Abstract— The fatigue behavior of asphalt mixtures is the basis of the design of road structures. The heterogeneous nature of the material and the thermo-viscoelastic behavior of the bitumen could make the interpretation complex. Damage is defined based on the concept of effective stress associated with the equivalence deformation assumption. This concept translates both the direct and semi-direct methods. Modeling the mechanical behavior leads to define the materials in order to know its viscoelastic behavior in the frequency domain and thus, the complex modulus test.

Index Terms— Fatigue, damage, thermomechanical, asphalt, bituminous, viscoelastic

I. INTRODUCTION

The bituminous coated materials are reconstituted from granules (0-6 mm for the studied mixture) that form the skeleton and whose cohesion is ensured by bitumen. The resulting mixture is a heterogeneous material from which the matrix imparts viscoelastic properties, but its characteristics are highly dependent on temperature. The modeling of characteristics’ thermal and time dependence is acquired in the field of small perturbations.

A. Loading

Although attempts have been carried out to simulate in the laboratory the realistic and random loading conditions [1], most of the time, the applied loading cycles are periodic and do not take into account the amplitude or frequency variations on real roadways. The main forms of stress signals used for fatigue testing are listed in Fig.1.

While some authors consider that the shape of the applied cyclic loading has little influence on the service life of materials, others consider it is very important to analyze the results of stress tests mainly through the influence of the rate of application of the load on the complex modulus [3].

B. Mechanical Properties

The mechanical properties can be deduced from the analysis of results of laboratory tests. These tests, which are standard when used in design, are made under defined conditions of temperature and load. They are performed on samples made up in the laboratory or eventually

The tests that characterize the mechanical properties of the coatings are many and varied, it is nevertheless possible to classify them into different categories. A first classification into three categories, which are more related to the interpretation of the test rather than its nature, is suggested by BONNOT (1973, 1984) [4] and [5]. A second classification proposed by DI BENEDETTO (1990) introduced two test classes: homogeneous tests and non-homogeneous tests [6].

C. Fatigue of bituminous mixtures

The fatigue phenomenon of asphalt mixtures is characterized by its breaking after repeated application of a large number of loading (in practice greater than $10^5$) whose amplitude is less than the resistance to the instantaneous breaking of the material.

By definition, a test tube is called tired when his unit reached half of its initial value measured at the first loading cycle in the same test conditions (temperature and frequency). Life $N_1$ of a test tube is the number of cycles corresponding to a modulus of rigidity equal to half the initial modulus of the same test tube (Figure I.2).

The classic test is one of the most widespread fatigue criteria. The life $N_f$ determined from this test is used for the design of pavements.

According to the solicitation level and the number of applied cycles, the asphalt has three types of behavior (Fig.3):
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Figure 3: Tests on coated homogeneous (temperature 20 °C) [7]

- For low number of loads and deformations of a few percent, the test is a deformability test and a highly nonlinear behavior is observed.
- For loads consisting of a few hundred cycles and low distortion (<10^{-4}) the behavior is considered a first approximation, linear viscoelastic behavior.
- When loading several thousand cycle and under low deformation, damage phenomena appear, the material fatigue.

Bituminous mixes are submitted, on the road, to short-term stresses at each passage of a heavy vehicle. Thus, the floor, which behaves like a rigid material resting on a deformable support, is subjected to bending forces. Tensile stresses then appear at the base of the floor and are repeated at each wheel arch. Since the asphalt is less resistant to traction and compression, in the case of passing a large number of vehicles, cracks are initiated on the basis of the floor due to the fatigue phenomenon.

1) Timing Factors
The reality differs from the theory and requires timing (1), (2)

Where
- \( \varepsilon_{0} \) Calibration temperature ,
- \( E^{1/2} = \text{cst} \):
- \( k_{e} \) correlation coefficient lab / field \( k_{e} \in [1; 1.3] \):
- \( k_{r} \) risk coefficient (reliability);
- \( k_{s} \) default coefficient of lift of the platform \( k_{s} \in [1; 1.2] \);
- \( k_{d} \) coefficient of discontinuity and thermal gradient.

D. Damage
Damage as it was defined, based on the concept of effective stress associated with the deformation equivalence hypothesis, is not directly accessible to measurements, possible evaluation methods being [6]:
- The semi-called direct method where we measure the variation of the surface or the volume of microvoids or density of the material;
- Indirect methods where damage is measured through its effect on the coupling deformation / damage.

1) Semi-direct methods
This method requires physical observation tools (microscope,…) for measuring the surface or the volume of microvoids, and in all cases, it poses serious practical difficulties. Also, the theoretical difficulties appear, (mechanical interpretation of delicate measurements) [9].

2) Indirect Methods
Measurements by the coupling deformation-damage resulting from the modification of the mechanical properties caused by the damage (measuring characteristics of elasticity, viscoelasticity, plasticity, Viscoplasticity, etc.).

These measures are thus closely linked to the chosen model to describe the behavior of the material. We find in [9], [10], measurement techniques for elasticity, plasticity and Viscoplasticity.

We intend to analyze the possibilities of measuring action through the viscoelastic model as it was conditioned above. The damage is linked to the history of the stress which makes complex determination in every load case. We are interested in sinusoidal loads which we will use later.

\[
\sigma(\tau) = \int_{-\infty}^{T} E(t - \tau) \varepsilon(t) \, dt \quad (3)
\]

Suppose a sinusoidal deformation in stress for different stages of damage, it will be assumed that the stress response is sinusoidal (deformation fatigue test) and shifted with respect to deformation of an angle \( \varphi \).

So we have:

\[
\varepsilon(t) = \varepsilon_{0} R_{\varepsilon}(e^{i\omega t}) = R_{\varepsilon}[\varepsilon^{*}(\omega)] \quad (4)
\]

and

\[
\sigma(t) = \sigma_{0} R_{\sigma}(e^{i\omega t}) = R_{\sigma}[\sigma^{*}(\omega)] \quad (5)
\]

Equation (5) then gives:

\[
\frac{\sigma(\tau)}{1-D(\tau)} = \int_{-\infty}^{T} \varepsilon_{0} R_{\sigma}[\omega e^{i\omega t}] E(t - \tau) \, dt \quad (6)
\]

This expression (7) becomes:

\[
\frac{\sigma(\tau)}{1-D(\tau)} = \varepsilon_{0} R_{\sigma}[e^{i\omega t} \cdot i\omega, \omega_{0}] e^{i\omega t} E(u), du \quad (7)
\]

We recognize there the LAPLACE-CARSON transform \( E(t) \) where the argument is \( i\omega \) (complex modulus of undamaged material).

\[
\frac{\sigma(\tau)}{1-D(\tau)} = \varepsilon_{0} R_{\sigma}[e^{i\omega t}, \omega_{0}, \omega] e^{i\omega t} E(u), du \quad (8)
\]

Assuming that the phase difference is very little influenced by the phenomenon of micro cracking (assumption implicitly assumed in the stress tests), \( 1-D(\tau) \) is considered non-periodic variable but slowly over time. We can write complex variable between:

\[
\sigma^{*}(\tau) = [1-D(\tau)], E^{*}(\omega) \varepsilon^{*} = [1-D(\tau)], E(\omega), \varepsilon^{10}, \varepsilon^{*}(\tau) \quad (9)
\]

Let

\[
\frac{E(\omega, t)}{1-D(t)} = \frac{E(\omega)}{(1-D(t))} \quad (10)
\]

and

\[
E^{*}(\omega, t) = E^{*}(\omega), e^{i\varphi} \quad (11)
\]

\( E^{*} \) represents the complex modulus of the material at time \( t \).

Then we have

\[
D(t) = 1 - \frac{E^{*}(\omega, t)}{E(\omega)} \quad (12)
\]
The damage is to be deduced from the modification of the complex modulus of the material assuming the argument $\phi$ to be constant. This expression is only valid in isothermal conditions. Temperature rises influencing the complex modulus are to be considered in any assessment of the damage through this characteristic in the asphalt-coated materials.

II. MECHANICAL MODELING OF THE BEHAVIOR OF ASPHALT MIXTURES

Mechanical modeling of the behavior of the material must be performed in view of the structure calculation. It does not represent the behavior of the material but its behavior in the structure, that’s in its context of use; loading, speed loads, temperatures and environment.

A. Materials

1) Aggregates: Aspect of the grading curve
It seems that the grading curve is not an easy parameter to study since it can’t be considered solely insofar as when the filler content and the bitumen content are a must. However, all the results show that continuous grading curve is better than a discontinuous curve.

The European standard defines the aggregate as the "granular material used in construction. An aggregate can be natural, artificial or recycled."

- the natural aggregate is the aggregate of mineral origin which has not undergone any processing other than mechanical. In this category we settle rock aggregates, such as limestone, porphyry, trap ...;
- the artificial aggregate is the aggregate of mineral origin resulting from an industrial process comprising thermal or other changes. In this category we arrange the processed aggregates, such as expanded shale, expanded clay, expanded mica (vermiculite) ...;
- the recycled granulate is a granulate resulting from the processing of inorganic materials previously used in construction. In this category fall the aggregates such as crushed concrete, the milled asphalt mix...

2) II.1.2 Fines
The fines, also called fines or fines addition, is a fine aggregate, of approximately 0-125 µm and intended to perform, to load various products of the construction industry BPW (Building and Public Works).

It support:
- the artificial asphalt in doses close to 10%, for its power stiffening;
- bituminous coated in neighboring doses of 5 to 6% in the countries of northern Europe, 3-4% in the countries of southern Europe, for its power stiffening;
- hydraulic cement concretes, in doses close to 50 to 80 kg per m³, as a substitution of the cement.

3) II.1.3 Hydrocarbon binder
A hydrocarbon binder is generally an adhesive material (a binder) comprising bitumen, tar, or both. This combined element together with aggregates provides "embedded materials."

B. Characterization of viscoelastic behavior of asphalt mixtures

The asphalt is a viscoelastic material that is to say, its mechanical behavior varies with the conditions of stress: speed, load time, temperature. To characterize the changing behavior of asphalt mix from the state "liquid" to that of a viscoelastic solid, the test measuring their complex modules was developed by Huet[11] It determines the intrinsic characteristics of materials (modulus and phase angle) for a large number of temperature / frequency pairs.

After describing this test and from standard results, we will see that the asphalt mixtures verify the time / temperature equivalence principle and it is then possible to know the properties of the coated at any temperature by building its main curve.

1) characterization in the frequency domain
The characterization of the behavior of bitumen in the frequency domain is done through the trial known as the complex modulus.

That’s to apply to an asphalt specimen $\theta$ for a given temperature, a sinusoidal stress of a frequency $f = \omega / 2\pi$ in the field of small deformations. In the case of a linear viscoelastic behavior, Mandel [12] showed that the response to such request is also a sine wave but phase-shifted by an angle $\phi$.

In practice, a deformation $\varepsilon(t, \omega, \theta) = \varepsilon_0 \sin(\omega t)$ is applied to a test piece of bitumen and response stress $\sigma(t, \omega, \theta) = \sigma_0 \sin(\omega t + \phi)$ is recorded.

The complex modulus $E^*(\omega, \theta)$ is defined by the ratio between the stress $\sigma(t, \omega, \theta)$ and the deformation $\varepsilon(t, \omega, \theta)$.

We can write:

$$E^*(\omega, \theta) = \frac{\sigma(t, \omega, \theta)}{\varepsilon(t, \omega, \theta)}$$

This complex module can also be written in the forms of equations:

$$E^*(\omega, \theta) = |E^*(\omega, \theta)| e^{j\phi(\omega, \theta)}$$

and

$$E^*(\omega, \theta) = E_1(\omega, \theta) + i E_2(\omega, \theta)$$

Where :

- $|E^*(\omega, \theta)|$ and $\phi(\omega)$ are the norm and the phase angle of the complex modulus of the material under the conditions of the test [14], [15];
- $E_1(\omega, \theta)$ represents the dynamic elastic modulus which reflects the instantaneous response of the material. This is the real part of the complex modulus. It quantifies the stored elastic energy; $E_1$ is reversible module associated with the elastic portion of the material;
- $E_2(\omega, \theta)$ represents the loss modulus which represents the viscosity (irreversible module). It quantifies the energy dissipated by internal friction as a result of a solicitation. This dissipated energy is transformed into heat which increase the temperature within the material under cyclic loading.

2) trapezoidal Specimens
As all flexural tests, it is a non-homogeneous test; the trapezoidal specimen is embedded at its large base and biased to its top. Solicitations may be exercised in strength or displacement.
The trapezoidal shape of the test piece is chosen to obtain a maximum deformation outside the mounting region of the sample loaded in cantilever beam. Rupture is generally performed in the vicinity of 1/5 of the total height of the specimen. Great experimental disparity exists in the values of this height. These trapezoidal specimens have a large base of 50 mm, a small base of 25 mm, a height of 250 mm and a thickness of 25 mm. [15].

These sample dimensions allow considering the asphalt samples as continuous and homogeneous media.

Figure 4: (a) Trapezoidal specimen for 2PB and (b) 2 Point Bending Test

The maximum values of stress and deformation $\sigma_{\text{max}}$ and $\varepsilon_{\text{max}}$ achieved by the specimen are calculated from geometric characteristics of samples using formulas from a strength of materials calculation.

\[
\varepsilon_{\text{max}} = \frac{2h}{(b-c)E}\n
\]

\[
\sigma_{\text{max}} = \frac{2h}{2\varepsilon(b-c)}F
\]

The development of these expressions to calculate the module $E$ then allowing the calculation of $E_1$ and $E_2$:

\[
E_1 = \gamma \left[ \frac{F}{b} \cos \phi + \mu a^2 \right] \quad E_2 = \gamma \left[ \frac{F}{b} \sin \phi \right]
\]

Where $F$ is the force measured, $\omega$ the angular frequency, $\gamma$ is the form factor depending on the dimensions of the specimen, $D$ movement at the top of the specimen, $M$ the mass of the sample, $m$ is the mass of the mobile crew.

Phase angle, after a rapid increasing, looks having a constant behavior during the test. Almost for the whole test this values is about 47 degrees. (See Fig.8).

The applied and the recorded waveform look symmetrical around the point zero. Fig.7 shows the typical trend of the stiffness modulus. It is calculated considering the recorded stress divided by the applied strain. As it can be noticed, a three stage evolution process is recorded during a fatigue test. After a rapid evolution of stiffness (phase 1), due to the internal heating phenomenon, stiffness decrease seems more regular (phase 2). Fracture occurs in the final stage (phase 3) and it is characterized by an acceleration of stiffness drop [16].

Figure 6 Stiffness evolution during a fatigue test (160μe)

Figure 7 phase angle evolution during a fatigue test (160μe)

Fig.9 shows the decreasing of the dissipated energy for controlled displacement mode undertaken with the 2PB; instead Figure 10 shows the evolution of the hysteresis loop (dissipated energy) during a fatigue test.
Fatigue tests are undertaken on the same material, at the same loading conditions, same frequency (15 Hz) and same temperature (20°C); however, differences are observed in the determination of stiffness, phase angle, number of cycles of failure and dissipated energy.

Several reasons may explain these behaviours:
- In 2PB and shear stresses exist in the specimen;
- Different boundary conditions: fixed trapezoidal specimen that is glued at the top and at the bottom between two plates in 2PB;
- Presence of shear forces in the 2PB bending test;
- Other phenomena such as heating can influence the result differently in the two kinds of tests.

The most important point is to understand which testing machine simulated better the real fatigue phenomenon; which machine gives us the right response of the material. These are not easy questions, thus further analysis within a wider spectrum of temperatures and frequencies to them is needed.

Furthermore, dissipated energy (depending on strain stress and phase angle) is a good parameter to describe fatigue behavior of bituminous materials and the key point to better understand the mechanical response of the same material for different kinds of the test.

1) the complex modulus test

a) introduction

The test module allows the complex to model empirically the viscoelastic behavior of the asphalt-coated under cyclic loading.

The specimens are subjected to repeated stresses under zero-centered sinusoidal load. The sample complex modulus is measured at different frequencies and at different temperatures. These two parameters are set for each individual measure. Frequencies and temperatures usually tested depend on the test apparatus and capabilities of the materials tested; they vary between -20 °C and 40 °C and that of the frequency from 1 to 40 Hz.

According to the NF-EN-12697-26 standard, the deformations applied to asphalt specimen must be kept below 50.10⁶ m / m to prevent fatigue damage to the mix specimen tested. Note that by increasing the number of load cycles (up to several thousands) on the same complex modulus test (homogeneous or inhomogeneous) it leads to another type of test: fatigue test.

b) Experimental study

\( \mathbf{E}^* \) is a complex number composed of a real part \( \mathbf{E}_1 \) and an imaginary part \( \mathbf{E}_2 \). The \( \mathbf{E}^* \) can also be divided into two parameters in its vector form: its amplitude (\(|\mathbf{E}^*|\)) and argument (\(\phi\)).

The representations of the most used results of the complex modulus are those in the complex plane \( \mathbf{E}_2 = f(\mathbf{E}_1) \) also called representation Cole and Cole (Fig.12). In the space of Black \( \delta = f(|\mathbf{E}^*|) \) called Black curve (Fig.13) and representation of the main curve \( |\mathbf{E}^*| = f(\mathbf{E}_1, \phi) \) (Fig.14) at a given reference temperature.

Figure 9: Evolution of hysteresis loop during a fatigue test

Figure 10: Representation of the complex modulus

Figure 11: Representation in the Cole and Cole plan

B. 1.4.1 Measuring criteria

For asphalt mixes, horizontal bottom layers’ distortion is written by a law of the type:

\[
\varepsilon_{\text{adm}} = \frac{K \varepsilon_0 \left( \frac{N}{10^6} \right)^{-1/b}}{20}
\]

with \(-1/b\) of 5th order.

Figure 12: Black representation in the plan
1) representations of complex modulus results

a) Analysis of results in the complex plane and in the space Black

In Fig.13, which represents the variation of the viscous modulus as a function of the elastic modulus, it appears that the experimental points are placed very substantially on the same global curve, independent of temperature and frequency. On this curve, only one dependent on these factors is the curvilinear abscissa of the experimental point. The important consequence demonstrated by Huet [12] of this representation is that the frequency and temperature play interchangeable roles with respect to complex modulus. This curve tends toward the origin for increasing temperatures and decreasing frequencies, and towards an asymptotic point $E_\infty$, located on the real axis when the frequency approaches infinity and when the temperature decreases. Materials behave increasingly as pure elastic materials when the frequency becomes very great or when the temperature becomes very low. The shape of the curve in Fig.13 is roughly that of a circular arc, passing by $E_0$ (near 0) and $E\infty$, and having a significant asymmetry. Originally, the angle that the experimental curve makes with $E_\infty$ axis is much greater than what it makes with this axis at the other end. The fact that the curve seems to pass through the origin indicates a liquid behavior. This behavior has been observed on creep tests. This curve can be used to calibrate the rheological models. The representation in the complex plane, if it is accurate, however, has a drawback. Indeed, if coordinates are arithmetic, accuracy becomes lower for low values of the module. To improve calibration at low modulus values, we use most often the representation in the Black space. In this new representation, the experimental points fall around the same overall curve. The extrapolation of the curve to $E_0$ is much easier than the complex plane. This curve is used to display the low modulus area and also to reduce of the phase angle for the high temperatures. The argument of the complex modulus also known as phase angle believes under the same conditions, from 0 to a finite value below $\pi/2$. At constant temperature, the reverse occurs when the frequency increases. In this case the material is considered as rheologically simple.

b) Master curve Analysis

When analyzing the isothermal curves shown in Fig.14, we see that the same value of $|E'|$ can be obtained for different pairs (frequency, temperature). This property is called time-temperature equivalence property [17]. It is translated by writing $E'(\omega,T)$ in the form $E'(\omega,a_T)$. When the principle of time-temperature equivalence has been observed for a given material, it is possible to construct a single curve (log-log), where $F$ is the frequency of an arbitrarily chosen reference temperature $T_R$.

This curve is obtained by translation parallel to the frequency axis of each isotherm compared with the isotherm corresponding to the reference temperature till the superposition of the same order points. Thus the obtained curve is called the master curve at $T_R$ reference temperature. It allows obtaining values of modulus at frequencies inaccessible by experimentation. It is generally considered a reference temperature 15°C for coated-materials for the construction of this master curve.

![Figure 13: Design principle of a master curve at 15°C](image)

\[ \Delta H = \frac{\Delta T}{\Delta T_0} \left( 1 - \frac{T}{T_0} \right) \]  

$\Delta H$ is the apparent activation energy depending on the material; $R$ is the constant of perfect gas; $T$ and $T_R$ are the temperatures expressed in Kelvin.

- the relationship of William-Landel-Ferry (WLF): from the analysis of the rheological results on different materials [19] they proposed a law of translation factors variation $\log(a_T)$ between the materials...
experimental isotherms $T$ and a temperature of any reference $T_{ref}$:

$$\log(\alpha_T) = \frac{-c_1(T - T_0)}{c_2(T - T_0)}$$  \hspace{1cm} (22)

$C_1$ and $C_2$ are constants depending on the material studied and have the following properties:

$$C_1', C_2' = C_1 - C_2, C_1' = (T_{ref} - T_0)$$ \hspace{1cm} (23)

$T$ and $TR$ are temperatures expressed in Kelvin; $C_1$ and $C_2$ are obtained at $T_{ref}$, while $C_1'$ and $C_2'$ are obtained at $T_{ref}'$. An example of changing the translation coefficient obtained from the WLF relationship depending on the temperature is shown in Fig. 15.

![Figure 14](image)

**Figure 14**: translation coefficient as a function of temperature $T_{ref} = 15^\circ C$ with $C_1 = 32$ and $C_2 = 216$ of the WLF law

- The translation factor can also be described by a second order polynomial

$$\log(\alpha_T) = aT^2 + bT + c$$  \hspace{1cm} (24)

$a$, $b$, $c$ are regression coefficients.

## IV. REFERENCES


