Mathematical and Experimental Analysis of Energy Storage for Thermal Underwater Glider

Wenliang Zhang, Zhesong Ma

Abstract—Thermal underwater glider can harvest thermal energy from the temperature difference between surface sea water and deep sea water. PCM (phase change material) which is filled in underwater glider’s heat exchange tube solidifies and melts periodically with volume contraction and expansion when the thermal underwater glider ascends and descends between the surface and deep sea water, thus realizing energy conversion and storage. In this paper the numerical method of equivalent specific heat capacity was used to simulate the phase change process. Using the calculated transient temperature field, we can get the position of solid-liquid moving boundary and mass fraction of liquidus PCM versus time. The theoretical time for complete melting was much less than the experimental time, it was because of the existence of residual air within the phase-change energy storage system. The thermal conductivity of liquidus PCM needs to be modified, the correct coefficient was 0.7-0.9. Combining with the nonlinear model of phase-change energy storage system, system pressure versus time was draw. The curve showed good consistency with experimental data. Related parameters were identified, air solubility in liquidus PCM was 0.05, and residual air within the system was 0.01–0.03. The residual air had great influence on the melting time of PCM and stored energy of phase-change energy storage system. The correct coefficient of thermal conductivity was one key factor to determine the curve of system pressure versus time and its value was fixed by comparing theoretical and experimental melting time of PCM. The fixed value of thermal conductivity was empirical and needs to be further studied.

Index Terms—Underwater glider, Phase change, Residual air, Thermal energy

I. INTRODUCTION

Phase-change energy storage system is a prospective technique for the storage of thermal energy by using phase change material (PCM). During its phase change, PCM can absorb or release large amount of latent heat. This technique is wildly used in domestic and industrial domain like buildings, green houses, solar energy storage and etc.

PCM has been a main research topic for the recent 20 years [1]-[3]. In 2004, P. Lamberg et al. invested the melting and freezing processes in phase change material storage numerically and experimentally [4]. In 2005, H. Kumano et al. studied direct contact melting with hydrocarbon mixtures as the PCM [5]. In 2012, H. H. Al-Kayiem et al. studied thermal behavior of encapsulated phase change material energy storage [6]. Mostly, PCM is used as thermal energy storage medium and its volume change is neglected or avoided. The total volume of PCM contracts when it changes from liquid to solid phase and expands reversely. This feature can be utilized in phase-change driving mechanism like actuator and micro valve. L. Klintberg et al. developed a large stroke, high force actuator which used paraffin as phase change material [7]. R. Liu designed single-use, thermally actuated paraffin valves for microfluidic applications [8]. D. C. Webb et al. designed the first thermal underwater glider using phase-change driving mechanism [9]. Q. Kong et al. studied the phase change process for underwater glider propelled by ocean thermal energy numerically and experimentally [10]-[11]. Z. Hong-Wei et al. putted forward the application and improvement of the interlayer thermal engine powered by ocean thermal energy in an underwater glider [12].

In this paper, combining the numerical method of equivalent specific heat capacity and previously established nonlinear model of energy storage process, the air solubility in liquidus PCM and residual air within the system was identified according to experimental data. The theoretical curve of system pressure versus time was gained and showed good accordance with experimental curve. The experimental data also verified the numerical method indirectly.

II. PROCEDURE FOR PAPER SUBMISSION

The whole configuration of thermal underwater glider is shown in Fig. 1, the heat exchange tube which is filled with PCM is attached to the main body, and PCM can absorbs and releases heat through the tube wall with outside sea water during phase change process. The cross section of heat exchange tube is described in Fig. 2, the middle of heat exchange tube is rubber hose which is filled with oil and connected with hydraulic circuit. The remaining space of heat exchange tube is filled with PCM of constant mass. When PCM solidifies, oil can be absorbed into the rubber diaphragm from the oil tank with low pressure; when PCM solidifies, oil can be extruded into the accumulator thus increasing the system pressure and realizing energy conversion and storage.

In order to analysis the system pressure versus time during melting process, it needs to combine equivalent specific heat capacity method and the previously established nonlinear model of energy storage process for thermal underwater glider. Through the numerical method, we can get mass fraction of liquidus PCM versus time, and through the nonlinear model of energy storage for thermal underwater glider, we can get the system pressure versus mass fraction of liquidus PCM. After that, we can draw the curve of system pressure versus time and analysis the deviation between theoretical and experimental data and do the following parameter identification.
A. Equivalent specific heat capacity method

During the solidification process of PCM, the outer layer of PCM solidifies continuously, and the boundary of solid and liquid phase moves toward the center point of the cross section continuously. During the melting process, the condition is similar. This problem is called moving boundary problem or Stefan problem [13]. Except for some simple situation which can be solved by analytical method, in actual engineering application, this problem can only be solved using numerical methods [14]-[15].

In this paper, the melting process is simulated using the numerical method of equivalent heat capacity. Based on the calculated transient temperature field, the mass fraction of liquidus PCM versus time is gained consequently.

The melting process of PCM in heat exchange tube is a three-dimensional heat transfer process, and the solving zone also changes because of the density difference of liquidus and solidus PCM. In the following numerical simulation, the solving zone is assumed to be constant and meshed using fixed grid not moving grid for simplicity [16]. In order to simplify the numerical method, assumption is made as follows:

1) Ignore the heat resistance of heat exchange tube;
2) Axial temperature distribution of heat exchange tube is equal;
3) Phase change material is isotropic and homogeneous;
4) Ignore the effect of temperature on density and other thermal parameter of PCM;
5) Influence of system pressure on phase change process is also neglected.

For a control volume with constant mass, assume the phase change happens in a narrow temperature range \[T_m - \Delta T, T_m + \Delta T\]

\[
\rho c \frac{\partial T}{\partial t} = k \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right)
\]

For a control volume with constant mass, assume the phase change happens in a narrow temperature range \[T_m - \Delta T, T_m + \Delta T\]

\[
f_s = \begin{cases} 
1 & T < T_m - \Delta T \\
\frac{T - T_m}{2\Delta T} & T_m - \Delta T < T < T_m + \Delta T \\
0 & T > T_m + \Delta T 
\end{cases}
\]

Where \(T_s = T_m - \Delta T\). Its value range is \([0, 1]\). When \(f_s = 0\), it means the control volume is solid phase, when \(f_s = 1\), it means the control volume is liquid phase, and when \(0 < f_s < 1\), it means mash zone with solid and liquid phase.

For a control volume of constant mass, mass fraction \(f_s\) is also the volume fraction of solid phase.

Since the convection of liquidus PCM is remarkable during melting process [17]-[18], its effect should be taken consideration by introducing effective thermal conductivity.

\[
k_{eff} = 0.18 * R_0^{0.25}
\]

Using this numerical method, the transient temperature field can be calculated. Through the following process, the curve of mass fraction of liquidus PCM versus time in the heat exchange tube can be got.

1. Tracking the moving boundary of solid and liquid phase

For a meshed solving zone, when the temperature of two adjacent nodes meets the constraint \(T(i) > T_m > T(i + 1)\), then we can get the position of moving boundary by

\[
r_m = r(i + 1) + \frac{T_m - T(i + 1)}{T(i) - T(i + 1)} * \Delta r
\]

Where \(r(i + 1)\) and \(r(i)\) are the position of nodes \(i, i + 1\) in radial coordinate system respectively and \(r_m\) the position of moving boundary.

2. Mass fraction of liquidus PCM during melting process

Since the remaining space of heat exchange tube which is filled with PCM is annulus, and the melting process is simplified as a one-dimensional heat transfer problem, mass fraction of liquidus PCM during phase change process can be calculated by dividing the liquidus area with the whole annulus area.

\[
w = \frac{R_0^2 - S^2}{R_0^2 - R_i^2}
\]

Where \(R_0\) is the outer radius of the annulus area, and \(R_i\) the inner radius of the annulus area.

3. Modification of thermal conductivity of liquidus PCM

When comparing the theoretical time for complete melting with experimental time, the former is much less than the latter, the main reason is residual air within the system. It forms bubble in the beginning of melting process thus decreasing the melting process severely. The actual thermal conductivity of liquidus PCM is less than the theoretical one. It is needed to be modified by correction coefficient beta. The formula of beta is given by...
\[ \beta = \frac{T_r}{T_c} \]  

(7)

With \( T_r \) the whole time for complete melting in theory, and \( T_c \) the whole time in experiment.

**B. Nonlinear model of phase-change energy storage system**

Thermal underwater glider can harvest environmental energy from the temperature difference of surface sea water and deep sea water. The actual component includes heat exchange tube, connected with hydraulic circuit, the inner oil tank, accumulator, etc. It can be simplified as a phase-change energy storage system which is showed in Fig. 3. 4. The performance of the phase-change energy storage system is significant to harvest and conversion of thermal energy from the sea water. Because the system is inevitably mixed with a certain amount of air, and even a small amount air will also have a severe effect on the process of increasing pressure phase. Based on the previous theoretical analysis, the formula for solving system pressure at different stage is obtained.

The critical pressure when residual air within system does not exist is described by

\[ P_c = \frac{(s_0 + \frac{a}{1 - a})}{s_0 \cdot w} \]  

(8)

During increasing pressure phase 1, the system pressure is

\[ P_1 = -B + \frac{\sqrt{B^2 - 4A \cdot C}}{2A} \]  

(9)

During increasing pressure phase 2, the system pressure is

\[ P_2 = P_1 \cdot \frac{B(1 - \frac{\rho_0}{\rho_{\infty}}) + P_0 - (s_0 + \frac{a}{1 - a}) \cdot \frac{P_0}{P_1} \cdot \frac{T_c}{T_0} \cdot \frac{B}{w}}{P_0 - (s_0 + \frac{a}{1 - a}) \cdot \frac{P_0}{P_1} \cdot \frac{T_c}{T_0} \cdot \frac{B}{w}} \]  

(10)

Where

\[ A = \frac{w}{B} \]  

(11)

\[ B = (s_0 \cdot \frac{T_c}{T_0} + \frac{\rho_0}{\rho_{\infty}} \cdot \frac{P_0}{B} - 1) \cdot w + (s_0 + \frac{a}{1 - a}) \cdot \frac{P_0}{P_1} \cdot T_0 \]  

(12)

\[ C = -(s_0 + \frac{a}{1 - a}) \cdot \frac{P_0}{P_1} \cdot \frac{T_c}{T_0} \]  

(13)

At the turning point of increasing pressure phase 1 and increasing pressure phase 2, we can get the constraint equation \( P_1 = P_2 = P_c \), namely

\[ \frac{s_0 + \frac{a}{1 - a} \cdot \frac{P_0}{P_1} \cdot \frac{T_c}{T_0} \cdot \frac{B}{w}}{s_0 \cdot w} = B(1 - \frac{\rho_0}{\rho_{\infty}}) + P_0 - (s_0 + \frac{a}{1 - a}) \cdot \frac{P_0}{P_1} \cdot \frac{T_c}{T_0} \cdot \frac{B}{w} \]  

(14)

1) When the value of residual air is zero, air solubility in liquidus PCM is

\[ s_0 = \frac{B(1 - \frac{\rho_0}{\rho_{\infty}}) + 1)w_c \cdot \frac{1}{B \cdot T_0 \cdot \frac{T_c}{P_0}}}{P_0 \cdot (1 - \frac{\rho_0}{\rho_{\infty}}) + 1} \]  

(15)

2) When the value of air solubility is already calculated, we can get the value of residual air by

\[ a = \frac{X - s_0}{X - s_0 + 1} \]  

(16)

Where

\[ X = \frac{B}{P_0} \left( \frac{1 - \rho_0}{\rho_{\infty}} + 1 \right) w_c \cdot s_0 \]  

(17)

**III. EXPERIMENT AND ANALYSIS**

Hexadecane is selected as PCM for thermal underwater glider because of its suitable melting temperature and certain amount of density difference. Other features of hexadecane are as follows: it is compatible with the heat exchange tube, its performance is stable for long duration and it is safe without toxicity or corrosivity. Detail property of hexadecane is listed in Table 1.

<table>
<thead>
<tr>
<th>Property</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melting temperature ( T_m )</td>
<td>℃</td>
<td>16-18°</td>
</tr>
<tr>
<td>Latent heat of fusion ( L )</td>
<td>KJ/kg</td>
<td>236</td>
</tr>
<tr>
<td>Thermal conductivity ( k )</td>
<td>W/m K</td>
<td>0.313/0.14</td>
</tr>
<tr>
<td>Specific heat ( c )</td>
<td>J/kg K</td>
<td>1.64/2.09</td>
</tr>
<tr>
<td>Density ( \rho_{\infty} / \rho )</td>
<td>Kg/m³</td>
<td>864/773</td>
</tr>
</tbody>
</table>

The actual melting temperature of PCM is a constant value, it is transformed into a narrow range which is required by the numerical method.

The research team of Tianjin University had done a series of experiments on Qiandao Lake of Zhejiang province in September 2013. The temperature distribution of the testing water is shown in Fig. 5. The surface water temperature is about 27℃, and the bottom water temperature is about 10.5℃. At the beginning of testing, underwater glider was set aside on the bottom of the lake for a long enough time. After PCM in the heat exchange tube completely solidifies, rapidly raises it to the surface and set aside. PCM begins to melts with volume expansion. Since oil was extruded from the rubber hose of heat exchange tube to the accumulator, the system pressure increases continuously until PCM completely melts. Underwater glider automatically records the system pressure versus time during the whole melting process. The possible melting and temperature change of PCM during underwater glider ascending from bottom to surface water is ignored since time is little.

**A. Raw experimental data**

The curve of system pressure versus time can be divided into three sections roughly as shown in Fig. 6 and Fig. 7. Taking Fig. 7 as an example, after thermal underwater glider ascends to the surface water, PCM begins melting. Because the existence of residual air within the system, system pressure increases slowly until it reaches the value of critical pressure, and this stage is named as increasing pressure phase 1. When system pressure exceeds the critical pressure, residual air dissolves completely in melted PCM, and system...
pressure increases sharply until it reaches the set value of high pressure tank, this stage is named as increasing pressure phase 2. After that, system pressure keeps constant as the set value of high pressure tank and oil is transferred into high pressure tank continuously until PCM melts completely, this stage is named as energy storage phase. These three stages are divided by two turning points.

In this paper, only increasing pressure phase 1 and increasing pressure phase 2 were analyzed, and mainly focus on the turning point of those two stages. When simulating the melting process using the numerical method of equivalent specific heat capacity, the influence of system pressure on phase change is ignored, namely the property of PCM keeps constant no matter what the value of system pressure is.

The experimental data is showed in Table 2. Among the 8 groups, the first 7 groups are raw experiment data, group #8 is calculated by mean of corresponding item respectively.

**B. Related parameter identification**

Taking group #8 as an example, using the experimental and theoretical value ($T_t=2909$) of whole time for PCM completely melting, then correct coefficient beta can be calculated. After modifying the thermal conductivity of liquidus PCM, the numerical method of equivalent specific heat capacity was applied again to simulate the phase change process. The curve of mass fraction of liquidus PCM versus time was drawn.

In experimental curve, after finding the turning point and its corresponding time, we can get the corresponding mass fraction of liquidus PCM in the above simulated curve. It means although PCM melts continuously, the system pressure didn’t show obvious increase until certain amount of PCM melts because of the existence of residual air in the system.

According to the above two steps, calculate the corresponding mass fraction of liquidus PCM for group #1–#7, as shown in Fig. 8. When assuming the typical value of air solubility is 0.08, calculated results showed residual air for all groups are negative, indicating the actual value of air solubility is less than 0.08. Here, according to the calculated value of mass fraction in #1–#3 team, assume the value of residual air for these three team is zero, then by Equation (15), we can get the value of air solubility for these three teams. The calculated values show good consistency. For the following #4–#8 team, using the already calculated value of air solubility and Equation (16), we can get the value of residual air respectively. The detail results of air solubility and residual air for all the teams are shown in Table 3.
Table 2. Raw experimental data

<table>
<thead>
<tr>
<th>Testing group</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time for complete melting (s)</td>
<td>413</td>
<td>334</td>
<td>348</td>
<td>347</td>
<td>371</td>
<td>358</td>
<td>341</td>
<td>3594</td>
</tr>
<tr>
<td>Time for turning point (s)</td>
<td>851</td>
<td>997</td>
<td>952</td>
<td>697</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steady system pressure (MPa)</td>
<td>12</td>
<td>12.1</td>
<td>12.2</td>
<td>12.5</td>
<td>12.1</td>
<td>12.4</td>
<td>12.1</td>
<td>7</td>
</tr>
</tbody>
</table>

*Group #8 is not actual raw experimental data, the corresponding item is calculated by average.

Table 3. Parameter identification.

<table>
<thead>
<tr>
<th>Testing group</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>air solubility ( s_0 )</td>
<td>0.052</td>
<td>0.052</td>
<td>0.052</td>
<td>0.052</td>
<td>0.052</td>
<td>0.052</td>
<td>0.052</td>
<td>0.052</td>
</tr>
<tr>
<td>residual air ( a )</td>
<td>0</td>
<td>3</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>0.010</td>
<td>0.016</td>
</tr>
</tbody>
</table>

*Group #1~#3 are assumed to have zero value of air solubility.

C. Theoretical curve of system pressure versus time

Taking group #8 as an example, based on the correction coefficient beta and numerical method, we can draw the curve of mass fraction of liquidus PCM versus time during PCM melting process. In the beginning PCM melts quickly, then its melting rate gradually slows down. The melting rate decreases continuously during the melting process. The whole time for complete melting is 3557s using modified thermal conductivity of liquidus PCM.

Based on the two identified parameters of air solubility and residual air within the system, the curve of system pressure versus mass fraction of liquidus PCM is obtained. When the mass fraction of liquidus PCM is 0.482, it changes from increasing pressure phase 1 to increasing pressure phase 2, and the system pressure increases sharply after the turning point.

Combining these two curves, we can draw the curve of system pressure versus time. As shown in Fig 9, when the time is 697.2s, it changes from increasing pressure phase 1 to increasing pressure phase 2, and the system pressure at this turning point is 0.254MPa. The turning point and changing trend of this curve showed a very good consistency with #1~#7 experiment data. It also verified the numerical method and nonlinear model indirectly.

IV. CONCLUSION

The mass fraction of liquidus PCM versus time during melting process is gained using the numerical method. Combining the nonlinear model of phase-change energy storage system, the curve of system pressure versus time is obtained and showed a good consistency with experiment. It also verified the numerical method indirectly. The identified parameters are as follow:

1) During the melting process of PCM, residual air within the system can severely decreases the melting rate, resulting the whole time for complete melting in experiment much more than that in theory. It needs to modify the thermal conductivity of liquidus PCM. The value range of correction coefficient beta in this paper is 0.7~0.9;

2) The step to identify the relevant parameters is established. The calculated value of air solubility in liquidus PCM is 0.05, and the value range of residual air within the system is 0.01~0.03;

The correct coefficient of thermal conductivity is empirical and needs to be further studied.

NOMENCLATURE

\( \rho \) Density (kg/m^3)
\( c \) Specific heat (J/kg \cdot K)
\( L \) Latent heat of fusion (J/kg)
\( k \) Thermal conductivity (W/m \cdot K)
\( f \) Particle fraction of liquid/solidified PCM
\( T \) Temperature (K)
\( t \) Time (s)
\( r \) Radial coordinate (m)
\( L \) Latent heat (kJ/kg)
\( T_m \) Melting temperature (K)
\( R_y \) Rayleigh number

Fig. 9 System pressure versus time
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R_c Diameter of heat exchange tube (m)
R_i Diameter of rubber diaphragm (m)
β Correction coefficient
ω Mass fraction of liquid/solidified PCM
s_0 Air solubility under standard condition
P Pressure (Pa)

Subscripts

eq Equivalent
l Liquid phase
s Solid phase
0 Reference / atmosphere
l Low
h High

PCM Phase change material
PCES Phase-change energy storage

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REFERENCES


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