Elastico-Viscous MHD Free Convective Flow Past an Inclined Permeable Plate with Dufour Effects in Presence of Chemical Reaction

Kamal Debnath

***Abstract*— An attempt has been made to study the unsteady MHD two-dimensional free convective elastico-viscous fluid past an inclined permeable plate with Dufour effects in presence of chemical reaction. The elastico-viscous fluid flow is characterized by Walters liquid (Model B'). The perturbation scheme is used to solve the governing equations of the fluid motion. The approximate solutions have been derived for the velocity profile, temperature field, concentration field, skin friction, rate of heat transfer and rate of mass transfer. It is noticed that the flow field is considerably affected by the elastico-viscous parameter. The velocity profile and the approximate skin friction coefficient have been presented graphically to observe the elastico-viscous effects for various values of the flow parameters involved in the solution.**

***Index Terms*— Chemical Reaction, Dufour effects, MHD, Skin friction**

I. INTRODUCTION

The analysis of free convection flow adjacent to inclined surface bounded by an extensive body of fluid is of considerable importance in micrometeorology and industrial applications. Investigation of chemical reaction, heat and mass transfer in MHD elastico-viscous fluid flow for an electrically conducting fluid past a heated surface has attracted the interest of many researchers in view of its applications in many engineering problems such as plasma studies, petroleum industries, MHD power generator, cooling of nuclear reactors, MHD propulsion thermo fluid dynamics, boundary layer control in aerodynamics, etc. Process involving heat and mass transfer in porous media are often encountered in the chemical industry and in reservoir engineering in connection with thermal recovery process.

A comprehensive review of the studies of heat transfer mechanism through porous media has been made by Neild and Bejan [1]. The study of natural convection flow of inclined plate has been presented by the authors Ganesan and Palani [2] and Sparrow and Husar [3]. Free convection boundary layer flow over a horizontal and slightly inclined surface has been studied by Pera and Gebhart [4]. Karkus [5] has applied the perturbation technique to study the natural convection flow adjacent to inclined isothermal and finite-length surfaces. Hossain et al. [6] have investigated the free convection flow from an isothermal plate inclined at a small angle to the horizontal. Raptis et al [7] have investigated the viscous flow over a non-linearly stretching sheet in presence of chemical reaction and magnetic field. Rajesh et al [8] have studied the chemical reaction and radiation effects on MHD flow past an infinite vertical plate with variable temperature. Diffusion thermo and radiation effects on MHD free convective heat and mass transfer flow past an infinite plate in the presence of chemical reaction of first order has been studied by Raveendra Babu et al [9]. The heat and Mass Transfer in MHD Boundary Layer Flow past an inclined plate with viscous dissipation in porous medium has been analyzed by Singh P K [10]. Considering the model of elastico-viscous fluid, many scientists have solved problems of engineering interests viz. Saxena and Dubey [11], Choudhury and Mahanta [12],Choudhury and Dey [13], Choudhury and Dhar [14].

The aim of the present work is to study the diffusion thermo (Dufour effect) and radiative effects on MHD unsteady free convective flow of elastico-viscous fluid past an inclined permeable plate characterized by Walters liquid (Model B') . The effects of the elastic-viscous fluid on velocity component and skin friction coefficient have been shown graphically with the combination of other flow parameters involved in the solution.

The constitutive equation for Walters liquid (Model B') is

$$\sigma^{ik} = -pg_{ik} + 2\eta_0 e^{ik} - 2k_0 e'^{ik} \qquad (1)$$

where σ^{ik} is the stress tensor, p is isotropic pressure, g_{ik} is the metric tensor of a fixed co-ordinate system x^i, v^i is the velocity vector, the contravariant form of e'^{ik} is given by

$$e'^{ik} = \frac{\partial e^{ik}}{\partial t} + v^m e^{ik}_{,m} - v^k_{,m} e^{im} - v^i_{,m} e^{mk} \qquad (2)$$

It is the convected derivative of the deformation rate tensor e^{ik} defined by

$$2e^{ik} = v^i_{,k} + v^k_{,i} \qquad (3)$$

Here η_0 is the limiting viscosity at the small rate of shear which is given by

$$\eta_0 = \int_0^\infty N(\tau)\,d\tau \text{ and } k_0 = \int_0^\infty \tau N(\tau)\,d\tau, \qquad (4)$$

$N(\tau)$ being the relaxation spectrum as introduced by Walters [15]-[16]. This idealized model is a valid approximation of Walters liquid (Model B') taking very short memories into account so that terms involving

$$\int_0^\infty \tau^n N(\tau)\,d\tau,\ n \geq 2 \qquad (5)$$

have been neglected.

The mixture of Polymethyl Methacrylate and pyridine at 250 C containing 30.5 gm of polymer per litre and having density 0.98 gm/ml. fits very nearly to this model. For this mixture, the relaxation spectrum as given by Walters is

$$N(\lambda) = \sigma\eta_0\delta(\lambda) + \frac{1-\sigma}{\beta}\eta_0 \qquad (0 \leq \lambda \leq \beta)$$
$$= 0 \qquad \tau < \beta$$

where $\sigma = 0.13$, $\eta_0 = 7.9$ poises (gm/cm. sec), $\beta = 0.18$ sec and $\delta(\lambda)$ is the Dirac's delta function so that k0=0.60557 gm/cm.

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II. Mathematical Formulation

The region of unsteady two-dimensional free convective hydromagnetic flow of an elastico-viscous fluid past an infinite inclined permeable plate has been analyzed in presence of chemical reaction. A uniform magnetic field B_0 is applied normal to the plate. Here the mechanism of heat and mass transfer in connection with Dufour effect are studied. Since the plate is of infinite length, all physical variables are functions of $\bar{y}$ and $\bar{t}$ only. Let x'-axis is taken along the inclined plate and y'-axis is taken normal to the plate. Let u' and v' be the velocity components in x' and y' direction respectively. Let α be the small angle made by the plate with the vertical so that sinα=0 as shown in Figure1.

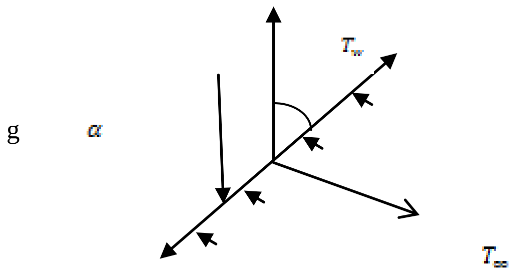

Figure 1: Geometry of the problem

Under the above assumptions the flow is governed by the following set of equations:

Equation of continuity:

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0 \qquad (6)$$

$$\bar{v} = constant = -v_0, \qquad v_0 > 0$$

Equation of motion:

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{k_0}{\rho}\left[\frac{\partial^3 \bar{u}}{\partial \bar{t}\partial \bar{y}^2} + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3}\right] + g\beta(\bar{T} - \bar{T}_\infty)\cos\alpha + g\bar{\beta}(\bar{C} - \bar{C}_\infty)\cos\alpha - \frac{\nu}{\bar{K}}\bar{u} - \frac{\sigma B_0^2}{\rho}\bar{u} \qquad (7)$$

Energy equation:

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{k}{\rho C_p}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{D_m K_T}{C_s C_p}\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} \qquad (8)$$

Concentration equation:

$$\frac{\partial \bar{c}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{c}}{\partial \bar{y}} = D_m\frac{\partial^2 \bar{c}}{\partial \bar{y}^2} - K_1(\bar{c} - \bar{c}_\infty) \qquad (9)$$

With relative boundary conditions:

$$\bar{u} = U, \quad \bar{v} = v(t), \quad \bar{c} = \bar{c}_\omega \quad \text{at } \bar{y} = 0$$
$$\bar{u} = 0, \quad \bar{T} = \bar{T}_\infty, \quad \bar{c} = \bar{c}_\omega \quad \text{as } \bar{y} \to \infty \qquad (10)$$

The non-dimensional parameters are:

$$y = \frac{\bar{y}U}{\nu}, t = \frac{\bar{t}U^2}{\nu}, u = \frac{\bar{u}}{U}, \theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty}, \phi = \frac{\bar{c} - \bar{c}_\infty}{\bar{c}_w - \bar{c}_\infty},$$
$$G_r = \frac{g\beta\nu(\bar{T}_w - \bar{T}_\infty)}{U^3}, G_m = \frac{g\bar{\beta}\nu(\bar{c}_w - \bar{c}_\infty)}{U^3}, P_r = \frac{\mu C p}{K},$$
$$S_c = \frac{\nu}{D_m}, D_u = \frac{D_m K_r(\bar{c}_w - \bar{c}_\infty)}{C_s C_p \nu}, M = \frac{\sigma B_0^2 \nu}{\rho U^2},$$
$$K = \frac{\bar{K}U^2}{\nu^2}, K_r = \frac{\nu K_1}{U^2}, k = \frac{k_0 U^2}{\rho \nu^2}, L = \frac{v_0}{U} \qquad (11)$$

where G_r is the Grashof number for heat transfer, G_m is the Grashof number for mass transfer, B_0 is the magnetic field, β is the volumetric co-efficient of expansion for heat transfer, $\bar{\beta}$ is the volumetric co-efficient of expansion for mass transfer, g is the acceleration due to gravity, $\bar{T}$ and $\bar{T}_\infty$ are the temperature of the fluid inside the thermal boundary layer and the fluid temperature in the free stream respectively, $\bar{c}$ and $\bar{c}_\infty$ are the corresponding concentrations, D_m is the co-efficient of mass diffusivity, K_r is chemical reaction parameter, K is the permeability parameter.

Introducing the non-dimensional parameters in the equations (7), (8), (9), we get

$$\frac{\partial u}{\partial t} - L\frac{\partial u}{\partial y} = +\frac{\partial^2 u}{\partial y^2} - k_1\left(\frac{\partial^3 u}{\partial t\partial y^2} - L\frac{\partial^3 u}{\partial y^3}\right) + G_r\theta\cos\alpha + G_m\phi\cos\alpha - \left(M + \frac{1}{K}\right)u \qquad (12)$$

$$\frac{\partial \theta}{\partial t} - L\frac{\partial \theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} + D_u\frac{\partial^2 \phi}{\partial y^2} \qquad (13)$$

$$\frac{\partial \phi}{\partial t} - L\frac{\partial \phi}{\partial y} = \frac{1}{S_c}\frac{\partial^2 \phi}{\partial y^2} - K_r\phi \qquad (14)$$

where k_1 is the visco-elastic parameter.

The boundary conditions of the problem in the dimensionless form are

$$u = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{at } y = 0$$
$$u = 0, \quad \theta = 0, \quad \phi = 0 \quad \text{as } y \to \infty \qquad (15)$$

III. Method of Solution

We assume the solutions of the non-linear partial differential equations (12) to (14) of the following form

$$u = u_0(y) + k_1 u_1(y), \qquad \theta = \theta_0(y) + k_1\theta_1(y)$$
$$\phi = \phi_0(y) + k_1\phi_1(y) \qquad (16)$$

Using (16) in the equations (12), (13) and (14) and equating the coefficient of like powers of k_1 and neglecting the highest powers of k_1, we get

$$u_0'' + Lu_0' - \left(M + \frac{1}{K}\right)u_0 = -G_r\theta_0\cos\alpha - G_m\phi_0\cos\alpha \qquad (17)$$

$$u_1'' + Lu_1' - \left(M + \frac{1}{K}\right)u_1 = -G_r\theta_1\cos\alpha - G_m\phi_1\cos\alpha - Lu_0''' \qquad (18)$$

$$\theta_0'' + P_r L\theta_0' = -P_r D_u\phi_0'' \qquad (19)$$

$$\theta_1'' + P_r L\theta_1' = -P_r D_u\phi_1'' \qquad (20)$$

$$\phi_0'' + LS_c\phi_0' - S_c K_r\phi_0 = 0 \qquad (21)$$

$$\phi_1'' + LS_c\phi_1' - S_c K_r\phi_1 = 0 \qquad (22)$$

The modified boundary conditions are

$$u_0 = 1, \theta_0 = 1, \phi_0 = 1, u_1 = 0, \theta_1 = 0, \phi_1 = 0 \ at\ y = 0$$
$$u_0 = 0, \theta_0 = 0, \phi_0 = 0, u_1 = 0, \theta_1 = 0, \phi_1 = 0 \ as\ y \to \infty \qquad (23)$$

Solving equations (21) and (22) and using boundary conditions (23), we get

$$\phi_0 = e^{-\alpha_2 y} \qquad (24)$$

$$\phi_1 = 0 \qquad (25)$$

Solving equations (19) and (20) and using boundary conditions (23), we get

$$\theta_0 = A_1 e^{-\alpha_3 y} - A_2 e^{-\alpha_2 y} \qquad (26)$$

$$\theta_1 = 0 \qquad (27)$$

Solving equations (17) and (18) and using boundary conditions (23), we get

$$u_0 = A_3 e^{-\alpha_2 y} - A_4 e^{-\alpha_3 y} + A_5 e^{-\alpha_5 y} \qquad (28)$$

$$u_1 = A_7 e^{-\alpha_2 y} - A_8 e^{-\alpha_3 y} + A_{10} e^{-\alpha_5 y} \qquad (29)$$

The constant are obtained but not presented here for the sake of brevity.

IV. RESULTS AND DISCUSSION

The velocity profile for the fluid flow is given by

$$u = A_3 e^{-\alpha_2 y} - A_4 e^{-\alpha_3 y} + A_5 e^{-\alpha_5 y} + k_1\left(A_7 e^{-\alpha_2 y} - A_8 e^{-\alpha_3 y} + A_{10} e^{-\alpha_5 y}\right) \quad (30)$$

The skin friction coefficient at the plate in non-dimensional form is given by

$$\tau = \left[\frac{\partial u}{\partial y} - k_1\left\{\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y}\right) - L\frac{\partial u}{\partial y}\right\}\right]_{y=0} \quad (31)$$

The non-dimensional form of the rate of heat transfer coefficient in terms of Nusselt number is given by

$$N_u = \frac{1}{P_r}\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = \frac{1}{P_r}\left(\frac{\partial \theta_0}{\partial y} + k_1\frac{\partial \theta_1}{\partial y}\right)_{y=0} \quad (32)$$

The non-dimensional form of the rate of mass transfer coefficient in terms of Sherwood number is given by

$$S_h = \frac{1}{S_c}\left(\frac{\partial \phi}{\partial y}\right)_{y=0} = \frac{1}{S_c}\left(\frac{\partial \phi_0}{\partial y} + k_1\frac{\partial \phi_1}{\partial y}\right)_{y=0} \quad (33)$$

The purpose of this study is to bring out the effects of elastic-viscous parameter on the unsteady MHD free convective flow past an inclined permeable plate with Dufour effects in presence of chemical reaction. The elastico-viscous effects is exhibited through the non-dimensional parameter k_1 .The corresponding results for Newtonian fluid are obtained by setting $k_1 = 0$.

To understand the physics of the problem the fluid velocity u is depicted against y in Fig. 1 to Fig. 6 for different values of Prandtl number P_r, Magnetic number M, Grashof number for heat transfer G_r, Grashof number for mass number G_m, Dufour number D_u, Chemical reaction K_r, Schmidt number Sc and elastico-viscous parameter k_1. The numerical calculations are carried out for Permeability parameter K=2 and L=2. Fig. 1 and Fig. 2 reveals that an increase in the Grashof number does not bring any significant change in the flow pattern but with the rising effect of visco-elasticity the fluid velocity enhances in comparison with Newtonian case. It is also observed that the fluid velocity diminishes due to the rise of magnetic parameter (Fig. 1 and Fig. 3) and Dufour number (Fig. 1 and Fig. 4) in both Newtonian and non-Newtonian cases. The variation of chemical reaction (Fig. 1 and Fig. 5) and Schmidt number (Fig. 1 and Fig. 6) shows that the fluid velocity enhances in both Newtonian and non-Newtonian cases.

Fig. 7 to Fig. 12 depict the variation of the shearing stress for different values of the flow parameters involved in the solution. Fig. 7 reveals as Prandtl number increases viscous drag declines for both Newtonian and non-Newtonian parameter. It is observed from Fig. 8 that the increasing values of Magnetic parameter amplify magnitude of shearing stress. Fig. 9 illustrates that the rising values of Dufour number increase the shearing stress for both Newtonian and non-Newtonian fluid flow phenomenon. The variation of shearing stress against chemical reaction K_r (Fig. 10) shows that the growth of visco-elastic parameter subdues the shearing stress. Fig. 11 shows that the magnitude of shearing stress increases with amplified values of Grashof number for cooling plate (G_r>0) but it decreases with the increasing values of elastico-viscous parameter in comparison with Newtonian fluid.. The reverse pattern of flow is observed for the heated plate (G_r<0) (Fig. 12). It is further noticed that the rate of heat and mass transfer coefficients are not significantly affected by the elastico-viscous parameter.

V. CONCLUSIONS

The unsteady two-dimensional free convective elastico-viscous fluid past an inclined permeable plate with Dufour effects in presence of chemical reaction has been investigated for different values of flow parameters. In the analysis, the following conclusions are made:

- The fluid velocity diminishes due to the rise of magnetic parameter and Dufour number in both Newtonian and non-Newtonian cases.
- The speed of both Newtonian and non-Newtonian fluids through the inclined permeable plate enhances with the increasing values of chemical reaction and Schmidt number.
- The growth of Grashof number does not bring any significant change in the flow pattern of the elastic-viscous fluid.
- Viscous drag declines for both Newtonian and non-Newtonian fluid with the increase of Prandtl number and chemical reaction parameter.
- The increasing values of Magnetic parameter amplify magnitude of shearing stress.
- The rising values of Dufour number increase the shearing stress for both Newtonian and non-Newtonian fluid.
- The shearing stress enhances with amplified values of Grashof number for cooled and heated plate.

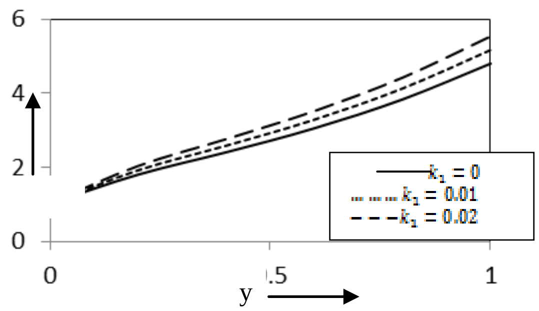

Fig: 1. Variation of u against y for P_r=5, G_r=6, G_m=7, M = 3, K = L= 2, D_u=0.03, K_r = 0.6, S_c=3

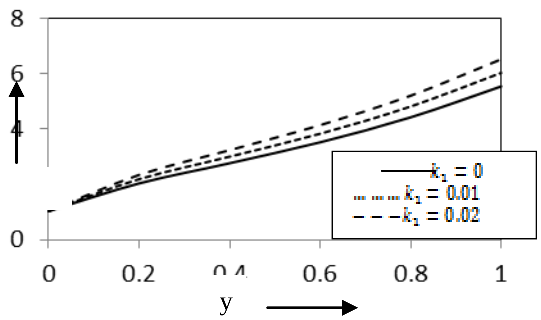

Fig: 2. Variation of u against y for P_r=5, G_r=10, G_m=7, M = 3, K = L= 2, D_u=0.03, K_r = 0.6, S_c=3.

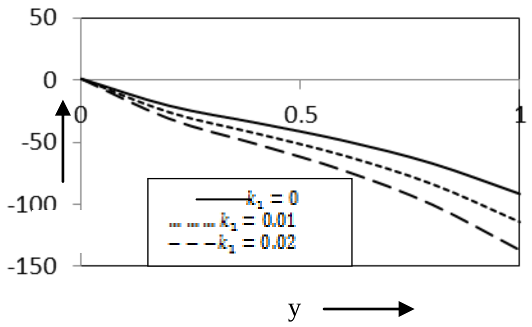

Fig: 3. Variation of u against y for P_r=5, G_r=6, G_m=7, M = 5, K = L= 2, D_u=0.03, K_r = 0.6, S_c=3

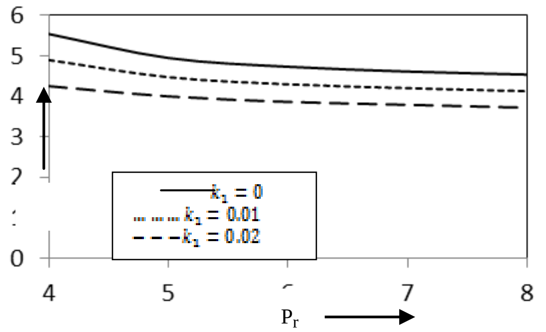

Fig: 7. Variation of τ against P_r for G_r = 3, G_m=8, M = 3, K =L=2, D_u=0.03, K_r=0.6, S_c=3.

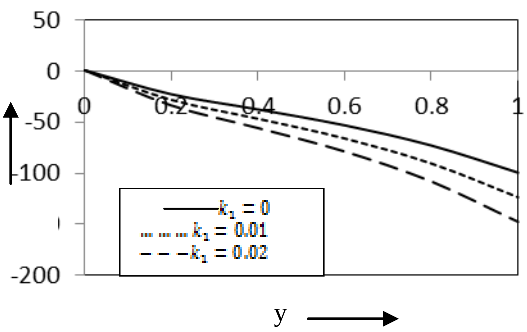

Fig: 4. Variation of u against y for P_r=5, G_r=6, G_m=7, M = 3, K = L= 2, D_u=0.05, K_r = 0.6, S_c=3.

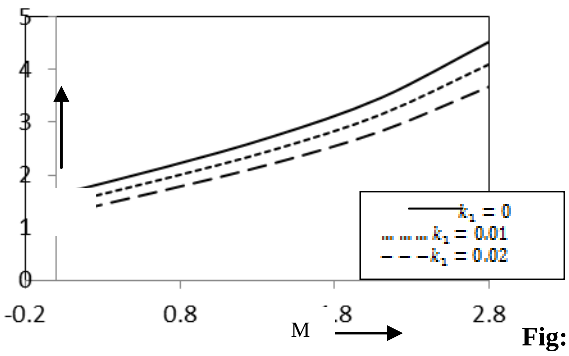

Fig: 8. Variation of τ against M for P_r=5, G_r = 3, G_m=8, K=L=2, D_u=0.03, K_r=0.6, S_c=3.

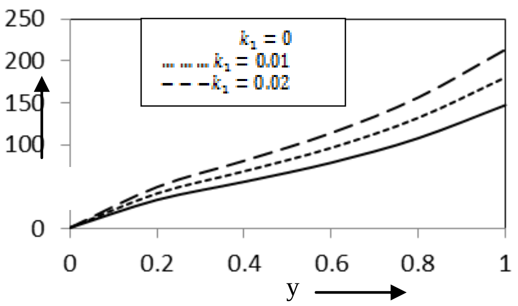

Fig: 5. Variation of u against y for P_r=5, G_r=6, G_m=7, M = 3, K = L= 2, D_u=0.03, K_r = 0.9, S_c=3.

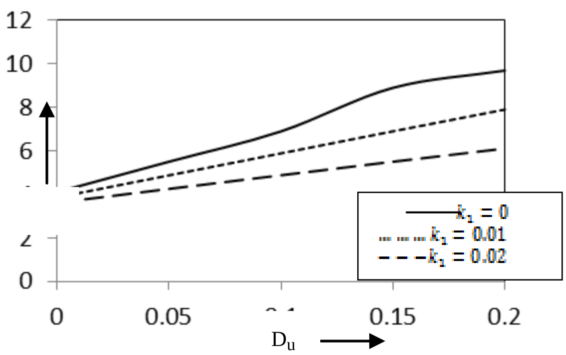

Fig: 9. Variation of τ against D_u for P_r=5, G_r = 3, G_m=8, M=3, K=L=2, K_r=0.6, S_c=3.

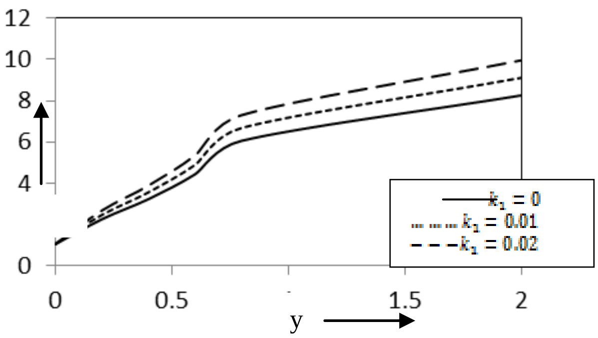

Fig: 6. Variation of u against y for P_r=5, G_r=6, G_m=9, M = 3, K = L= 2, D_u=0.03, K_r = 0.6, S_c=4.

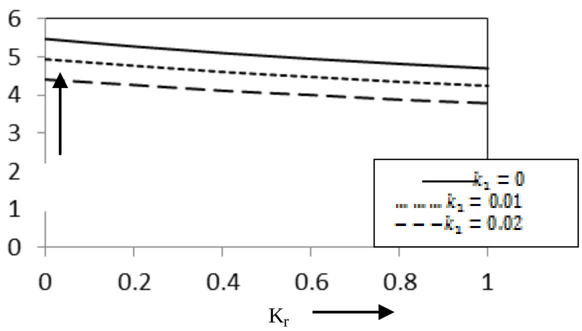

Fig: 10. Variation of τ against K_r for P_r=5, G_r = 3, G_m=8, M = 3, K =L=2, D_u=0.03, S_c=3.

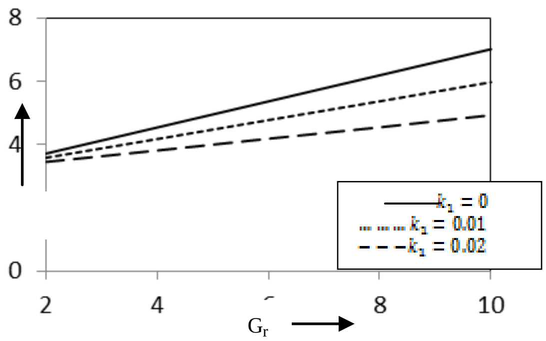

Fig: 11. Variation of τ against G_r (> 0) for P_r=5, G_m=8, M = 3, K =L=2, D_u=0.03, K_r=0.6, S_c=3.

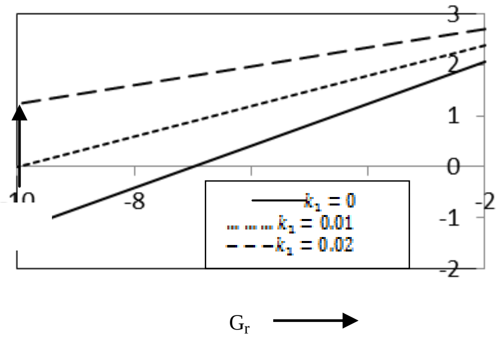

Fig: 12. Variation of τ against G_r (< 0)for P_r=5, G_m=8, M = 3, K =L=2, D_u=0.03, K_r=0.6, S_c=3.

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