Spectrum Sensing Schemes for Cognitive Radio With Induced Cyclostationarity And Optimum Detector For Cyclic Prefix OFDM With Modulation Scheme

Archana Tiwari, Sanjiv Mishra

Abstract— Cognitive Radio offers a solution by utilizing the spectrum holes that represent the potential opportunities for non-interfering use of spectrum which requires three main tasks Spectrum Sensing, Spectrum Analysis and Spectrum Allocation. Spectrum sensing involves obtaining the spectrum usage characteristics across multiple dimensions such as time, space, frequency, and code and determining what type of signals are occupying the spectrum. In this project, OFDM based Cognitive Radio and Spectrum Sensing methods namely Energy Detection Based Spectrum Sensing and Cyclostationary Spectrum Sensing are discussed. A higher probability of detection can be achieved by using the proposed optimum detector for induced cyclostationarity as compared to energy detection and simple cyclic prefix detection. Also the performance scales with number of subcarriers and induced correlation.

Index Terms— Cognitive radio, Spectrum Sensing Schemes, OFDM, Cyclic prefix, Induced cyclostationarity

I. INTRODUCTION

Cognitive radio is an intelligent wireless communication system that is aware of its surrounding environment (i.e., outside world), and uses the methodology of understanding by-building to learn from the environment and adapt its internal states to statistical variations in the incoming RF stimuli by making corresponding changes in certain operating parameters (e.g., transmit-power, carrier-frequency, and modulation strategy) in real-time, with two primary objectives in mind: highly reliable communications whenever and wherever needed and efficient utilization of the radio spectrum. Cognitive radio (CR) technology is a new way to compensate the spectrum shortage problem of wireless environment. It enables much higher spectrum efficiency by dynamic spectrum access. It allows unlicensed users to utilize the free portions of licensed spectrum while ensuring no interference to primary users’ transmissions. Cognitive radio arises to be tempting solution to the spectral congestion problem by introducing opportunistic usage of the frequency bands that are not heavily occupied by licensed users. FCC define cognitive radio as, “A radio or system that senses its operational electromagnetic environment and can dynamically and autonomously adjust its radio operating parameters to modify system operation, such as maximize throughput, mitigate interference, facilitate interoperability, access secondary markets”. Hence, one main aspects of cognitive radio is related to autonomously exploiting locally unused spectrum to provide new paths to spectrum access. In cognitive radio terminology, PU can be defined as the user who has license to use a specific part of the spectrum. On the other hand, secondary users (SU) or CR users do have license to use the spectrum but can use the spectrum. This paper presents a CP-OFDM signal with induced cyclostationarity, as a vector matrix model. It develops an optimum maximum likelihood test statistic for signal detection that exploits the cyclostationary property by mathematically quantifying the correlation introduced through induced cyclostationarity. As the test statistic obeys no closed form probability distribution, different SNR scenarios are simulated. It is found that the probability of detection achieved by the proposed optimum detector for a Neyman-Pearson scenario is better than energy detection and simple cyclic prefix detector. Also, the detection probability increases with higher induced correlation for a nominal tradeoff in bit rate. Hence, it offers a better and flexible approach to spectrum sensing in cognitive radio. Probability distribution, different SNR scenarios are simulated. It is found that the probability of detection achieved by the proposed optimum detector for a Neyman-Pearson scenario is better than energy detection and simple cyclic prefix detector. Also, the detection probability increases with higher induced correlation for a nominal tradeoff in bit rate. Hence, it offers a better and flexible approach to spectrum sensing in cognitive radio.

II. OFDM MODEL

Orthogonal frequency-division multiplexing (OFDM) is a method of encoding digital data on multiple carrier frequencies. OFDM has developed into a popular scheme for wide band digital communication, used in applications such as digital television and audio broadcasting, DSL, Internet access, wireless networks, power line networks, and 4G mobile communications. In this we introduce the cyclic prefix OFDM model. Base band signal is a cyclic prefix OFDM signal (t) having N orthogonal subcarriers. The duration of the OFDM symbol equals T = Ts+ Tcp, where Tsis the duration of input message transmitted in one OFDM symbol and Tcp is the length of cyclic prefix. The signal (t) may be represented as a sum of Nc statistically independent sub-channel quadrature amplitude modulated (QAM) signals.

\[ s(t) = \sum_{k=-\infty}^{k=\infty} d_n k g(t - kT)e^{j2\pi \frac{f_k}{f_s} t} \]

where d_n is the independent and identically distributed (i.i.d.) input message sequence and g(t) is the pulse shaping square waveform of duration T. It should be noted that here n denotes the subcarrier number and k denotes the OFDM symbol number. The samples (m) obtained by sampling thek-the
transmitted symbol at a frequency \( f_{\text{sam}} = Nc/T \) can be written as follows.

\[
s_k(m) = \sum_{n=0}^{N_c+1} \sum_{k=0}^{N_p-1} d_{n,k} g(m) e^{j2\pi \frac{km}{N_p}} \tag{2}
\]

Here \( N_{cp} = f_{\text{sam}} T_{cp} \), and the total length of the digital OFDM symbol is \( N = N_c + N_{cp} \).

III. 111-SPECTRUM SENSING TECHNIQUE

A major challenge in cognitive radio is that the secondary users need to detect the presence of primary users in a licensed spectrum and quit the frequency band as quickly as possible if the corresponding primary radio emerges in order to avoid interference to primary users. This technique is called spectrum sensing. Spectrum sensing and estimation is the first step to implement Cognitive Radio system [5]. We can categorize spectrum sensing techniques into direct method, which is considered as frequency domain approach, where the estimation is carried out directly from signal and indirect method, which is known as time domain approach, where the estimation is performed using autocorrelation of the signal. Another way of categorizing the spectrum sensing and estimation methods is by making group into model based parametric method and period gram based nonparametric method. Another way of classification depends on the need of spectrum sensing as stated below [13]: For any cognitive radio, spectrum sensing is an important activity through which a radio identifies unused spectrum resources in multiple domains like time, frequency, code etc. This activity protects the licensed users from any harmful interference from unlicensed devices and is a prerequisite to sustainable operation of a cognitive radio. For ex. IEEE 802.22 standard [11] for accessing unused TV white spaces has outlined elaborate spectrum sensing constraints in this section we discuss two major technique for spectrum sensing, namely energy detection and cyclostationary detection and how they apply specifically to OFDM

A. Energy detection

In this scheme, signal is allowed through band pass filter of the bandwidth \( W \) and is integrated over time interval. The outcome from the integrator block is then compared to a precalculated threshold. This comparison is used to find the existence or absence of the primary user. The threshold value of energy detection can be fixed or variable based on the channel conditions and threshold value depends on SNR ratio. The ED is called to be the Blind signal detector because it ignores the structure of the signal and properties of the signal. It estimates the presence of the signal by comparing the energy received with a known threshold derived from the statistics of the noise derived from SNR. Logically, signal detection can be reduced to formalize as a hypothesis test, a simple identification problem. It is a non-coherent detection method that finds the primary signal based on the technique is shown below.

For energy detection, the test statistic for signal decision is

\[
E = \sum_{i=1}^{K N} |y(i)|^2 \tag{3}
\]

The probability distribution of \( E \) can be derived using central limit theorem [2]. Considering \( P \) to be the power of signal(\( \eta \)) which is equal to its variance, the expression for the detection and false alarm probabilities is given by [12]

\[
P_f = Q\left( \frac{\lambda - K N \sigma_w^2}{\sqrt{K N \sigma_w^4}} \right),
\]

\[
P_d = Q\left( \frac{\lambda - 2 K N (\sigma_w^2/2 + P)}{\sqrt{4 K N (\sigma_w^2/2 + P)^2}} \right) \tag{4}
\]

Energy detection is one of the simplest techniques for spectrum sensing. But is has been shown to be non-optimum when correlation exists in the transmitted signal.

B. Cyclostationary Detection

Cyclostationary feature detection needs high computation complexity, the best detection point is determined through simulation analysis on different detection points, and then we intend combination detection method using multiple detection points to obtain better performance. Output validate the effectiveness of the suggested method Cyclostationary feature detection can be able to have high detection probability under low SNR, actually, it requires high computation complexity. In reality, based on channel and a given location, the licensed users’ signal parameters are known and the SNR is changing gradually, so we assume that we can obtain the licensed users’ signal type and SNR before making detection. Using of the licensed users’ prior knowledge like properties of signal, we only makes detections in some specific frequencies and cycle frequencies, and multiple combine detection points to increase the performance further. And then given the PD required by licensed users, the probability of false alarm (PFA) under different SNRs is implemented. Through the threshold adjustment, we decrease the PFA to make better use of spectrum hole when the SNR is high and increase the PFA to avoid interference to the licensed users when the SNR is low.

IV. THE PRINCIPLE OF CYCLOSTATIONARITY

Modulated signals are in general coupled with repeating spreading, cosine carrier, over-sampling etc., outcome in built-in periodicity. When the signal’s mean and autocorrelation exhibit periodicity, i.e., \( \text{mx}(t + T) = \text{mx}(t) \), \( \text{Rx}(t + T, u + T) = \text{Rx}(t, u) \)

We name this signal a second order cyclic statistics process. The auto-correlation of signal \( x(t) \) is given a

\[
\text{Rx}(t, \tau) = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{k=-N}^{N} x(t + \frac{\tau}{2} + n + T) x(t + \frac{\tau}{2} + nT)
\]

As \( \text{Rx}(t, \tau) \) is periodic with period \( T_0 \), it can be expressed as a Fourier series representation.
\[ R_\alpha(t, \tau) = \sum_{m=-\infty}^{\infty} R(x^{m/T}(\tau) e^{j2\pi m t/T} \right \}
\]

\[ R^{\alpha}(\tau) = \left( \frac{1}{T} \right) \int_{-\infty}^{\infty} R(x(t, \tau) e^{-j2\pi \alpha t} dt \right \}
\]

Where is the second-order cycle frequency equals to the spectrum coherence function (SCF) can be obtained from as

\[ Sx^{\alpha}(f) = \int_{-\infty}^{\infty} R_x(x(t), e^{-j2\pi \alpha t} dt = \frac{1}{T} \int_{\mathbb{R}} x(t+\frac{\alpha}{2}) \cdot x(t) dt = 4\]

Where \( x(t) \) is the Fourier transform of signal \( x(t) \) from (4) we can find that \( Sx(f) \) is the correlation of the signal spectrum different types of signals have different spectrum correlation features. The SCF of the Gaussian white noise is given. These figures illustrate 17]

**V. INDUCED CYCLOSTATIONARITY AND OPTIMUM DETECTOR FOR CP-OFDM**

**A. Induced Cyclostationarity in OFDM**

Induced Cyclostationarity is a feature that can be intentionally embedded in the signal which may be easily detected and generated [9]. Such features can be introduced in an OFDM signal, giving rise to additional cyclostationary signatures that can aid reliable signal acquisition and detection. These are more useful than inherent cyclostationary features such as cyclic prefix because they can be introduced at any desired cyclic frequency and easily manipulated to distinguish the Signal of Interest (SOI) from other interfering users. In order to induce cyclostationarity, we create correlation in the OFDM symbol of Eq. 1 as shown in Eq. 6

\[ d_n = d_{n+p}, = n, n1, + 1, \ldots, n1 + M - 1 \]

It can be seen that all the subcarriers from \( n1 \) to \( n1 + M - 1 \) are mapped to another subset \( p \) subcarriers apart as shown in Fig. 1. This introduction of statistical dependence between certain subcarriers results in cyclostationary behavior of the OFDM signal [9].

The following section discusses the vector matrix model for the CP-OFDM signal used to derive an optimum detector for signal \( r \) contains the vector \( s \) added to the \( L \times 1 \) dimension noise vector \( w \).

\[ r = s + w \]

We assume that symmetric QAM is used in OFDM subcarrier modulation and hence the vector \( s \) is zero mean. Also the received sequence length \( L \) is large enough to allow the application of central limit theorem. The AWGN is zero mean and with variance \( \sigma^2 \).

Two hypotheses can be formulated; viz. H0 (with no transmitted signal) and H1 (with transmitted signal present). The probability distribution of the complex vector \( r \) under both the hypotheses is given by [8]

\[ p(r | H_0) = \frac{1}{\pi L / \sigma^2} \exp \left( -\frac{\|r\|^2}{\sigma^2} \right), \]

\[ p(r | H_1) = \frac{1}{\pi L \det (\Sigma_r)} \exp \left( -r^H \Sigma_r^{-1} r \right), \]

Where \( L \) is the received vector. In order to derive the elements of the matrix \( \Sigma_r \), first, the derivation is presented for a simpler case of \( K = 1 \) using a vector matrix model. Let \( q \) denote a length \( N_c \times 1 \) OFDM symbol before adding cyclic prefix and \( sk \) denote the corresponding cyclic prefixed OFDM symbol of length \( (N_c + Ncp) \times 1 \). Then \( sk \) is defined as

\[ U = 0 \]

From Eq. 11, the co-variance matrix of \( r \) under H1. In order to derive the elements of the matrix \( \Sigma_r \), first, the derivation is presented for a simpler case of \( K = 1 \) using a vector matrix model. Let \( q \) denote a length \( Nc \times 1 \) input message sequence \( [d0,0, d1,0, \ldots dNc, 0, d0, 1, \ldots, dNc, K] \). Then the \( L \times 1 \) length vector of transmitted cyclic prefixed OFDM symbols can be written as \( s = Tq \) where the matrix \( T \) is defined

\[ T = \begin{bmatrix} U & 0 & \cdots & 0 \\ 0 & U & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & U \end{bmatrix} \]

From Eq. 11, the co-variance matrix of the transmitted signal can be written as

\[ \Sigma_s = \{ ss^H \} E \{ Tq(Tq)^H \} = TE \{ qq^H \} TH \]

Where \( (\cdot) \) denotes the Hermitian operator and \( E[\cdot] \) denotes Expectation operator. Let the covariance matrix of \( q \) be \( \Sigma_q \). Then the covariance matrix of the received vector \( r \) can be written as

\[ \Sigma_r = \sigma_q^2 I + T \Sigma_q T^H \]

Case 1: When no Induced Cyclostationarity is present

In this case, there exists no correlation structure in the message sequence and its covariance matrix is an identity matrix. Hence the covariance matrix of the received vector can be simply written as

\[ \Sigma_r = \sigma_q^2 I + \sigma^2 TT^H \]
Case 2: When Induced Cyclostationarity is present: When induced cyclostationarity is inserted into the OFDM signal, the elements of vector \( q \) are no longer independent and hence its covariance matrix is no longer an identity matrix and the matrix \( \Sigma_q \) must be determined from first principle.

C. Derivation of Covariance Matrix \( \Sigma_q \) for the Induced Cyclostationary Case:

Consider a single OFDM symbol duration with normalized samples before the addition of CP in Eq. 2.

\[
q_k(m) = \frac{1}{\sqrt{N_c}} \sum_{n=0}^{N_c-1} d(n) e^{j \frac{2\pi}{N_c} mn}, \quad m = 0, 1, \cdots, N_c-1
\]  

(15)

The symbols \( n \) are independent and identically distributed (i.i.d.) with a power of \( 2\sigma^2 \). Let the vector \( q_k \) be the \( N_c \times 1 \) vector \([0, 1, \cdots, q(N_c-1)]'\). Suppose that we Induce cyclostationary signature in \( q_k \) by mapping the last \( M \) subcarriers to the first \( M \) subcarriers \((\text{put } n_1 = 0, p = N_c - M \text{ in Eq. 6})\) i.e. \( d(n) = d(N_c - M + n), n = 0 \text{ to } M-1 \). Then, we can partition \( q_k(m) \) as,

\[
\begin{align*}
q_k(m) &= \frac{1}{\sqrt{N_c}} \left[ a_M^m + b_{N_c-2M}^m + c_M^m \right],
\end{align*}
\]

Where

\[
\begin{align*}
a_M^m &= \sum_{n=0}^{N_c-1} d(n) e^{j \frac{2\pi}{N_c} mn}, \\
b_{N_c-2M}^m &= \sum_{n=M}^{N_c-1-M} d(n) e^{j \frac{2\pi}{N_c} mn}, \\
c_M^m &= \sum_{n=0}^{N_c-1-M} d(n) e^{j \frac{2\pi}{N_c} mn}.
\end{align*}
\]

The vector \( q_k \) can be written as

\[
q_k = \begin{bmatrix}
a_M^2 + b_{N_c-2M}^2 + c_M^2 \\
a_M^1 + b_{N_c-2M}^1 + c_M^1 \\
\vdots \\
b_{N_c-1-M}^2 + b_{N_c-1-2M}^2 + c_{N_c-1-M}^2
\end{bmatrix}
\]

(17)

Let the element at \( l1th \) row and \( l2th \) column of the \( Nc \times Nc \) covariance matrix \( \Sigma_q \) of \( q_k \) be denoted by \( t(l1, l2) \). Then,

\[
\begin{align*}
t(l1, l2) &= E \left\{ \left[ a_M^{l1} + b_{N_c-2M}^{l1} + c_M^{l1} \right] \left[ a_M^{l2} + b_{N_c-2M}^{l2} + c_M^{l2} \right]^* \right\} \\
t(l1, l2) &= E \left\{ \left[ a_M^{l1} + b_{N_c-2M}^{l1} + c_M^{l1} \right] \left[ a_M^{l2} + b_{N_c-2M}^{l2} + c_M^{l2} \right]^* \right\}
\end{align*}
\]

(18)

This term is made up of nine product terms. The terms in \( aM \) and \( cm \) are correlated with each other because of the induced correlation structure, but none of them are correlated with \( bM \) owing to the independence of zero mean source symbols. This leaves only five product terms. Sample calculation of two of the terms has been shown here where \( W = e^{-j 2\pi} \)

\[
E \left\{ \left( a_M^{l1} \right) \left( b_M^{l2} \right)^* \right\} = \frac{1}{N_c} E \left\{ \left( \sum_{n=0}^{N_c-1-M} d(n) W^{n l1} \right) \left( \sum_{n=0}^{N_c-1-M} d^*(n) W^{n l2} \right) \right\}
\]

(19)

\[
E \left\{ \left( a_M^{l1} \right) \left( b_{N_c-2M}^{l2} \right)^* \right\} = \frac{1}{N_c} E \left\{ \sum_{n=0}^{N_c-1-M} d(n) W^{n l1} \right\} \times \frac{1}{N_c} E \left\{ \sum_{n=0}^{N_c-1-M} d^*(n) W^{n l2} \right\}
\]

(20)

Adding all the five product terms, we get the general term of \( \Sigma_q \) as

\[
E \left\{ \left( a_M^{l1} \right) \left( b_M^{l2} \right)^* \right\} = \frac{W_{Ml2}}{N_c} \sum_{n=0}^{N_c-1-M} W^{n l1} \frac{W^{n l2}}{N_c} \times E \left\{ d(n) d^*(n) \right\}
\]

(21)

For the case when \( l1 = l2 \), the diagonal term \( t(l1, l2) \) can be found as

\[
t(l1, l1) = \frac{\sigma_a^2}{N_c} \left( N_c + 2M \cos \left( \frac{2\pi M l1}{N_c} \right) \right)
\]

(22)

For the case of \( l1 \neq l2 \), Eq. 23 can be simplified to

\[
t(l1, l2) = \frac{\sigma_a^2}{N_c} \left( \sum_{n=0}^{N_c-1-M} W^{l1-1-n} \left( l1 \right) \left( l2 \right) W^{l1-1-n} \left( l1 \right) \left( l2 \right) \right)
\]

(23)

\[
W = e^{-j 2\pi}
\]

After plugging in this general term to obtain the matrix \( \Sigma_q \), the covariance matrix \( \Sigma_q \) can be found by using Eq. 27 as individual blocks of OFDM symbols are independent and
uncorrelated and possess the same covariance matrix. The matrix $\Sigma_q$ can then be inserted into Eq. 13 to obtain the covariance matrix $\Sigma_r$ for a CP-OFDM signal with induced cyclostationarity.

$$\Sigma_q = \begin{bmatrix}
\sum_q & 0 & \cdots & 0 \\
0 & \sum_q & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \sum_q
\end{bmatrix}_{K N_c \times K N_c}$$  

(27)

D. The Optimum Detector for CP-OFDM
To detect the CP-OFDM signal optimally, a maximum likelihood test statistic can be constructed by defining the likelihood ratio and finding its optimum value that satisfies the constraint of given false alarm probability. The test statistics shown in Eq. 28. The optimum time domain test statistic for induced cyclostationarity in OFDM can be found by substituting the value of $\Sigma_r$ from Eq. 13 into Eq. 28

$$A_{optimum} = \log \frac{p(r|H_1)}{p(r|H_0)} = \log \left( \frac{\sigma^2_u}{\delta(\Sigma_r)} \right) - r^H \left( \Sigma_r^{-1} - \frac{1}{\sigma^2_u} I \right) r$$  

(28)

Similarly when induced cyclostationarity is absent the optimum detector is found by combining Eq. 14 and Eq. 28. Axell and Larsson in [8] derive Eq. 28 for the case of CPOFDM without induced cyclostationarity and state that there is no closed form expression for the distribution of this test statistic. Hence the threshold $n_{optimum}$ for the test statistic must be computed empirically. It should be noted that the test statistic in Eq. 28 is also applicable when no cyclic prefix is present in the signal. This shows that energy detection is optimum only when cyclic prefix is not present.

$$A_{optimum} = \sum_{i=0}^{K N_c-1} |r_i|^2$$

VI. SIMULATION RESULTS
To show the improvement in signal detection with the proposed detector for induced cyclostationarity, an OFDM signal with number of subcarriers $N_c=32$ and cyclic prefix length $Ncp=8$ is simulated. Cyclostationarity is induced by mapping first $M$ Subcarrier to last $M$ subcarrier. value $M=7$ are chosen. 4-QAM modulation is used and signal variance $\sigma^2_x$ is varied between $1$ and $0.01$ and noise variance $\sigma^2_w$ is chosen as unity giving an SNR range from $0dB$ to $-20dB$. The false alarm probability is fixed to $0.05$. The simulation is carried out for 10 complete OFDM symbol durations i.e. $L=10(Nc+Ncp)$ samples and the results are averaged over 1000 Monte Carlo runs.

For Second subplot The whole experiment is repeated for $N_c=48$ and $Ncp=12$. It can be seen that the proposed optimum test statistic provides.

We can see from the graphical result Better probability of detection find than any other techniques. This is at a cost of a nominal tradeoff in bit rates shown in Table I that can be countered by increasing the value of $N_c$. Increasing the number of subcarriers scales the performance of each technique equally while reducing bit rate loss. The additive improvement for induced cyclostationarity however remains similar in both cases and independent of $N_c$.

VII. CONCLUSION
This paper presents the performance of the optimum detector for cyclic prefixed OFDM with induced cyclostationarity in the context of spectrum sensing in cognitive radio. Through a vector matrix modeling approach for CP-OFDM.

FRACTIONAL LOSS OF BITRATE FOR VALUES OF $M$ FOR $N_c=48$ & $N_c=32$

<table>
<thead>
<tr>
<th>$M$</th>
<th>Fractional Loss of Bit Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>0.21875</td>
</tr>
<tr>
<td>48</td>
<td>0.14583</td>
</tr>
</tbody>
</table>

covariance matrix for a CP-OFDM signal with induced cyclostationarity derived from first principle and the maximum likelihood test statistic is developed. The resultant probability of detection for a Neyman Pearson test is compared with energy detection and cyclic prefix detection techniques. Simulation results show that the proposed detector outperforms both techniques. The improvement increases with $M$ along with a nominal tradeoff in bit rate as shown in Table I that can be countered by increasing the number of subcarriers. As the number of subcarriers $N_c$ increases the detection performance improves and further improvement can be obtained by increasing the mapped subcarrier subset $M$. Thus, the proposed optimum detector using induced cyclostationarity provides an improved and flexible spectrum sensing approach in OFDM as compared to energy detection and cyclic prefix detection techniques.

REFERENCES


