Investigations of the Energy Processes in the Wheel – Rail Contact Directed at Improving the Safety of Railway Transportation

Joanna Bril, Edward Rydygier

Abstract—**The paper presents a method of modeling energy processes that uses the concept of one−period energy applied to the study on a wheel–rail rolling contact. It was assumed that from the wheel on the rail operates discontinuous periodic force. The dynamic wheel−track system is described by a linear set of differential equations of second order. Solving this set of ordinary differential equations the computer simulations are done for various mechanical conditions. The limits and benefits of the established method are discussed. It has been shown that this method can support diagnostics of railway vehicles and track. The modeling studies presented in this article are the part of a study on the transport safety and the improvement of comfort of journey.**

Index Terms— **Modeling rail vehicle–track system, one−period energy, wheel−rail contact**

I. Introduction

Along with the progress of civilization growing man wants to move farther and faster. In order to fulfill this need of move are being developed easier, cheaper and better ways of transportation. Transportation undoubtedly facilitates a man's life, but also raises the negative side effects, such as environmental pollution, negative impact on human health (e.g. noise, vibration), collisions and accidents including fatal events.

Transportation has a very large impact on the economy and social life, but with the progress of civilization is deteriorating transport safety. Transport safety can be considered in many aspects. In general, the term 'transport safety' includes any action to avoid damage to the goods during the carriage. It is not only the care of cargo during the transport, but also the use of electronic systems to improve safety, the development of transport logistics and appropriate education of staff [3]. In this article we will be dealt with the technical aspect of rail safety. It is presented an original way of modeling contact problems which supports the diagnostics of track and rolling stock, and thereby contributes to the improvement of the technical safety of railway transportation.

II. Technical safety of railway transport

Analyzing the state of rail safety there are listed the following factors affecting the level of safety [3]:

- condition of the railway infrastructure,
- condition of the rolling stock,
- traffic management,
- technology and service conditions, including transport dangerous goods,
- professional and proper performance of duties by employees directly related to the operation and safety of traffic on railway lines and leading railway vehicles,
- supervision by superiors over the work of operating the railway.

Considering the technical aspect of railway transport safety it can be assessed that the most common abnormalities that may result in an accident, and even a train derailment, are the exceeded storage life of the superstructure elements and excessive wear of the elements in rolling contact. Therefore diagnostics investigations of track and rolling stock have a fundamental role in ensuring the safety of transport of passengers and goods. The important issue in a railway diagnostics is to study the effects of contact wheel and rail [9], [11], [15]. Because it is often due to technical reasons it is not possible to obtain a complete set of measurement data required to diagnose the rail and wheel deformation caused by the impact of a rail vehicle on the track, the mathematical modeling is needed to support experimental research.

The analysis of the contact wheel−rail can be carried out using various methods [7], [9], [10], [12], [15]. During the past decade has increased the need for new requirements that should be included in the conceptual work on the development of methods for modeling the dynamic rail vehicle−track. These requirements are primarily related to the reduction of operating costs, new diagnostic techniques and eliminating the negative impact on the environment. The research carried out by specialized research centers focused mainly on increasing the speed of trains, but were neglected ensure transport safety issues and improve ride comfort [7], [8], [9], [13]. Recently, many research centers and academic institutions in the world are carried out studies on different areas of the rail vehicle dynamics, taking into account the high demands on comfort and ensuring the security of transport, with particular regard to the operation of trains with higher speeds. New research challenges require innovative engineering solutions, better understanding of the technical issues and the use of new computer-aimed techniques of calculations [1], [5], [7], [10], [11], [12].

In this paper the method of modeling the contact wheel−rail based on the methods used in the study of electrical circuits using one period energy concept. This method was adapted for modeling the dynamics of the contact wheel−rail on the assumption that in the vertical plane of the rail the normal force has a character of periodic signal. The developed method is a quantitative assessment of energy transferred to the rail in one period.

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III. SYSTEM DESCRIPTION

The issue of contact between wheel and rail is an essential element of the dynamic test rail vehicle–track. Modeling contact wheel–rail covers two aspects:

– geometric or kinematic relations of the wheel-rail contact,

– mechanical relations used to calculate the forces acting in the contact area.

Knowledge of the rail vehicle dynamics enables a value of mechanical factors, such as the stresses in the contact area with the rail wheels or tangential slip forces under various conditions. On the basis of the mechanical factors acting in contact wheel–rail can assess their impact on fatigue and wear of the material of rail and wheel.

Nowadays rail vehicles have a conventional wheel sets consisting of two wheels attached to a common axis (Fig. 1). Railway vehicle can rest on bogies including two wheel sets. The wheels are rigidly connected with the common axis and both rotate at the same speed, and the contact forces are symmetrically distributed on both wheels [8]. Contact forces are the main cause of damage to the rail.

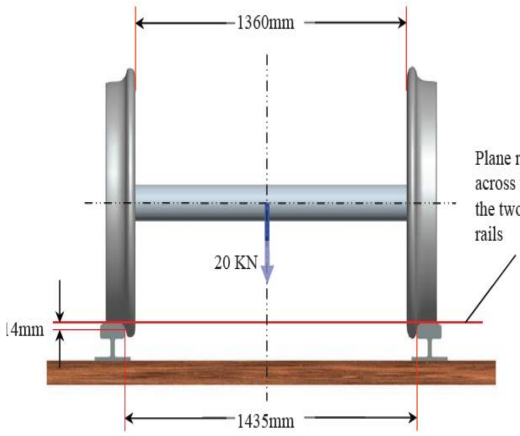

Fig. 1. Convenient wheel set on rails located on sleepers

Wear of the rail in rolling contact with the wheel depends on the development of cracks due to the action of stress force and tangential forces in the contact area. Also the rail wheel slippage has an influence on a rail wear. In addition, the wear rate also depends on a large number of interdependent factors, such as:

- vehicle configuration (wheelbase, axle load, wheel diameter),
- suspension design,
- geometry of the track,
- wheel profiles (nominal profile and after use),
- rail profiles (nominal profile and after use),
- friction between wheel and rail,
- track cant,
- forces of acceleration and braking,
- material properties of wheels and rails.

The contact area between a railway wheel and rail is small compared with their overall dimensions and its shape depends not only on the rail and wheel geometry but also on how the wheel meets the rail influence. The size and shape of the contact wheel and rail can be determined by different techniques [7], [9]. In this paper it is assumed that contact area has roughly 1 cm^2.

IV. DYNAMICAL MODEL OF WHEEL–RAIL CONTACT

Building a dynamic model of contact wheel-rail contact there was considered a elastic rolling contact and it was assumed that the material properties of the wheels and the rails are the same, i.e., these bodies are in a quasi-identical contact. In the test case an analysis of contact problem was reduced to study the interaction between wheel and rail in the vertical plane to the direction of the track. The investigated system was described by the discrete model and divided into segments containing one sleeper. It was assumed that the sleepers lie on a rigid substrate, the tract is described by a linear two-layer model and the deformation is small [8]. Schematic model is shown in Fig. 2.

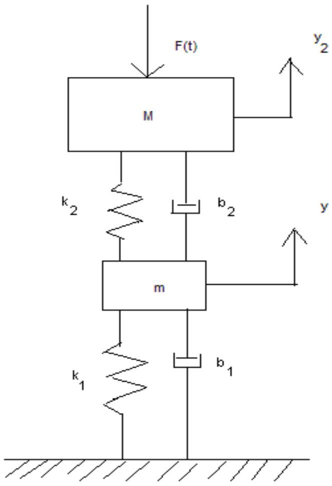

Fig. 2. Discrete model of a wheel–track system

With the above given assumptions, the equations of motion for the wheel–track system can be written as the following set of ordinary differential equations:

$$\begin{aligned} & m\frac{d^2y_1}{dt^2} + b_2\frac{d(y_1-y_2)}{dt} + k_2(y_1-y_2) + b_1\frac{dy_1}{dt} + k_1y_1 = 0 \\ & M\frac{d^2y_2}{dt^2} + b_2\frac{d(y_2-y_1)}{dt} + k_2(y_2-y_1) = F(t)\,, \end{aligned} \tag{1}$$

where $y_1 = y_1(t)$, $y_2 = y_2(t)$ are the displacement in the vertical plane to the track, the mass m and the dynamic parameters (damping and stiffness) b_1, k_1 are associated with a sleeper, the mass M and the dynamic parameters b_2, k_2 are related with the rail.

The system of equations (1) can be presented in the following matrix form

$$\begin{bmatrix} m & 0 \\ 0 & M \end{bmatrix}\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} + \begin{bmatrix} b_2+b_1 & -b_2 \\ -b_2 & b_2 \end{bmatrix}\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} + \tag{2}$$

$$+\begin{bmatrix} k_2+k_1 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ F(t) \end{bmatrix}.$$

It is assumed that the test system is in a steady state, and from the wheel runs on the rail a vertical discontinuous force having the nature of the periodic signal

$$F(t) = f(t+T)\,, \tag{3}$$

where the period $T = \Delta x/\mathrm{v}$, Δx means the distance between axles along the track, v is a velocity of the train.

The time course of force $F(t)$ in the case of a vehicle on the bogies can be described in the following form:

$$F(t) = f(t+T) = \begin{cases} F_0 & for \quad 0 \le t \le T_1 \\ 0 & for \quad T_1 \le t \le T_2 \\ F_0 & for \quad T_2 \le t \le T_3 \\ 0 & for \quad T_3 \le t \le T \end{cases} , \qquad (4)$$

where F_0 is the amplitude of the signal, T_k for k = 1, 2, 3 are time intervals corresponding to intermittent impact force to the rail.

The time course of the force $F(t)$ acting during a one period in the case of a vehicle with bogies (4) is illustrated in Fig. 3.

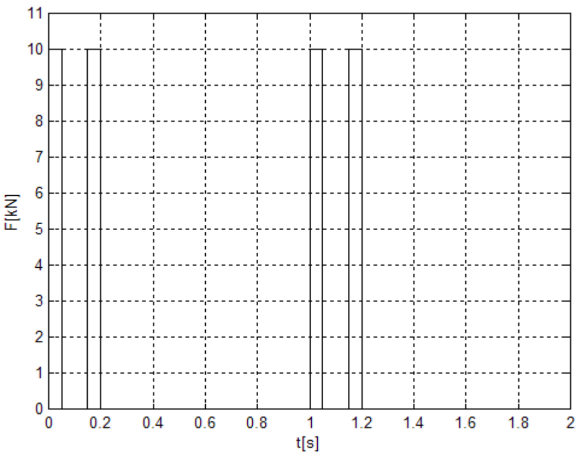

Fig. 3. Loading normal force

The discontinuous periodic loading force shown in Fig. 3 has the amplitude F_0 = 10 kN and the period T = 1 s.

V. ONE-PERIOD ENERGY CONCEPT

The concept of one-period energy is used in a real-time analysis of energy processes in electric circuits in the periodic state. Energy processes can then examine taking into account changes in the instantaneous voltage and current, which are associated with the element of the circuit during one period, and the value of the transferred energy can be evaluated by analyzing the surface area covered by the loop on phase plane of energy. For example, considering the dynamic terminal circuit operating in the non-sinusoidal periodic state (period T), for which the force signal voltage $v(t) = v\,(t + T)$, and the response current $i(t) = i(t + T)$, the energy transferred from the source $v(t)$ to the receiver for a time interval $\Delta t = nT$, $n \in N$ can be designated by the term [14]

$$W(\Delta t) = nW_T\,, \qquad (5)$$

where W_T is one-period energy, i.e. the energy supplied to the receiver during one period of forcing and response.

For periodic signal to force the energy taken from the source by the receiver in a given time interval $\Delta t = nT$ can be reduced to determine the one period energy W_T , and then multiplying this value by n. In the case of testing the effects of discontinuous periodic force $F(t)$ in the area of wheel–rail contact (3), one period energy can be determined from the formula

$$W_T = \int_0^T Fw\,dt = \int_{y(0)}^{y(T)} F\,dy_2\,, \qquad w = \frac{dy_2}{dt}. \qquad (6)$$

The form of expression (6) shows that the area of a loop in energy phase plane with coordinates ($F\,(t)$, $y_2(t)$) determines the energy W_T transferred to a rail in one period. In the case of sinusoidal signal, the loop in the energy phase plane takes the shape of an ellipse.

Using the principle of solve the differential equations, a system of two ordinary differential equations of second-order of two variables (1) was converted into a system of four first-order differential equations of four variables using the substitution [6]

$$\frac{dy_1}{dt} = y_3(t) \qquad \frac{dy_2}{dt} = y_4(t)\,. \qquad (7)$$

Obtained after the substitution (7) a new system of ordinary differential equations was solved numerically using the function *ode23* from the MATLAB library, which allowed to determine the time course of the displacement $y_2(t)$.

VI. RESULTS

For the train speed v = 80 km/h, assuming the length of the contact area $\Delta x = 10^{-2}$ m and using the formula $\Delta x = v\Delta t_s$, the interval of acting the force is estimated as Δt_s = 0.5 ms. Referring to the force as a function illustrated in Figure 4, the signal time intervals are as follows: T_1 = 0.0005 s, $T_2 = T_3 = T$ = 1 s. It means that the rail vehicle with single wheel sets is taken to considerations. The values of the masses and dynamic parameters associated with damping and elasticity are taken from the work [1], [8]. Value of the force amplitude F_0 = 110 kN was assumed.

M = 264 kg, $b_2 = 9.8 \cdot 10^4$ Ns/m, $k_2 = 2.8 \cdot 10^8$ N/m,
m = 308 kg, $b_1 = 1.65 \cdot 10^5$ Ns/m, $k_1 = 1.0 \cdot 10^8$ N/m.

Solving the system of equations (1) the time course of the displacement $y_2(t)$ was determined for a time interval corresponding to one period T = 1 s. The time course of the $y_2(t)$ in the interval (0, 0.0005) s corresponding to the acting of vertical force in a contact area is illustrated in Fig. 4, while the time course of $y_2(t)$ in the range (0.0005, 1) s is shown in Fig 5.

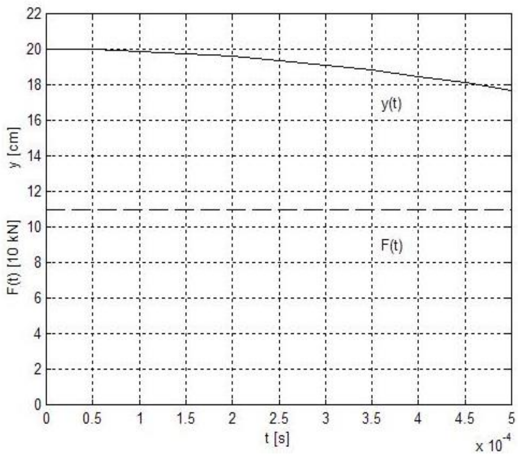

Fig. 4. Displacement for F =110 kN

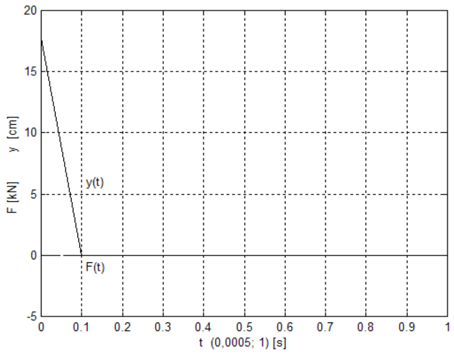

Fig. 5. Displacement for F = 0

In the coordinate system ($F(t)$, $y_2(t)$) the one–period energy loop takes the form shown in Fig. 6.

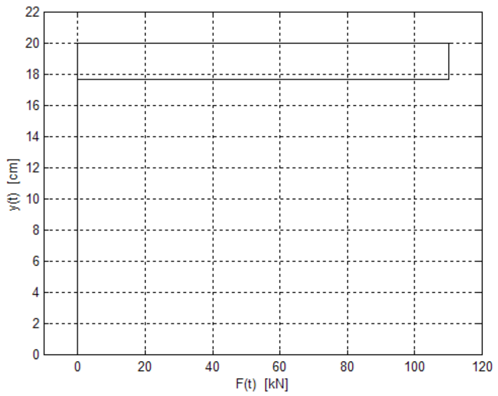

Fig. 6. One−period energy loop, case of the elastic sleeper

From the plot of the loop in phase space energy (Fig. 6) the value of one period energy transferred to rail can be determined as a result of the action of the normal force $F(t)$ in one period T. Taking into account the rectangular surface area of the resulting loop, W_T = 110 kN · 2.3 cm = 2.53 kJ.

Also the calculations were performed for the masses and dynamic parameters taken from the works [8], [15]

M = 264 kg, $b_2 = 6.0 \cdot 10^4$ Ns/m, $k_2 = 1.0 \cdot 10^6$ N/m,
m = 200 kg, $b_1 = 2.0 \cdot 10^5$ Ns/m, $k_1 = 7.0 \cdot 10^7$ N/m.

For this data set the time course of the displacement $y_2(t)$ in time interval (0, 0.0005) s corresponding to the vertical pressure force acting at the contact area is illustrated in Fig. 7, while the time course of $y_2(t)$ in the interval (0.0005, 1) s is shown in Fig. 8.

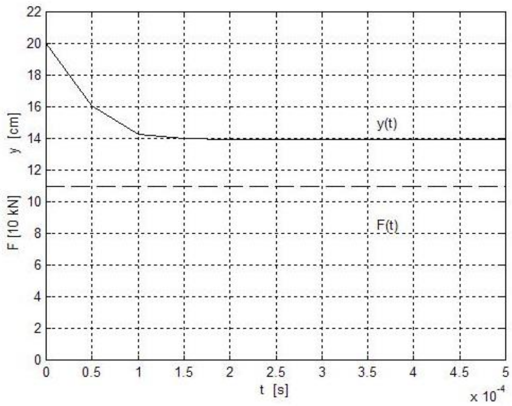

Fig. 7. Displacement for F = 110 kN

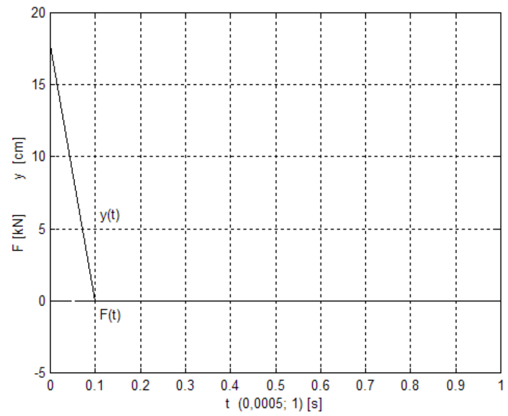

Fig. 8. Displacement for F = 0

One period energy loop in this case takes the form shown in Fig. 9.

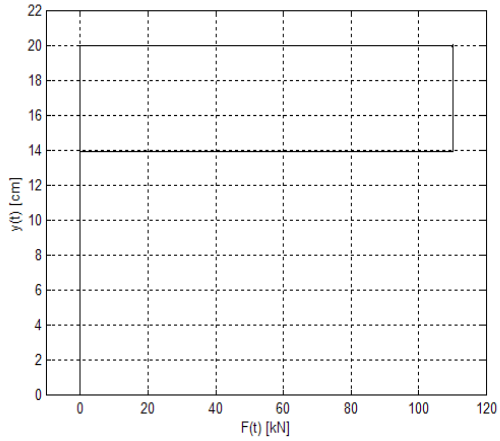

Fig. 9. One–period energy loop, the case of elastic sleeper

From the plot of the energy loop (Fig. 9) the amount of energy transferred to a rail was determined W_T = 6.66 kJ as an effect of acting the normal force $F(t)$ during the period T = 1 s.

The established model of system allows simulations of various parameters of the track, for example, in the case of a rigid foundation on a rigid substrate. Then, the calculations were done including only the parameters associated with the rail $b_2 = 6.0 \cdot 10^4$ Ns/m, $k_2 = 1.0 \cdot 10^6$ N/m. In this case, the time course of the displacement $y_2(t)$ in the interval (0, 0.0005) s is illustrated in Fig. 10. The time course of $y_2(t)$ in the range (0.0005, 1) s is shown in Fig. 11.

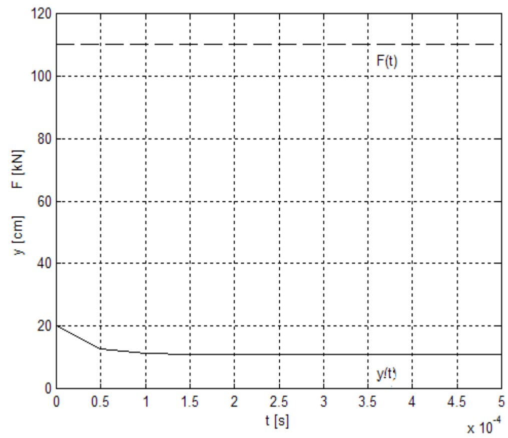

Fig. 10. Displacement (F = 110 kN, rigid sleeper)

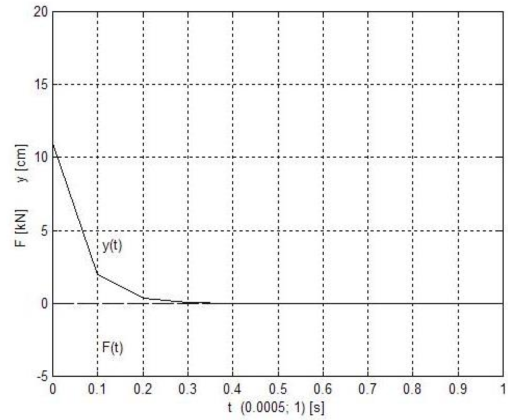

Fig. 11. Displacement (F = 0, rigid sleeper)

One period energy loop in this case takes the form shown in Fig. 12.

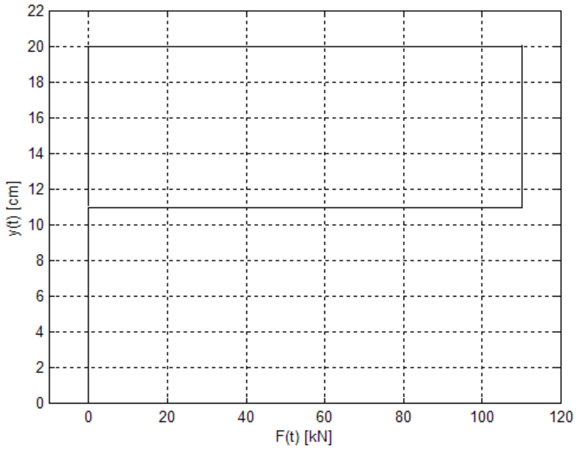

Fig.12. One–period energy loop, case of the rigid sleeper

From the plot of the energy loop the amount of energy transferred to a rail was determined W_T = 9.9 kJ as an effect of acting the normal force $F(t)$ during the one period T = 1 s.

VII. CONCLUSION

It the paper was shown that the method of modeling energy processes with the use of one period energy concept is useful in studies of contact wheel–rail assuming that the vertical force acting on a rail from a wheel is discontinuous and periodic. The advantage of this method is a complete elimination of the frequency analysis, which introduced simplifications in the identification of the contact effects. Another advantageous property of this method is to determine the amount of energy transferred to the rail on the basis of an analysis of the relevant loop diagrams in phase space energy.

The linear model of contact wheel–rail is used in a limited frequency range. Because the frequency as the inverse unit of the period depends on the speed of the train, then in the case of larger frequency the model should be rebuild by introducing stiffness and damping coefficients depending on frequency. Furthermore, to study the relationship of the speed it should be taken into account the excitation induced by imbalances of the track.

Determination of the energy transferred to the rail in rolling contact wheel–rail can be used in diagnostics of rail and the vehicle in the assessment of the degree of wear of the material of rail and wheel, as the cyclic load on the wheel and rail contact area causes cracks develop around sites of destruction. More precise results may be obtained by adjusting the coefficients of stiffness and damping due to the change in the contact material produced during the operation. The use of the established method in assisting the diagnosis has an impact on improving transport safety.

The unfavorable conditions that are detrimental for ride comfort are:

- low damping,
- instability in the vibration of bogie induced by feedback of self-excited wheel vibration with eigenmode vibrations of rail and sleeper,
- resonance eigenmode vibrations of vehicle components with periodic vibrations of loading force.

After modifications the established method for study energy processes in contact wheel–rail can also be used for research into improving ride comfort.

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