Modified Product Group delay Spectral Estimation Using Auto-Regressive (AR) Model

Dr.K.Nagi Reddy, Sreelekha.D , Sreenu.A, C.Tousif , M. Mohan,S Poorna saie

***Abstract*— In this paper a method of spectrum estimation of group delay using Auto- Regressive (AR) model is proposed. The method described is based on the properties of negative derivative of Fourier Transform (FT) phase function. The results obtained from the proposed method called modified product group delay are compared with the, Traditional group delay proposed by Hema.A Murthy proposedin 1992, modified group delay proposed by Rajesh M.Hegde in 2007 and product group delay proposed by Donglai Zhu and Kuldip K. Paliwal in 2004 . This proposed method suppresses the spikes generated due to noise in spectrum and also provides better resolution. In the estimates obtained using this method the resolution properties of the periodogram estimate are preserved while the variance is reduced. This method works for even high noise levels (SNR = 0 dB or less).**

***Index Terms*— Auto-Regressive method (AR Method), group delay function, Product group delay, Spectral Estimation.**

I. INTRODUCTION

The objective of this chapter is to explore an approach to spectrum estimation from the Fourier transform phase of the signal. The method described is based on the properties of the second order derivative of Fourier Transform phase function. Various attempts [1]-[2] have been made to demonstrate the spectrum estimation based on the properties of the negative derivative of the FT phase function, also called group delay function. The most important properties of the group delay function are the additive and high resolution properties [3]. Here the resolution refers to the sharpness of the peaks in group delay function, which is due to FT magnitude function behavior of the group delay function.

Traditionally, the phase spectrum of the signal has been ignored, primarily because only the principal values of the phase can be estimated from the Fourier transform. For the phase to be used, the phase function will have to be unwrapped to produce a continuous estimate [3]. On the other hand, the group delay function [6] (defined as the negative derivative of the phase function), which has properties similar to the phase, can be computed directly from the signal.

The most important properties of the group delay function are the additive and high resolution properties. Here resolution refers to the sharpness of the peaks in the group delay function which is due to the squared Fourier transform magnitude function behavior of the group delay function near the peaks. The key idea behind the new spectrum estimation method is that the properties of the group delay functions for noise and an auto regressive process are distinct. Group delay is an important feature of the signal that can help in enhancing the signal quality in noisy conditions. Previous research works have revealed the usefulness of group delay in many applications. Recent studies on speech perception have revealed the importance of the phase of speech Signal. In order to overcome the problem of spikes in group delay, some researchers have suggested solutions such as modified group delay and product spectrum[4]-[5].

II. GROUP DELAY

The Group delay is defined as the negative derivative of the Fourier transform of a signal. Computing of group delay is difficult due to wrapping of the phase function [7]. Calculation of Group Delay is being done as follows.

$$X(\omega) = \sum_n x(n)e^{-j\omega n} \qquad (1)$$

The X(ω) as a function of phase and magnitude can be expressed as,

$$X(\omega) = |X(\omega)|e^{j\emptyset(\omega)}$$

Now the group delay function can be expressed as

$$\tau(\omega) = \frac{-d\emptyset(\omega)}{d\omega}$$

$$\tau(\omega) = \frac{X_R(\omega)Y_R(\omega)+X_I(\omega)Y_I(\omega)}{|X(\omega)|^2} \qquad (2)$$

where X_R is Real part of the Fourier Transform of x(n), X_I is Imaginary part of the Fourier Transform of x(n), Y_R is Real part of the Fourier Transform of nx(n) and Y_I is Imaginary part of the Fourier Transform of nx(n)

2.1 Autoregressive Process in Noise (Estimation of the AR Spectrum)

$$x_1(n) = s(n) + u(n)$$

$$s(n) = -\sum_{k=1}^{4} a_k s(n-k) + Ge(n) \qquad (3)$$

Where the excitation e(n) is a white Gaussian noise of variance unity and u(n) is an additive noise with variance dependent upon the coefficients are:a_1=-2.760, a_2 = 3.809, a3 = -2654, and a_4=0,924. And the signal used is a two sinusoids in noise (estimation of frequencies of the sinusoids)

$$x_2(n) = \sqrt{10}\exp\left[j2\pi(0.10)n\right] + \sqrt{20}\exp\left[j2\pi(0.15)n\right] + u(n) \qquad (4)$$

Where u(n) is additive white Gaussian noise with the variance dependent upon the SNR. These examples are similar to the ones used in [8] for discussion of periodogram estimates.

We assume a sampling frequency of 10 kHz and number of samples N=512 for AR spectrum estimation(4). Different realizations of x_1(n) and x_2(n) are obtained by using different noise sequence each time. Spectral estimation calculated and

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presented by using (3) , different method like MGDF, and product group delay function are compared here.

III. Product Group Delay:

In order to make the GDF meaningful, a modification to the GDF has been proposed by replacing the power spectrum $|X(\omega)|^2$ with the cepstrally smoothed power spectrum $|S(\omega)|^2$ in (1). This gives the Modified Group Delay Function (MGDF) as follows [4].

$$\tau_p(\omega) = \frac{X_R(\omega)Y_R(\omega)+X_I(\omega)Y_I(\omega)}{|S(\omega)|^2} \quad (5)$$

We define the product spectrum Q(ω) as the product of the power spectrum and the GDF as follows

$$\begin{aligned} Q(\omega) &= |X(\omega)|^2 \tau_p(\omega) \\ &= X_R(\omega)Y_R(\omega) + X_I(\omega)Y_I(\omega) \end{aligned} \quad (6)$$

The product spectrum [5] is influenced by both the magnitude spectrum and the phase spectrum. It enhances the region at the formants over the Modified Group Delay Function and has an envelope comparable to that of the power spectrum.

In order to prevent the spikes on the group delay of signal, we will use a modified group delay function with different parameters as follows:

$$\tau(\omega) = \frac{X_R(\omega)Y_R(\omega)+X_I(\omega)Y_I(\omega)}{S(w)^{2a}} \quad (7)$$

$$\tau_P(\omega) = \tau(\omega)|\tau(\omega)|^{b-1} \quad (8)$$

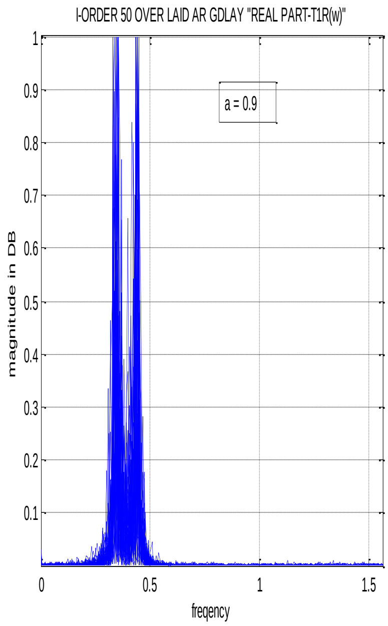
Where S(ω) is cepstrally smoothed spectrum of X(ω) in order to reduce spikes in group delay function. The parameter A and B are varied accordingly and estimated the spectrum .

IV. Results:

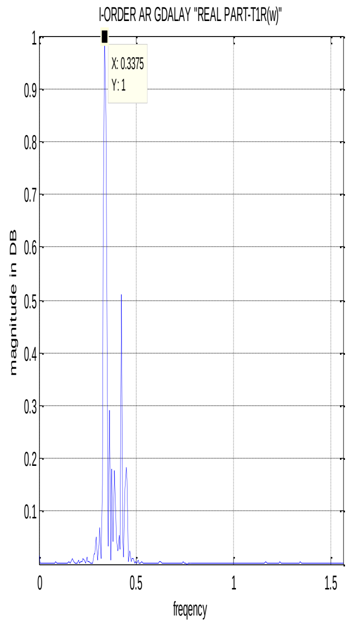
The results of the group delay function, modified group delay function product group delay and modified group delay function are presented for different values of the parameters **a** and **b** .The Fig. 1(a), 1(b) and 1(c) are 50 overlaid , single realization and averaged spectrum estimation of AR Process respectively for the parameter a=0.9.similar results are depicted in Fig. 1(d), 1(e) and 1(f).From these , average spectrum is giving a smoothed spectrum estimation.

The Fig. 2(a), 2(b) and 2(c) are 50 overlaid , single realization and averaged spectrum estimation of AR Process for modified group delay method respectively for the parameter a=1 and b=1 .Similar results are illustrated in Fig. 2(d), 2(e) and 2(f).

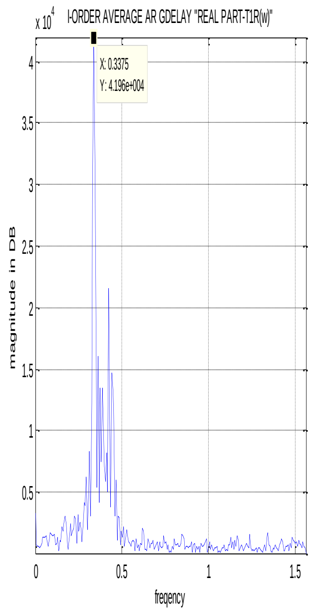
The spectrum estimation of Modified group delay with a=0.9 ,b=0.4 and a=1,b=0.4 has been depicted in the Fig. 3(a), 3(b) and 3(c) are 50 overlaid , single realization and averaged spectrum estimation of AR Process respectively for the parameter a=0.9.similar results are depicted in Fig.3(d),3(e) and 3(f).Based on the estimations carried out it is observed that the modified group delay method is giving better and smooth spectral characteristics when compared with the traditional group delay[10] and product group delay method.



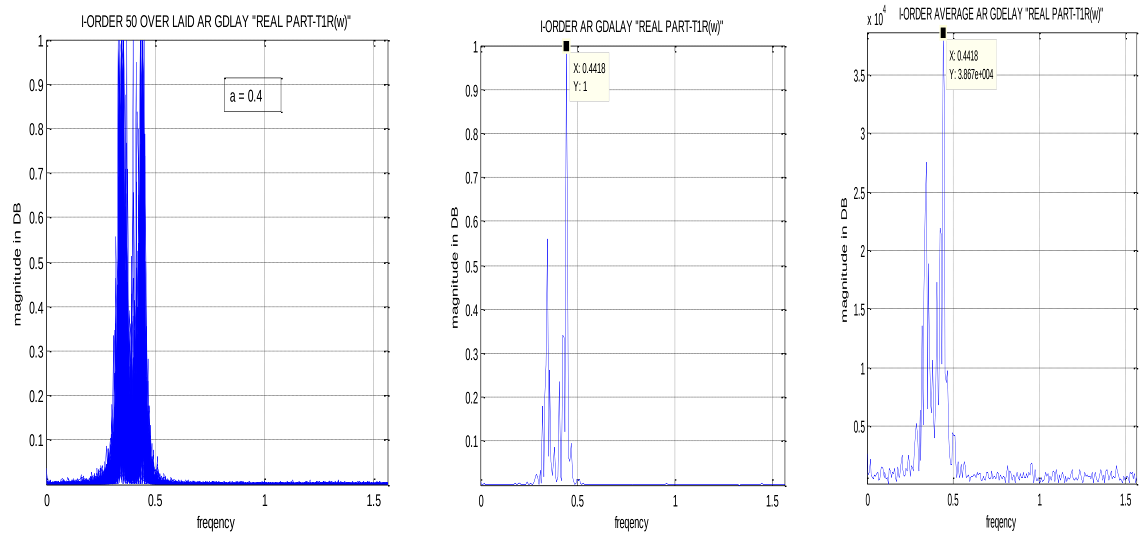
1(a)



1(b)

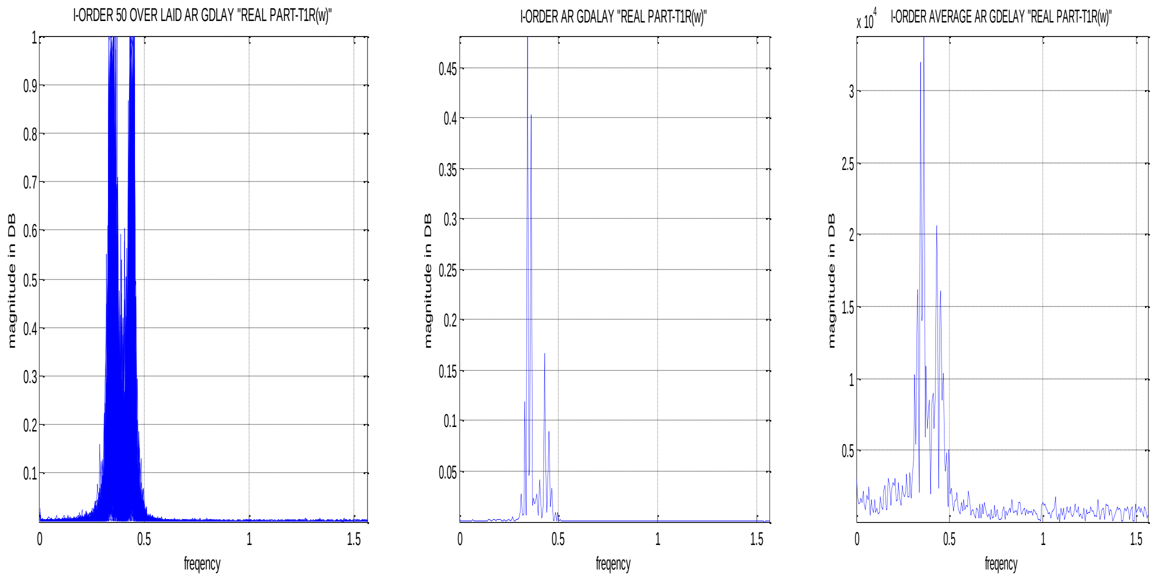


1(c)

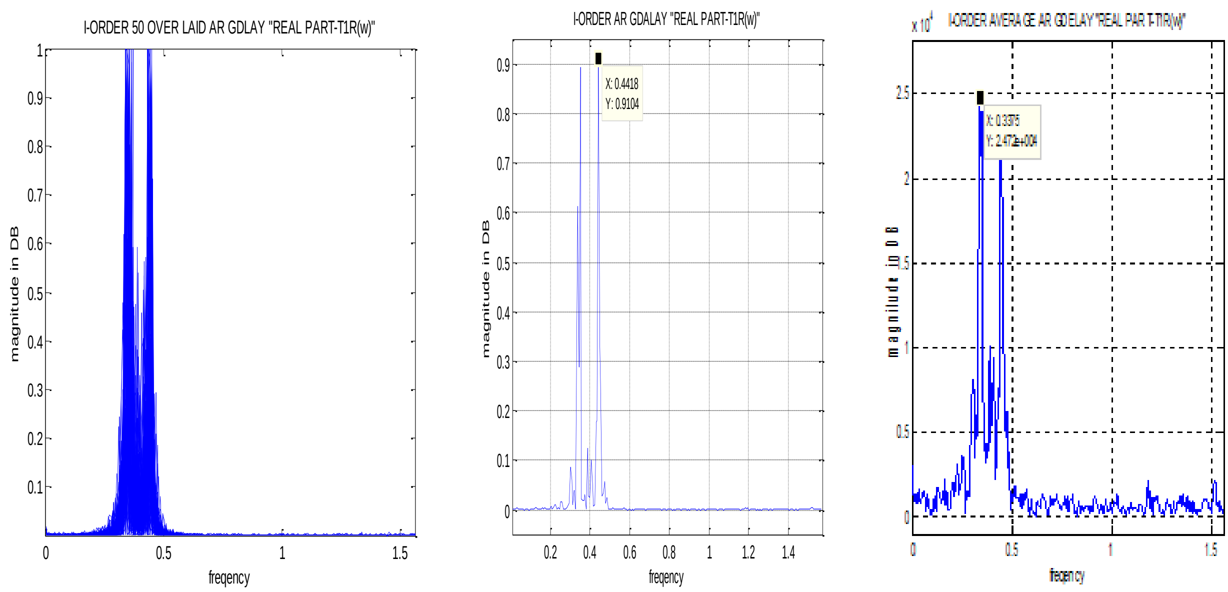


1(d) 1(e) 1(f)

Fig.1 Spectrum estimation product group delay of AR process for A=0.9 and 0.4 respectively

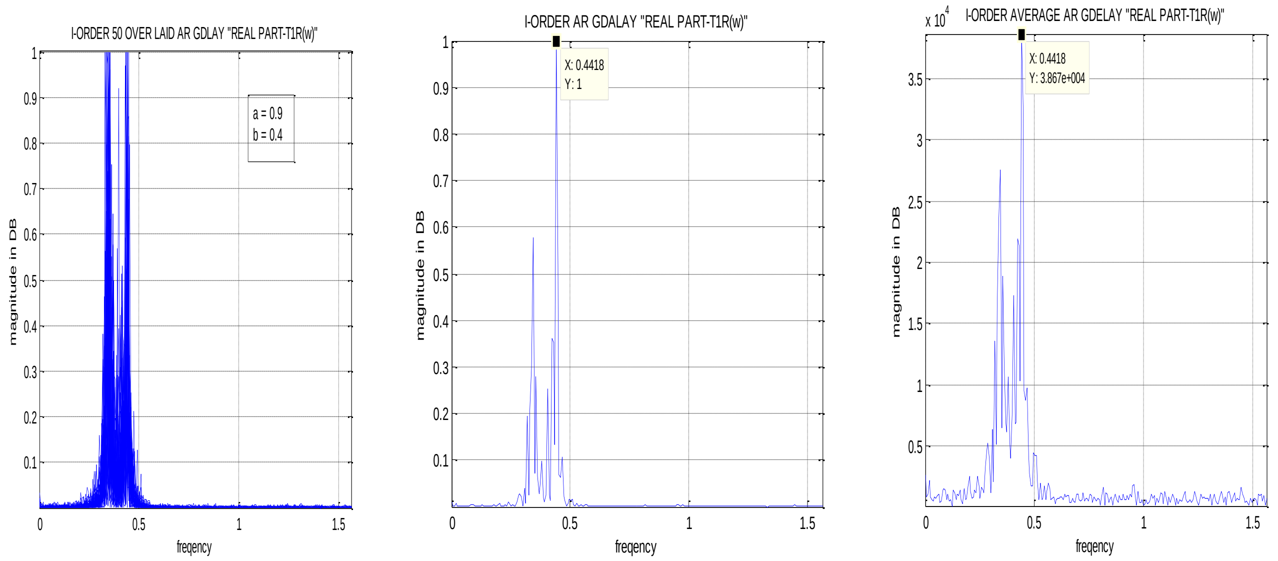


2(a) 2(b) 2(c)

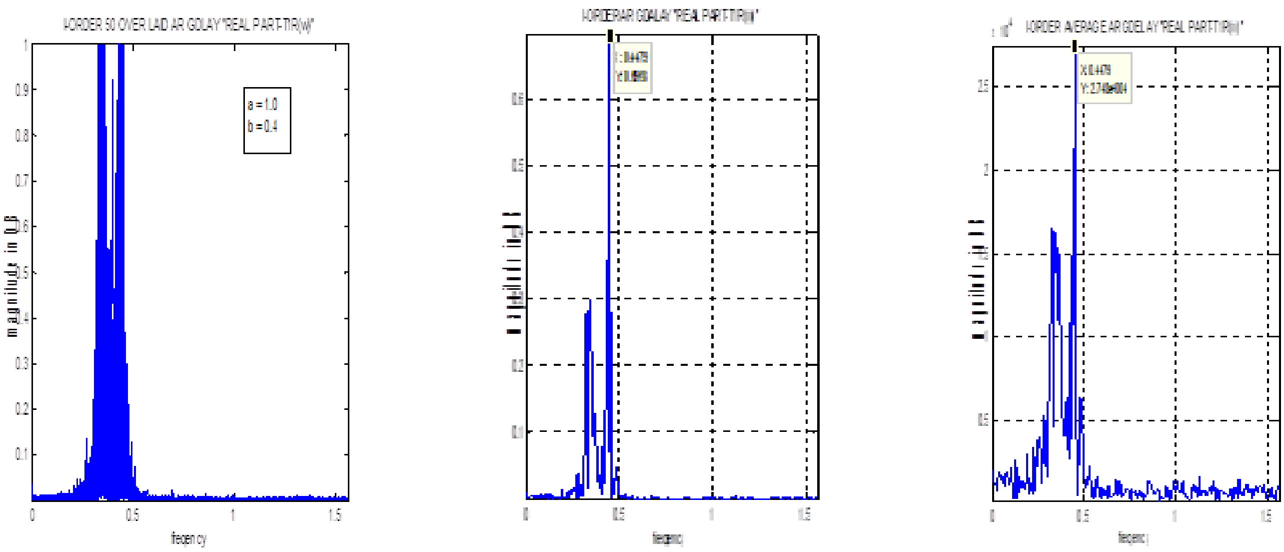


2(d) 2(e) 2(f)

Fig2 spectrum estimation modified product group delay of AR process for A=1 and b=1 respectively



3(a) 3(b) 3(c)



3(d) 3(e) 3(f)

Fig 3 spectrum estimation modified product group delay of AR process for a=9.0 ,b=0.4 and a=1, b=0.4 Respectively

V. Conclusion:

In this paper we have introduced the spectral estimation of the first order modified product group delay function using Auto-Regressive (AR) model. This method has been compared with normal group delay, modified group delay and product group delay. The proposed method provides better resolution with reduced variance and also suppresses the spikes generated due to noise in the spectrum compared to first order group delay functions to a great extent. This method works even for high noise levels (SNR = 0 dB or less). Based on the estimations carried out it is concluded that the modified group delay method is giving better and smooth spectral characteristics when compared with the traditional group delay and product group delay method.

References

[1] B. Yegnanarayana and Hema A. Murthy "Significance of Group Delay Functions in Spectrum Estimation" IEEE Transactions on signal processing. Vol. 40. NO.9.pp 2281-2289, September 1992

[2] B. Yegnanarayana, "Formant extraction from linear prediction phase spectra," *J. Acoust. Soc. Amer.*, vol. 63, pp. 1638-1640, May 1978

[3] J. Tribolet, "A new phase unwrapping algorithm," *IEEE Trans.Acoust., Speech, Signal Processing*, vol. ASSP-25, no. 10, pp. 170–177, 1977

[4] Rajesh M.Hegde ,Hema A. Murthy and Venkata Rmana Rao Gadde " Significance of joint Features Derived from the Modified Group Delay Function in Speech Processing",EURASIP Journal on Audio, speech and Music Processing vol.2007, Article ID79032,pp.1-12

[5] D. Zhu and K. K. Paliwal, "Product of power spectrum and group delay function for speech recognition," in Proc. ICASSP, Montreal, Canada, May 2004

[6] A.V oppenheim and R.W Schafer ' ' "Digital signal Processing" Englewood cliff,NJ , Prentice –Hall

[7] A S R Reddy, K. Nagi reddy , S.Narayana Reddy, "Complex group

delay Function and its Derivatives for Spectrum Estimation of MST Radar signals ” , Journal of innovation in Electronics & Communication PP:158-162 January 2012.

[8] K. K. Paliwal and L. D. Alsteris, "Usefulness of phase spectrum in human speech perception," in Proc. Eurospeech, Geneva, Switzerland, Sep. 2003

[9] G. Farahani1, S.M. Ahadi1 and M.M. Homayounpoor ” use of spectral peaks in autocorrelation and group delay domains for robust speech recognition” pp-517-520 ,ICASSP 2006

[10] K. Nagi Reddy, Dr.S.Narayana Reddy, Dr.ASR Reddy “Significance of Complex group delay functions in Spectrum Estimation” pp :114-133, Signal & Image processing An International Journal(SIPIJ) Vol.2, No.1, March 2011

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