac{\gamma}{c}\bar{\Theta}^T\Phi H_pe)d\tau\\&=\int_t^{t+T_k}(\frac{\gamma^2}{c^2}e^TH_p\Phi^T\Phi H_p+\gamma\bar{\Theta}^T\Phi H_p^2)ed\tau\\&\quad-\frac{\gamma}{c}\int_t^{t+T_k}\bar{\Theta}^T\Phi H_p\Phi^T\bar{\Theta}d\tau\\&=Q_1-Q_2.\end{aligned} \quad (30)$$

From (18), we have

$$H_p=U_p^T\Lambda_pU_p\ge\varpi_{min}I. \quad (31)$$

Then

$$Q_2\ge\frac{\gamma}{c}\varpi_{min}\int_t^{t+T}\bar{\Theta}^T\Phi\Phi^T\bar{\Theta}d\tau. \quad (32)$$

From (13), it follows that

$$\frac{1}{\varepsilon^2}\int_t^{t+T}\bar{\Theta}^T\Phi\Phi^T\bar{\Theta}d\tau\ge\int_t^{t+T}\frac{\bar{\Theta}^T}{\|\bar{\Theta}(t)\|}\Phi\Phi^T\frac{\bar{\Theta}}{\|\bar{\Theta}(t)\|}\ge\alpha, \quad (33)$$

and

$$\int_t^{t+T}\bar{\Theta}^T\Phi\Phi^T\bar{\Theta}d\tau\ge\alpha\varepsilon^2. \quad (34)$$

From the PE condition (14), we have

$$Q_2\ge\frac{\gamma}{c}\alpha\lambda_{min}\varepsilon^2,\quad\forall t>T_1. \quad (35)$$

Since e, $\bar{\Theta}_0$, $\bar{\Theta}_f$, Φ_0, Φ_f are all bounded, there exists a positive constant M such that $l_1\le M\int_t^{t+T}\|e\|d\tau$. Then we have

$$Q_1 \le \frac{\gamma}{2c}\alpha\lambda_{min}\varepsilon^2, \quad \forall t > T_3. \tag{35}$$

From (29), (34) and (35), we have

$$\dot{\Psi}(\bar{\Theta}(t),t) \le -\frac{\gamma}{2c}\alpha\lambda_{min}\varepsilon^2 < 0, \quad \forall t > T_4, \tag{36}$$

where $T_4 = \max\{T_1, T_2, T_3\}$. Thus, there exists a time interval $[t_{k_j}, t_{k_j} + T)$ with $t_{k_j} > T_4$ such that $\dot{\Psi}(\bar{\Theta}(t),t) \le -\frac{\gamma}{4c}\alpha\varpi_{min}\varepsilon^2 < 0$ holds for $\forall t \in [t_{k_j}, t_{k_j} + T)$ due to the sign-preserving property of continuous function. Integrating $\dot{\Psi}(\bar{\Theta}(t),t)$ from t_{k_j} to $t_{k_j} + T$ and selecting $\varepsilon_1 = \frac{\gamma}{4c}\alpha\varpi_{min}\varepsilon^2 T$, we have

$$\Psi(\bar{\Theta}(t_{k_j}), t_{k_j}) - \Psi(\bar{\Theta}(t_{k_j} + T), t_{k_j} + T) > \varepsilon_1 \tag{37}$$

which contradicts (28). So, (26) holds. This completes the proof of (25).

Because of $\lim_{t\to\infty} e(t) = 0$, for any $\varepsilon > 0$ there exists a time T_5 such that

$$\|e(t)\| \le \sqrt{\frac{c}{2\gamma}}\varepsilon, \quad \forall t \ge T_5. \tag{38}$$

Since (25) holds, there exists a time $T_\varepsilon > T_5$ such that

$$\|\bar{\Theta}(T_\varepsilon)\| \le \frac{1}{\sqrt{2}}\varepsilon. \tag{39}$$

From the initial conditions $e(T_\varepsilon)$ and $\bar{\Theta}(T_\varepsilon)$, according to (16), (38) and (39), we have

$$\|\bar{\Theta}\| \le \sqrt{\frac{\gamma}{c}\|e(T_\varepsilon)\|^2 + \|\bar{\Theta}(T_\varepsilon)\|^2} \le \varepsilon, \quad \forall t \ge T_\varepsilon, \tag{40}$$

which implies (24). Therefore, the equilibrium point $(e, \bar{\Theta}) = 0$ is attractive. Since (23) and (24) hold uniformly for the initial time, it follows that the equilibrium point $(e, \bar{\Theta}) = 0$ is globally uniformly asymptotically stable.

V. SIMULATIONS

In this section, an example to illustrate the algorithm is given to verify our theory results. Consider a system consists of six agents, and assume that $\varphi_i^T(x_i, t) = [x\sin(t), x\cos(t)]^T$. Then the nonlinear dynamics of the six agents are

$$f_i(x_i, t) = [x\sin(t), x\cos(t)]\theta_i, \quad i = 1, 2, \cdots, 6, \tag{41}$$

and we pick $\theta_i = [\frac{\sqrt{5}}{5}, \frac{\sqrt{2}}{7}]^T$.

The six agents are divided into two groups. We assume that there is a virtual-leader in each group. The velocity of the virtual-leaders in the first group is

$$v_{0\eta(i)}(t) = [\sin(t), \cos(t)]\theta_{0\eta(i)}, \quad i = 1, 2, 3, \tag{42}$$

where we choose $\theta_{0\eta(i)} = [\frac{\sqrt{6}}{3}, \frac{\sqrt{3}}{5}]^T$. The velocity of the virtual-leaders in the second group is

$$v_{0\eta(i)}(t) = [\cos(t), \sin(t)]\theta_{0\eta(i)}, \quad i = 4, 5, 6, \tag{43}$$

and choose $\theta_{0\eta(i)} = [\frac{\sqrt{3}}{5}, \frac{\sqrt{11}}{3}]^T$.

We assume that all possible switching graphs are $\{G_1, G_2, G_3\}$ shown in **Fig. 1**. The switching graphs switch according to $G_1 \to G_2 \to G_3 \to G_1 \cdots$. However, each switching graph is time invariant and connected in the time intervals $[t_k^l, t_k^{l+1}), l = 0, 1, \cdots, l_k$. Under the control law (9) and the adaptive law (10), we choose $c = 0.8$, $\gamma = 400$. The simulation results are shown in **Fig. 2, 3** and **4**. **Fig. 2** shows that the states of ten agents converge to the trajectories of the two leaders, respectively, and **Fig. 3** shows that over time the tracking errors converge to zero. **Fig. 4** shows that the parameter convergence is also guaranteed. Note that the interconnection graph of six agents is connected, and the regression matrix Φ in this example satisfies the PE condition. Based on **Theorem 1**, it is expected that both consensus errors and parameter errors converge to zero globally, uniformly and asymptotically.

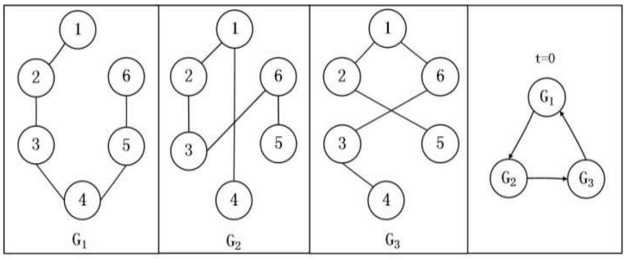

Fig. 1: The interconnection graph of six agents.

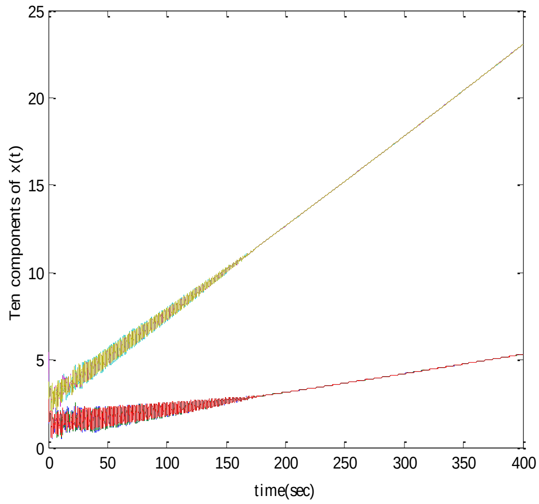

Fig. 2: The trajectories of six agents converge to the trajectories of the two leaders.

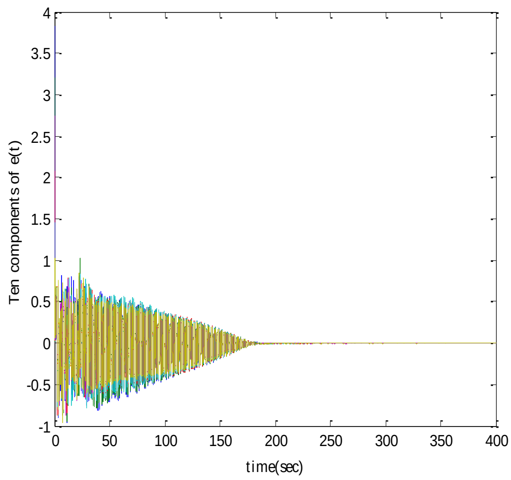

Fig. 3: Consensus errors converge to zero under (9) and (10).

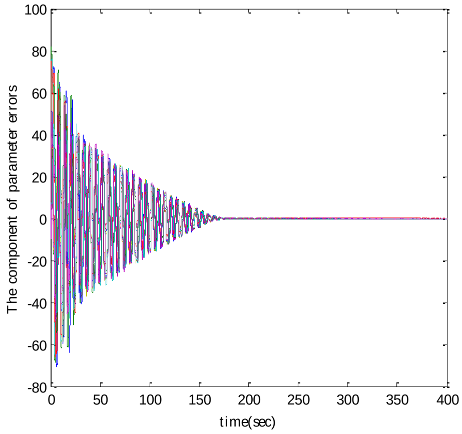

Fig. 4: Parameter errors converge to zero applying (9) and (10).

VI. Conclusion

In this paper, the group consensus problem is investigated for the first-order multi-agent systems with unknown and non-identical nonlinear dynamics. By using adaptive design method and pinning control, an adaptive group consensus algorithm is proposed. Algebraic graph theory is used to model the interconnection relations between agents, and we conduct stability analysis by using the Barbalat's lemma and Lyapunov techniques. The connectedness of the switching graphs property ensures group consensus achieving and the PE condition plays a key role the in the analysis of the parameter convergence. The simulation example illustrates our algorithm, and validates our theory.

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