

# Research of Fractional Order Fuzzy Speed Control on PMSM

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**Abstract**—In order to overcome the nonlinear characteristics of permanent magnet synchronous motor (PMSM), fractional order (FO) fuzzy proportional integral (PI) controller was designed based on the fractional order calculus and fuzzy technology for PMSM with the nonlinearity and time-varying parameters at work. Fuzzy control rules are designed in the light of running state of EV, Discrete fractional order fuzzy PI controller is obtained by using bilinear transform and continued-fraction expansion and fuzzy control rules. The simulation results show that the proposed method can make PMSM driving system to get good control performance and strong robustness.

**Index Terms**—Fractional order calculus; permanent magnet synchronous motor; fuzzy control.

## I. INTRODUCTION

In recent years, electric vehicles have been paying increased attention. The motor control is the key of the electric vehicles technologies, which has drawn the interest of researchers from across the globe.

Permanent magnet synchronous motor (PMSM) is increasingly used in EV due to its simple structures, high efficiency, low inertia, high power density and high torque-current ratio. In order to obtain good characteristics, some control strategies were proposed like PI control [1]-[2], adaptive control [3]-[4], sliding mode control [5]-[6], and intelligent control [7]-[8]. In particular, Fuzzy optimization gets addressed in PMSM control. Elmas C [9] proposed training parameters of fuzzy controller using four layers neural network. Fang and Yu [10]-[11] used back stepping and fuzzy optimization to overcome parameter uncertainty and load disturbance and obtain good control of position tracking. Choi [12] proposed a speed controller on the basis of Takagi-Sugeno fuzzy method.

Fractional order calculus extends integer order ones to non-integral order, which can model various real materials more adequately than integer order ones. This study introduces fractional order calculus into fuzzy PI control and establishes the structure of fractional order fuzzy proportional integral controller. Fuzzy reasoning rules are set on the basis of actual running states of EV. Fractional order calculus more really describes the physical object and spreads the stable region of the system parameters. The proposed method is applied to permanent magnet synchronous motor speed control in order to improve robustness of system against the

disturbances. Simulation results verify that the proposed control method has given good robust results and fast speed tracking performance.

## II. MATHEMATICAL MODEL OF THE PMSM

The mathematical model of the PMSM can be obtained according to the following assumption.

Assumption: Magnetic saturation and iron losses are neglected.

Assumption: The back emf is sinusoidal.

The electromagnetic torque model and speed dynamic model of PMSM can be described in the d-q frame.

$$T_e = \frac{3}{2} p_n \left[ \psi_f i_{sq} + (L_d - L_q) i_{sd} i_{sq} \right] \quad (1)$$

$$\frac{d\omega}{dt} = \frac{1}{J} (T_e - B\omega - T_m) \quad (2)$$

where,  $i_{sd}$  and  $i_{sq}$  are stator d-axis current and q-axis current respectively;  $\psi_f$  is amplitude of the flux induced by the permanent magnets of the rotor in the stator phases;  $\omega$  is angular speed of the motor;  $p_n$  is number of pole pairs;  $B$  is viscous friction coefficient of rotor and load;  $J$  is inertia of moment;  $T_m$  is the load torque.

## III. DESIGN OF FO FUZZY PI CONTROLLER

### A. Fractional order operator

Fractional order calculus is a basic subject with mathematics to study differential coefficient and integral of random order, which can be expressed in calculus operator of fractional order and integral order [13]

$$D_t^\lambda = \begin{cases} d^\lambda/dt^\lambda, & R(\lambda) > 0 \\ 1, & R(\lambda) = 0 \\ \int_a^t (d\tau)^{-\lambda}, & R(\lambda) < 0 \end{cases} \quad (3)$$

where,  $a$  and  $t$  are upper limit and lower limit of calculus operator respectively;  $\lambda$  is the order of calculus;  $R(\lambda)$  denotes real number.  $R(\lambda)$  is integral order calculus when  $\lambda$  is integer;  $D_t^\lambda$  denotes fractional differential when  $\lambda$  is real number greater than zero;  $D_t^\lambda$  denotes fractional integral when  $\lambda$  is real number less than zero.

Fractional order calculus extends the order number of calculus to fraction and more really describes the physical object and spreads the stable region of the system parameters. The structure of fractional order PI controller can be established and be expressed as

$$u = k_p e + k_i D_t^{-\lambda} e \quad (4)$$

where,  $k_p$  and  $k_i$  are proportionality coefficient and integral coefficient.

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Using Laplace Transform, transfer function for  $u$  and  $e$  is inferred as follow

$$G(s) = \frac{U(s)}{E(s)} = k_1 + k_2 s^{-\lambda} \quad (5)$$

In order to obtain discrete mathematical model for number control, the fractional order operator need to be discretized. The bilinear transformation and continued fractions expansion technique are used to obtain the approximate discrete model. When  $\lambda$  equals 1.2, sampling period equals 1ms and the order of the polynomial function equals 4, the discrete model follows.

$$Z(s^{-1.2}) \approx \frac{num}{den} \quad (6)$$

where,  $num$  and  $den$  are described as

$$num = -0.0035 - 0.0041z^{-1} + 0.0008z^{-2} + 0.0016z^{-3} + 0.0001z^{-4}$$

$$den = -15.54 + 21.75z^{-1} + 0.27z^{-2} - 7.33z^{-3} + z^{-4}$$

**B. Fuzzy controller**

In the fractional order PI controller, the parameters  $k_p$  and  $k_i$  are be adjusted by fuzzy controller. The frame of fuzzy fractional order PI controller is shown in Fig 1.  $\omega^*$  and  $\omega$  are speed reference and actual value respectively.  $e$  and  $e_c$  are speed error and speed error ratio respectively.  $\Delta k_p$  is variation of  $k_p$ .  $\Delta k_i$  is variation of  $k_i$ .

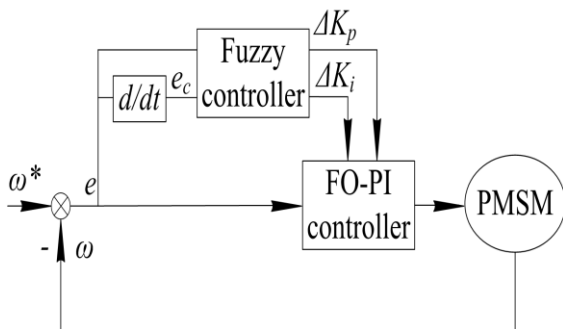


Fig. 1 fractional order fuzzy PI controller

The fuzzy controller performs fuzzy reasoning of fuzzy rules based on the input variables,  $e$  and  $e_c$ . The output variables of fuzzy controller are used to adjust the  $k_p$  and  $k_i$  to ensure the good static and dynamic characteristics of motor under various running states.

The domain of discourse of the input and output variables are set as  $\{-3, -2, -1, 0, 1, 2, 3\}$ , which is expressed as  $\{NB, NM, NS, ZE, PS, PM, PB\}$ .

The membership function of fuzzy net adopts the triangle function that converts the exact variables in input analytical space to variables in output fuzzy space..

According to step response characteristic of PMSM and the effect of  $k_p$  and  $k_i$  on the step response, fuzzy logic rules are made. Mamdani algorithm is used in the process of fuzzy reasoning in terms of the relation between the minimax value and maximin value of a function. Fuzzy decoupling of the fuzzy control adopts the method of weighted mean, which can take into account elements of all membership degrees and not cause to neglecting the effect of some part..

The fuzzy controller is shown in Table 1 and Table 2.

Tab. 1  $\Delta k_p$  fuzzy control

$e$	$e_c$						
	-3	-2	-1	0	1	2	3
-3	3	3	2	2	1	1	0
-2	3	2	2	2	1	0	1
-1	2	2	2	1	0	1	2
0	2	2	1	0	1	2	3
1	2	1	0	1	2	3	3
2	1	0	1	2	3	3	3
3	0	1	2	3	3	3	3

Tab. 2  $\Delta k_i$  fuzzy control

$e$	$e_c$						
	-3	-2	-1	0	1	2	3
-3	3	3	2	2	1	1	0
-2	3	2	2	2	1	0	1
-1	2	2	2	1	0	1	2
0	2	2	1	0	1	2	3
1	2	1	0	1	2	3	3
2	1	0	1	2	3	3	3
3	0	1	2	3	3	3	3

**IV. EXPERIMENT ANALYSIS**

To illustrate the effectiveness of the proposed method, the simulation test is be done for the PMSM in several operating conditions.

The parameters of the PMSM are follows: rated power equals 1.5kW; rated voltage equals 72V; rated current equals 25A; rated speed equals 2800r/min; number of poles  $p_n$  equals 4. The results of the fractional order fuzzy PI controller and the PI controller are compared to observe the performance of the proposed control method.

To verify performance, a reference step input of 1500 r/min is applied at  $t = 0.02$  s. Then, a load disturbance of 3Nm is used at  $t = 0.05$  s.

Figure 2 - 4 show dynamic step response curves of PMSM using the fractional order fuzzy PI controller and the PI controller respectively. While the rotor speed is changed from 0rpm to 1500rpm, the speed response provided by the proposed control method gives lower rise times. But the overshoots are reduced.

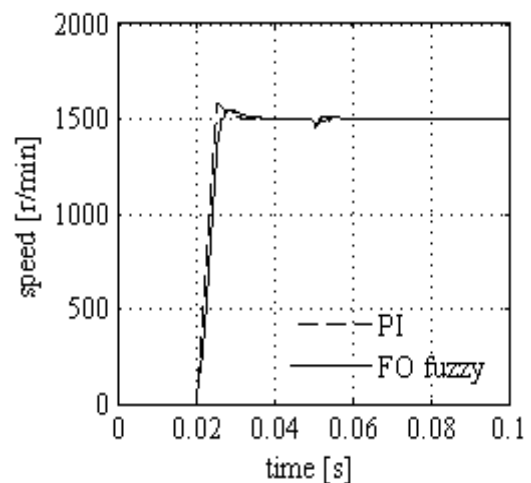


Fig. 2 the speed response of a PMSM

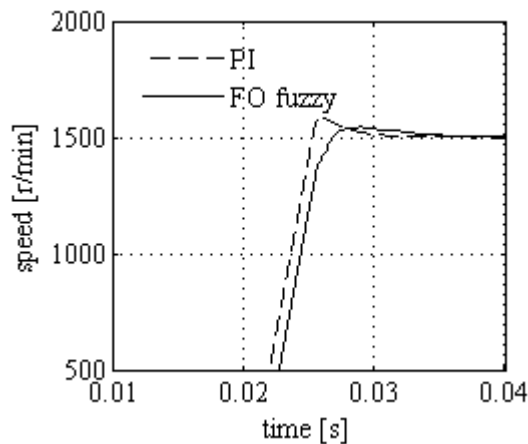


Fig.3 the speed response in the no-load state

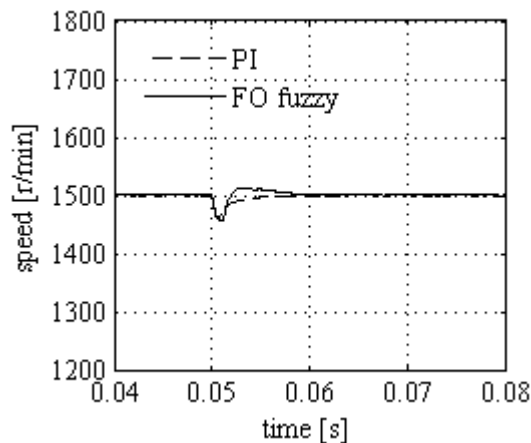


Fig. 4 the speed response under the load disturbance

## V. CONCLUSION

This paper proposes a new control algorithm for PMSM. Based on the PI controller, the fractional order operator is applied to integral loop and the fuzzy controller is used to adjust the parameters of the PI. The fractional order calculus can describe the real physical object more nearly and extend the range of stability domain. fuzzy logic rules have been designed to meet the motor state.

The results of the proposed scheme show that speed responses of this controller has the advantages of little excessive regulation and good stability.

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