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The Effect of Radiation on Boundary Layer Flow and Heat Transfer of Unsteady Dusty Fluid over a Stretching Sheet with Electrification of Particles

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Abstract:-The unsteady, viscous, incompressible, boundary layer flow with heat and mass transfer in the presence of radiation and electrification of particles over a stretching sheet is numerically studied. The approach is based on the two-phase formulation where both phases are treated as continuum. With the appropriate similarity transformations the governing partial differential equations are reduced into ordinary differential equations, and then the Runge-Kutta 4th-order method coupled with Shooting technique is applied to find the solutions of the problem using Fortran-77. A number of qualitative distinct potential scenarios are predicted. These include the effect of unsteady parameter, Radiation parameter, Electrification Parameter, Prandtl number, Eckert number, Volume fraction, Fluid interaction parameter etc. Their effects on the flow and heat transfer are investigated with help of tables and graphs. The investigation reveals that the electrification of particles contributes to enhance the temperature of particle phase. The effect of radiation enhances significantly the temperature of both phases. It is also found that the thermal and momentum boundary layer thickness decreases with the increase of unsteady parameter.

AMS classification 76T10, 76T15

***Index Terms*—Unsteady parameter, Radiation parameter, Electrification parameter, Volume fraction,**

Fluid-Particle interaction parameter, Boundary layer flow, Stretching sheet, Eckert number, Prandtl number, Shooting Technique.

I. INTRODUCTION

The flow and heat transfer over a stretching surface is important for its wide area of applications such as cooling of nuclear reactor, aerospace component production metal casting, food processing, lubrication, heat removal from nuclear fuel debris, the aerodynamic extrusion of plastic sheet, glass blowing, cooling or drying of papers, drawing plastic films, extrusion of polymer melt-spinning process and rolling and manufacturing of plastic films and artificial fibers cooling industry of dying etc. The importance of such flow problems involving high temperature regime lies in the fact that the mechanical properties of final product are influenced by stretching rate and the rate of cooling The rate of stretching is important as rapid stretching results in sudden solidification, thereby destroying the properties expected for outcomes. In fact radiative heat transfer cannot ignore in the industrial process involving high temperature regime and good working knowledge of it helps designing pertinent equipment.

Due to wide area of applications and simple geometry many researchers have studied boundary layer flow and heat transfer over a stretched surface and have presented the numerical and analytical solutions for various flow characteristics. Crane [11] has studied Flow past a stretching plate. Chen [4] investigated Laminar Mixed convection Adjacent to vertical continuity stretching sheet. Grubka et.al [7] investigated the temperature field in the flow over a stretching surface when subject to uniform heat flux. Vajravelu et.al. [22] have investigated the Hydromagnetic flow of a dusty fluid over a stretching sheet. Sharidan[20] presented similarity solutions for unsteady boundary layer flow and heat Transfer due to stretching sheet . K.M.Chakrabarti[10] has studied the Boundary Layer in a dusty gas. B.J. Gireesha et.al [1,2,3] have studied the effect of various parameter such as unsteady parameter , magnetic parameter, source/sink parameter, radiation parameter on the boundary layer flow and heat Transfer of a dusty fluid over a stretching surface. They have examined the Heat Transfer characteristics for two types of boundary conditions namely variable wall temperature and variable Heat flux. H.I.Anderson et.al.[8] have studied MHD flow of a power law fluid over a stretching sheet. R.N.Jat et.al.[16] have studied MHD Flow and Heat Transfer over an Exponential stretching Sheet with Viscous Dissipation and Radiation Effect . G.K.Ramesh et.al [5,6] have investigated the momentum and heat transfer characteristics in hydrodynamic flow of dusty fluid over a stretching sheet with non uniform heat source/sink and radiation. Paresh Vyas et.al.[15] have investigated the Dissipative MHD Boundary-Layer Flow in a Porous Medium over a Sheet Stretching Nonlinearly in the presence of Radiation .

In view of the above literature survey, since investigations were restricted to flow and heat transfer without taking both radiation and electrification of particles into account, an attempt has been made to study the effect of different flow parameters on unsteady boundary layer and heat transfer of a dusty fluid over a stretching sheet in presence of radiation and electrification of particles. Here, the particles will be allowed to diffuse through the carrier fluid i.e. the random motion of the particles shall be taken into account because of the small size of the particles. This can be done by applying the kinetic theory of gases and hence the motion of the particles across the streamline due to the concentration and pressure diffusion. We have considered the terms related to the heat added to the system to slip-energy flux in the energy equation of particle phase, The momentum equation for

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particulate phase in normal direction, heat due to conduction and viscous dissipation in the energy equation of the particle phase have been considered for better understanding of the boundary layer characteristics. The effects of volume fraction, fluid interaction parameter, radiation, electrification, unsteady parameter, on skin friction and heat transfer have been studied. The problem of two phase suspension flow is solved in the frame work of a model of a two-way coupling model or a two-fluid approach. The physical-computational difficulties pertaining to inclusion of radiation and electrification of particles are addressed with reasonable simplification using similarity transformations. The governing partial differential equations are reduced into system of ordinary differential equations and solved by Shooting Technique using Runge-Kutta Method with help of FORTRAN-77.

II. MATHEMATICAL FORMULATION AND SOLUTION:

Let us consider an unsteady two dimensional laminar boundary layer flow of a viscous, incompressible dusty fluid over a stretching sheet .The flow is caused by impermeable stretching sheet. A Cartesian coordinate system is used. The sheet is considered along the x-axis and y-axis being normal to it. Two equal and opposite forces are applied along the stretching sheet. The sheet being stretched with the velocity $U_w(x)$ along the x-axis, keeping the origin fixed in the fluid of ambient temperatureT_∞. Both the fluid and the dust particle clouds are suppose to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size. Rosseland approximation is assumed to account for radiative heat flux in presence of electrification of particles in the flow .

Soo (21) has studied the effect of electrification on the dynamics of a particulate system. At low temperature, electrification of solid particles occurs because of impact with the wall. Even a very slight charge on the solid particles will have a pronounced effect on concentration distribution in the flow of a gas-solid system. Although electric charge on the solid particles can be excluded by definition in theoretical analysis or when dealing truly with a boundless system, electrification of the solid particles always occurs when contact and separation are made between the solid particles and a wall of different materials or similar materials but different surface condition. The electric charges on the solid particles cause deposition of the solid particles on a wall in a more significant manner than the gravity effect and are expected to effect the motion of a metalized propellant and its product of reaction through a rocket nozzle and the jet at the exit of the nozzle. The charged solid particles in the jet of a hot gas also effect radio communications.

As a general statement, any volume element of charge species, with charge "e" experiences an instantaneous force given by the Lorentz force law,

$$\vec{f} = e\,\vec{E} + \vec{J} \times \vec{B}$$

Where $\vec{B}$ is the magnetic flux density. The current densities in corona discharge are so low that the magnetic force term $\vec{J} \times \vec{B}$ can be omitted, as this term is many orders of magnitude smaller than the Coulomb term$e\vec{E}$.

The ion drift motion arises from the interaction of ions, constantly subject to the Lorentz force with the dense neutral fluid medium. This interaction produces an effective drag force on the ions. The drag force is in equilibrium with the Lorentz force so that the ion velocity in a field $\vec{E}$ is limited to $k_m\vec{E}$, where k_m is the mobility of the ion species. The drag force on the ions has an equal and opposite reaction force acting on the neutral fluid molecules via this ion-neutral molecules interaction, the force on the ions is transmitted directly to the fluid medium, so the force on the fluid particles is also given by the above equation .

The governing equations of unsteady two dimensional boundary layer incompressible flows of dusty fluids are given by

$$\frac{\partial u_f}{\partial t} + \frac{\partial}{\partial x}\vec{F}(u_f) + \frac{\partial}{\partial y}\vec{G}(u_f) + H(u_f) = S(u_f, u_p, T, T_p) \quad (2.1)$$

$$\frac{\partial u_p}{\partial t} + \frac{\partial}{\partial x}\vec{F}(u_p) + \frac{\partial}{\partial y}\vec{G}(u_p) + H(u_p) = S_p(u_f, u_p, T, T_p) \quad (2.2)$$

Where $\vec{F}(u_f) = \begin{bmatrix} u \\ (1-\varphi)\rho u^2 \\ \rho c_p uT \end{bmatrix}$,

$$\vec{G}(u_f) = \begin{bmatrix} v \\ (1-\varphi)\rho uv \\ \rho c_p vT \end{bmatrix}, \quad \vec{F}(u_p) = \begin{bmatrix} \rho_p u_p \\ \rho_p {u_p}^2 \\ \rho_p u_p v_p \\ \rho_p c_s u_p T_p \end{bmatrix}, \quad \vec{G}(u_p) = \begin{bmatrix} \rho_p v_p \\ \rho_p u_p v_p \\ \rho_p {v_p}^2 \\ \rho_p c_s v_p T_p \end{bmatrix}, H(u_f) = 0\,, H(u_p) = 0$$

$$S(u_f, u_p, T, T_p) = \begin{bmatrix} 0 \\ \mu\frac{\partial^2 u}{\partial y^2} - \frac{\rho_p}{\tau_p}(u - u_p) + \varphi\rho_s\left(\frac{e}{m}\right)E \\ k\,(1-\varphi)\frac{\partial^2 T}{\partial y^2} + \frac{\rho_p c_s}{\tau_T}(T_p - T) + \frac{\rho_p}{\tau_p}(u_p - u)^2 + \mu(1-\varphi)\left(\frac{\partial u}{\partial y}\right)^2 + \varphi\rho_s\left(\frac{e}{m}\right)Eu_p - (1-\varphi)\frac{\partial q_r}{\partial y} \end{bmatrix}$$

$S_p(u_f, u_p, T, T_p)$

$$= \begin{bmatrix} 0 \\ \frac{\partial}{\partial y}\left(\varphi\mu_s \frac{\partial u_p}{\partial y}\right) + \frac{\rho_p}{\tau_p}(u - u_p) + \varphi\rho_s\left(\frac{e}{m}\right)\mathrm{E} \\ \frac{\partial}{\partial y}\left(\varphi\mu_s \frac{\partial v_p}{\partial y}\right) + \frac{\rho_p}{\tau_p}(v - v_p) \\ \frac{\partial}{\partial y}\left(\varphi k_s \frac{\partial T_p}{\partial y}\right) - \frac{\rho_p}{\tau_p}(u-u_p)^2 + \varphi\mu_s\left(u_p\frac{\partial^2 u_p}{\partial y^2} + \left(\frac{\partial u_p}{\partial y}\right)^2\right) - \frac{\rho_p c_s}{\tau_T}(T_p - T) + \varphi\rho_s\left(\frac{e}{m}\right)Eu_p - \varphi\frac{\partial q_{rp}}{\partial y} \end{bmatrix}$$

With boundary conditions

$$\left.\begin{array}{l} u = U_w(x,t) = \frac{cx}{1-at}, v = 0 \ as\ y \to 0 \\ \rho_p = \omega\rho, u = 0, u_p = 0, v_p \to v\ as\ y \to \infty \end{array}\right\} \quad (2.3)$$

In order to solve the temperature equations, we consider non –dimensional temperature boundary conditions as follows

$$\left.\begin{array}{l} T = T_w = T_\infty + T_0\frac{cx^2}{\nu(1-at)^2}\ at\ y = 0 \\ T \to T_\infty, T_p \to T_\infty\ \ as\ y \to \infty \end{array}\right\} \quad (2.4)$$

For most of the gases $\tau_p \approx \tau_T$, $k_s = k\frac{c_s}{c_p}\frac{\mu_s}{\mu}$ if $\frac{c_s}{c_p} = \frac{2}{3P_r}$

Introducing the following non dimensional variables in equation (2.1) and (2.2)

$$\left.\begin{array}{l} u = \frac{cx}{1-at}f'(\eta), v = -\sqrt{\frac{c\nu}{1-at}}f(\eta), \\ u_p = \frac{cx}{1-at}F(\eta), \ v_P = \sqrt{\frac{c\nu}{1-at}}\ G(\eta), \\ \eta = \sqrt{\frac{c}{\nu(1-at)}}\ y\ \ H(\eta) = \frac{\varphi\rho_s}{\rho} = \frac{\rho_p}{\rho} \\ = \rho_r, \ P_r = \frac{\mu c_p}{k}, \ \beta = \frac{1-at}{c\tau_p}, \epsilon = \frac{\nu_s}{\nu}, \\ \varphi = \frac{\rho_p}{\rho_s}, \mathrm{A} = \frac{a}{c}, E_c = \frac{c\nu}{C_pT_0}, \nu = \frac{\mu}{\rho} \\ M = \frac{(1-at)^2}{c^2x}\left(\frac{e}{m}\right)E,, \quad Ra = \frac{16T_\infty^3\sigma^*}{3kk^*} \\ ,\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \ \theta_p(\eta) = \frac{T_p-T_\infty}{T_w-T_\infty} \\ \text{Where } T - T_\infty = T_0\frac{cx^2}{\nu}\frac{1}{(1-at)^2}\theta, \\ T_p - T_\infty = T_0\frac{cx^2}{\nu}\frac{1}{(1-at)^2}\theta_p \end{array}\right\} \quad (2.5)$$

Using Rosseland approximation, the radiative heat flux q_r is modeled as :

$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}$,where σ^* is the Stefan-Boltzman constant and k^* is the mean absorption coefficient.

Assuming that the differences in the temperature within the flow to be sufficiently small so that T^4 can be expressed as linear function of the temperature, one can expand T^4 in a Taylor's series about T_∞ as follows

$$T^4 = T_\infty^4 + 4T_\infty^3(T - T_\infty) + 6T_\infty^2(T - T_\infty)^2 + \text{---} \quad (2.6)$$

By neglecting higher order terms beyond the first degree in $(T - T_\infty)$ we get:

$T^4 = -3T_\infty^4 + 4T_\infty^3 T$

Substituting this value in q_r, we get

$$\left.\begin{array}{l} \frac{\partial q_r}{\partial y} = -\frac{16T_\infty^3\sigma^*}{3k^*}\frac{\partial^2 T}{\partial y^2} \\ \text{Similarly we can write} \\ \frac{\partial q_{rp}}{\partial y} = -\frac{16T_\infty^3\sigma^*}{3k^*}\frac{\partial^2 T_p}{\partial y^2} \end{array}\right\} \quad (2.7)$$

Substituting for q_r and q_{rp} ,
The equations (2.1) and (2.2) become

$$H'(\eta) = -\left(H(\eta)F(\eta) + H(\eta)G'(\eta)\right)/\left(A\frac{\eta}{2} + G(\eta)\right) \quad (2.8)$$

$$f'''(\eta) = -f(\eta)f''(\eta) + \left(f'(\eta)\right)^2 + A\left(f'(\eta) + \frac{\eta}{2}f''(\eta)\right) - \frac{\beta H}{(1-\varphi)}\left(F(\eta) - f'(\eta)\right) - \frac{HM}{(1-\varphi)} \quad (2.9)$$

$$F''(\eta) = \left[A\left\{\frac{\eta}{2}F'(\eta) + F(\eta)\right\} + \left(F(\eta)\right)^2 + G(\eta)F'(\eta) + \beta\left(F(\eta) - f'(\eta)\right) - M\right]/\epsilon \quad (2.10)$$

$$G''(\eta) = \left[\frac{A}{2}\left(\eta G'(\eta) + G(\eta)\right) + G(\eta)G'(\eta) + \beta\left(f(\eta) + G(\eta)\right)\right]/\epsilon \qquad (2.11)$$

$$\theta'' = \frac{\begin{bmatrix} Pr(2f'\theta - f\theta') - \frac{2}{3}\frac{\beta}{1-\varphi}H[\theta_p - \theta] - \frac{1}{1-\varphi}PrE_c\beta H[F-f']^2 - PrE_c(f'')^2 \\ +\frac{A}{2}Pr\left(\eta\theta'(\eta) + 4\theta(\eta)\right) - \frac{Pr}{(1-\varphi)}E_cHMF(\eta) \end{bmatrix}}{(1+R_a)} \qquad (2.12)$$

$$\theta_p'' = \frac{\begin{bmatrix} \frac{A}{2}\left(\eta\theta_p'(\eta) + 4\theta_p(\eta)\right) + 2F(\eta)\theta_p + G(\eta)\theta_p'(\eta) + \beta\left(\theta_p(\eta) - \theta(\eta)\right) \\ +\frac{3}{2}E_cP_r\beta\left(f'(\eta) - F(\eta)\right)^2 \\ -\frac{3}{2}\epsilon E_cP_r\left(F(\eta)F''(\eta) + \left(F'(\eta)\right)^2\right) - E_c\frac{3}{2}P_rMF(\eta) \end{bmatrix}}{\left(\frac{\varepsilon}{P_r} + \frac{3}{2}\frac{1}{\gamma}R_a\right)} \qquad (2.13)$$

$$\left.\begin{aligned} & G'(\eta) = 0, f(\eta) = 0, f'(\eta) = 1, \\ & F'(\eta) = 0, \theta(\eta) = 1, \theta_p' = 0 \quad as\ \eta \to 0 \\ & \&\ f'(\eta) = 0, F(\eta) = 0, G(\eta) = -f(\eta) \\ & , H(\eta) = \omega, \theta(\eta) \to 0, \theta_p(\eta) \to 0 \\ & as\ \eta \to \infty \end{aligned}\right\} \qquad (2.14)$$

IV. SOLUTION METHOD:

Here in this problem the value of $f''(0), F(0), G(0), H(0), \theta'(0), \theta_p(0)$ are not known but $f'(\infty) = 0, F(\infty) = 0, G(\infty) = -f(\infty), H(\infty) = \omega, \theta(\infty) = 0, \theta_p(\infty) = 0$ are given. We use Shooting method to determine the value of $f''(0), F(0), G(0), H(0), \theta'(0), \theta_p(0)$ .We have supplied $f''(0) = \propto_0$ and $f''(0) = \propto_1$. The improved value of $f''(0) = \propto_2$ is determined by utilizing linear interpolation formula. Then the value of $f'(\propto_2, \infty)$ is determined by using Runge-Kutta method. If $f'(\propto_2, \infty)$ is equal to $f'(\infty)$ up to a certain decimal accuracy, then $\propto_2$ i.e $f''(0)$ is determined, otherwise the above procedure is repeated with $\propto_0 = \propto_1$ *and* $\propto_1 = \propto_2$ until a correct $\propto_2$ is obtained. The same procedure described above is adopted to determine the correct values $F(0), G(0), H(0), \theta'(0), \theta_p(0)$.

The essence of Shooting technique to solve a boundary value problem is to convert the boundary value problem into initial value problem. In this problem the missing value of $\theta'(0)$ and $f''(0)$ for different set of values of parameter are chosen on hit and trial basis such that the boundary condition at other end i.e. the boundary condition at infinity (η_∞)are satisfied.A study was conducting to examine the effect of step size as the appropriate values of step size $\Delta\eta$ was not known to compare the initial values of $F(0), G(0), H(0), \theta_p(0)$, $\theta'(0)$ and $f''(0)$ .If they agreed to about 6 significant digits, the last value of η_∞ used was considered the appropriate value; otherwise the procedure was repeated until further change in η_∞ did not lead to any more change in the value of $F(0), G(0), H(0), \theta_p(0)$, $\theta'(0)$ and $f''(0)$ .The step size $\Delta\eta$ =0.125 has been found to ensure to be the satisfactory convergence criterion of 1×10^{-6}.The solution of the present problem is obtained by numerical computation after finding the infinite value for η. It has been observed from the numerical result that the approximation to $\theta'(0)$ and $f''(0)$ are improved by increasing the infinite value of η which is finally determined as η =10.0 with a step length of 0.125 beginning from η = 0. Depending upon the initial guess and number of steps N. the value of $f''(0)$ and $\theta'(0)$ are obtained from numerical computation which is given in table – 1 for different parameters.

V. GRAPHICAL REPRESENTATION:

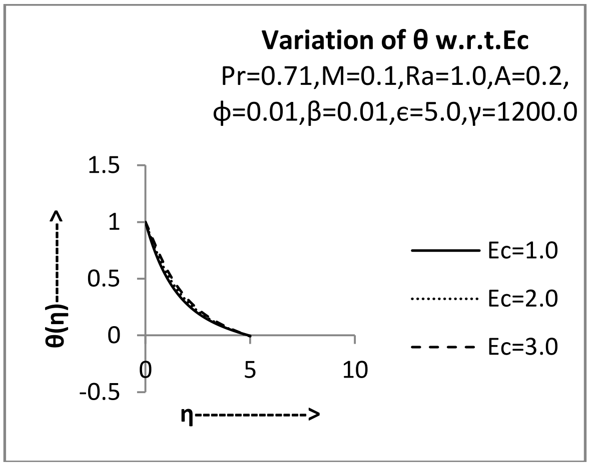


Fig-1

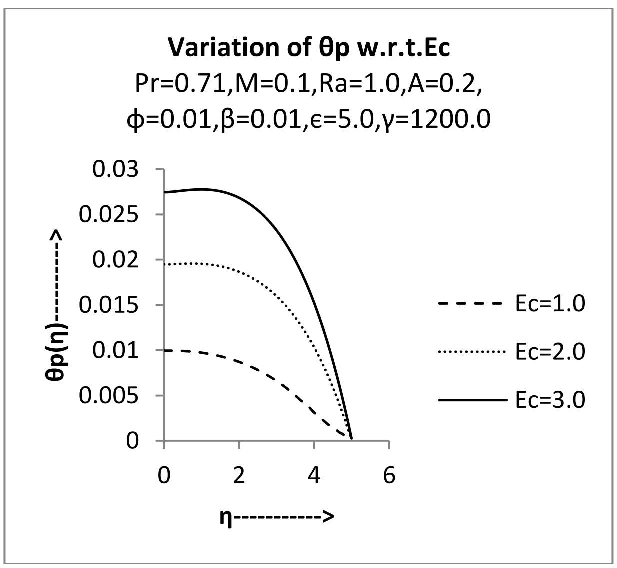


Fig-2

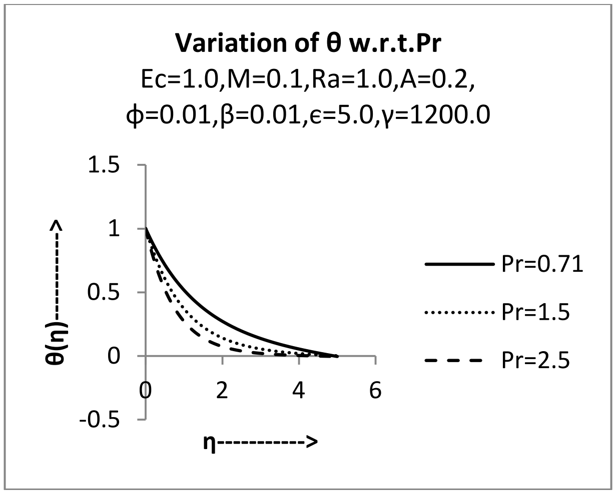


Fig-3

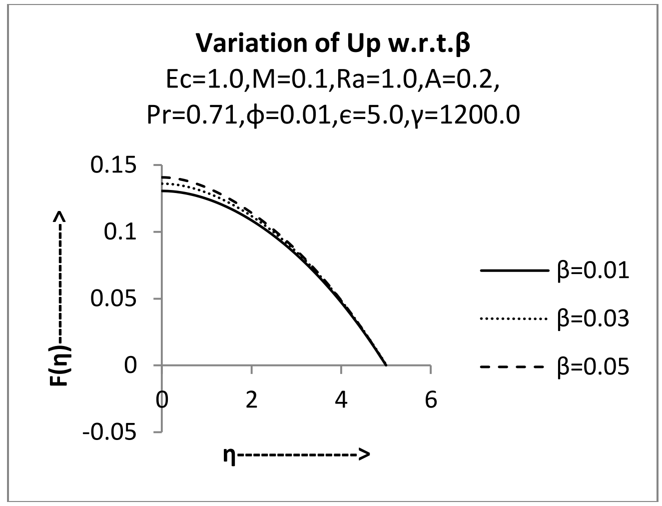


Fig-6

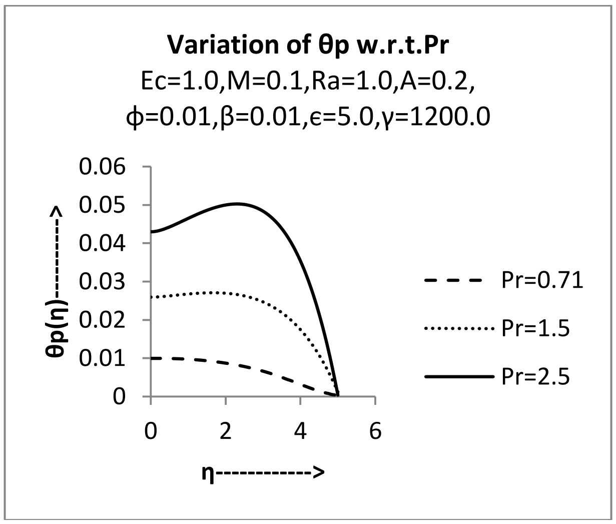


Fig-4

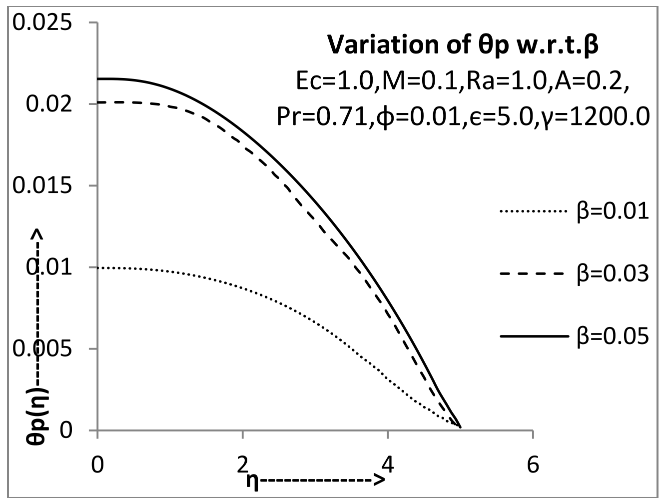


Fig-7

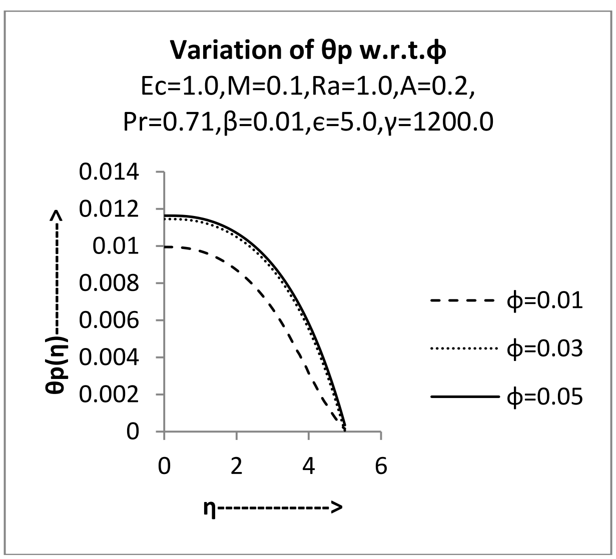


Fig-5

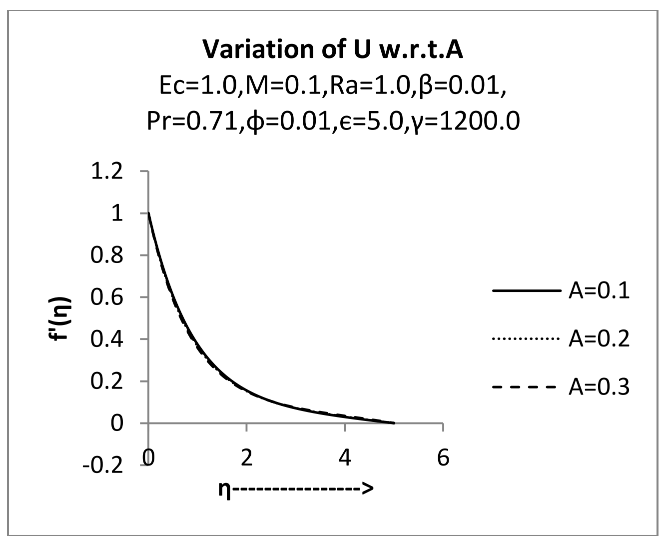


Fig-8

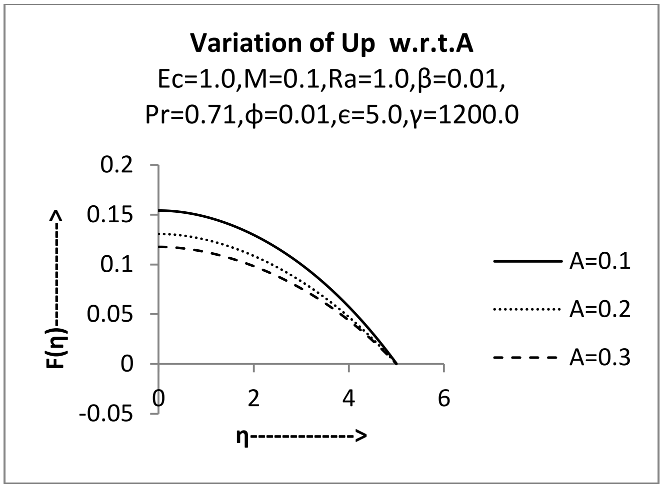


Fig-9

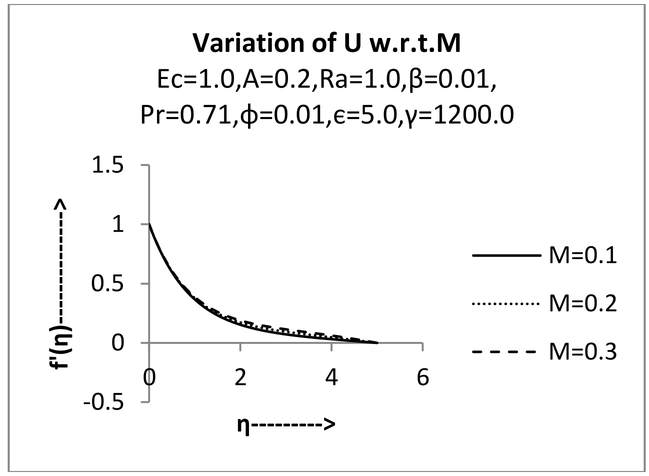


Fig-12

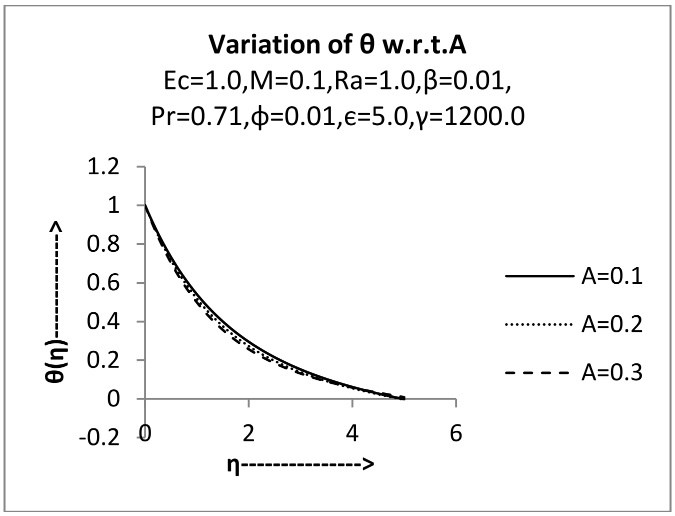
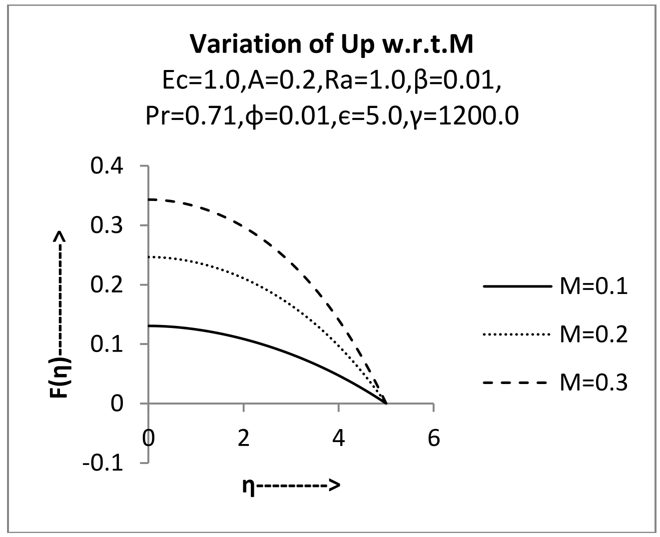


Fig-10



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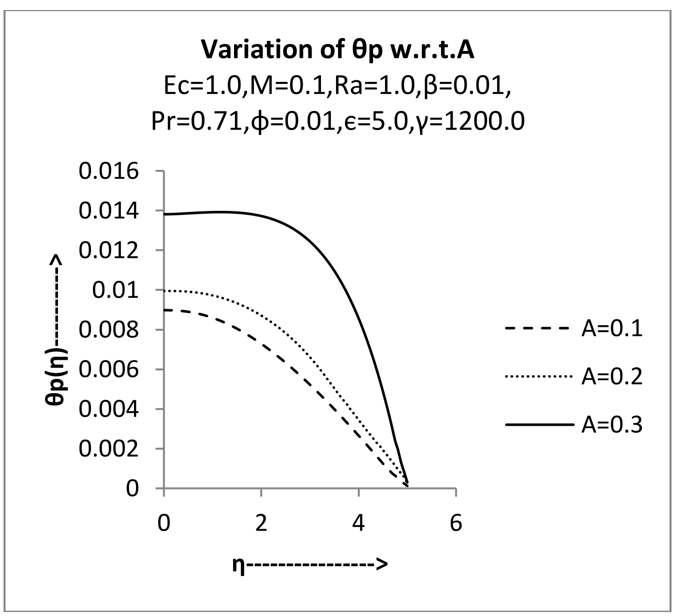
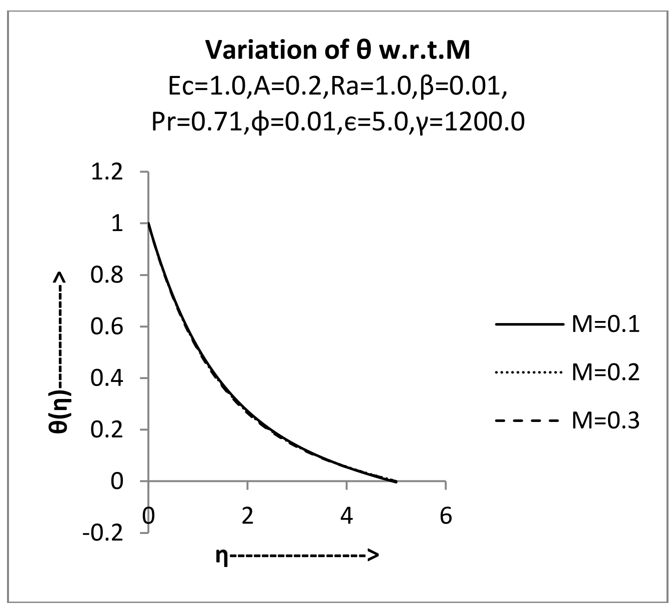


Fig-11



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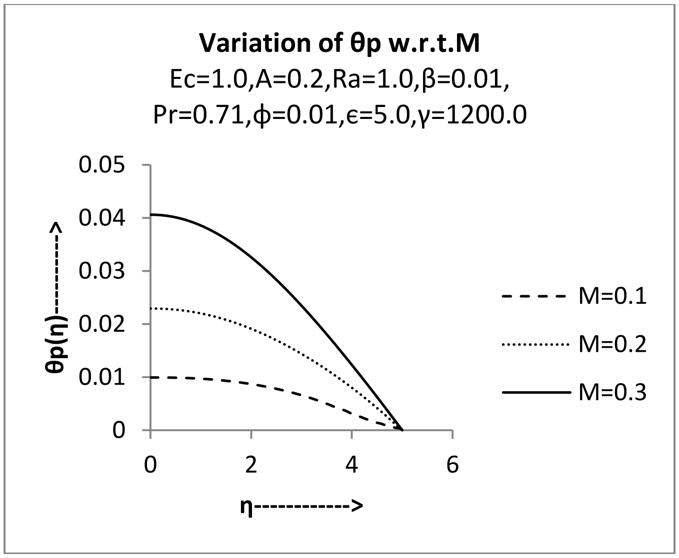

Fig-15

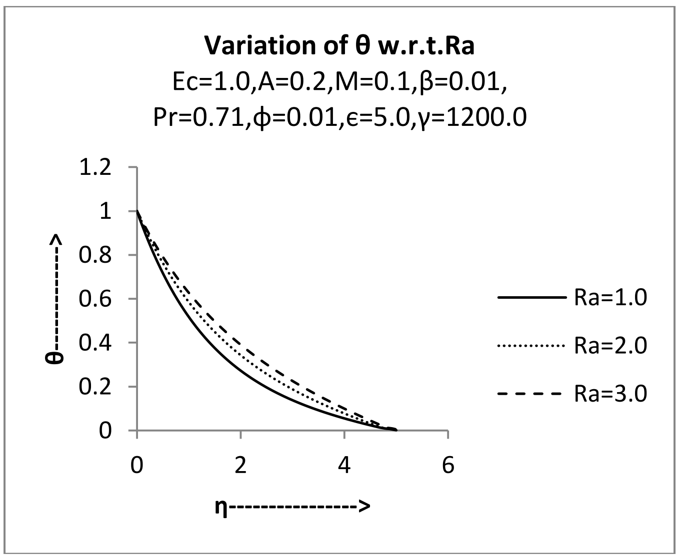

Fig-16

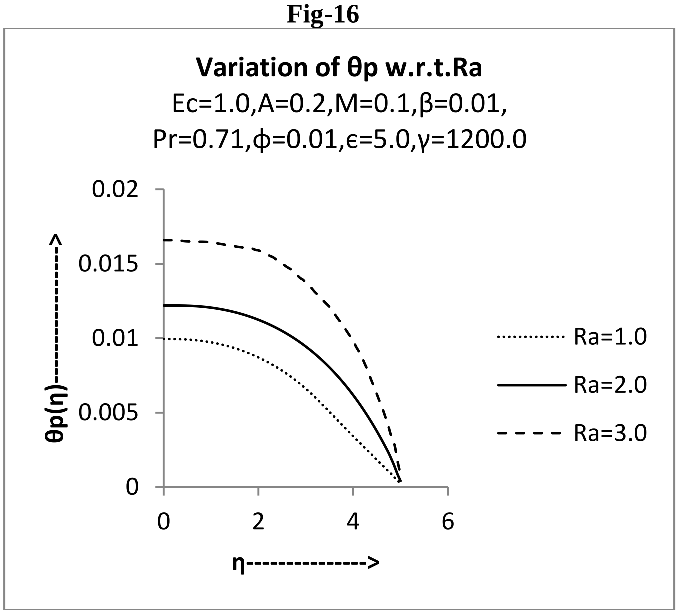

Fig-17

VI. RESULT AND DISCUSSION:

The set of non linear ordinary differential equations (2.8) to (2.13) with boundary condition (2.14) were solved using well known Runge-Kutta forth order algorithm with a systematic guessing of $F(0), G(0), H(0), \theta_p(0), f''(0)$ and $\theta'(0)$ by the shooting technique until the boundary condition at infinity are satisfied.The step size 0.125 is used while obtaining the numerical solution accuracy up to the sixth decimal place i.e. 1×10^{-6}, which is very sufficient for convergence. In this method we choose suitable finite values of $\eta \to \infty$ which depends on the values of parameter used. The computations were done by the computer language FORTRAN-77.The shear stress(Skin friction coefficient)which is proportional to $f''(0)$ and rate of heat transfer(Nusselt number) which is proportional to $\theta'(0)$ are tabulated in Table-1 for different values of parameter used .It is observed from the table that shear stress and rate of heat transfer decreases on the increase of Ec .The shear stress decrease for increasing values of Pr whereas rate of heat transfer is increasing for increasing values of Pr. The Nusselt number increase on the increasing of unsteady parameter ‘A’. The Nusselt number increase on the increase of electrification parameter M due to Lorentz force .The increasing value of Radiation parameter increases the Nusselt number.

Fig-1 and Fig-2 explain that the increasing values of Ec, increase the temperature of fluid phase and particle phase because the heat energy is stored in the fluid due to frictional heating, which shows effect on the boundary layer growth. Fig-3 depicts the effect of Pr on temperature profile of fluid phase. From the figure we observe that, when Pr increases the temperature of fluid phase decreases because of the increase of fluid heat capacity. But Fig-4 indicates the increase of particle phase temperature with increase of Pr.Fig-5 shows the effect of φ on temperature of particle phase. It is evident that increasing in volume fraction increases the temperature of particle phase due to collisions of more and more volume of particles generating heat energy. Fig-6 demonstrates the effect of β which infers that increasing of β increases the particle phase velocity.Fig-7 illustrates the increasing β, increases the temperature of dust phase.Fig-8 and Fig-9 demonstrate that velocity profile of fluid phase and particle phase respectively. It is observed that the velocity of both phases decrease on the increase of unsteady parameter ‘A’. This is because of fluid flow caused by stretching sheet.Fig-10 explains that the temperature of fluid phase decreases with increasing of unsteady parameter ‘A’. It means the temperature at a point of surface decreases significantly with the increase of unsteady parameter, which implies the increase of heat transfer rate $-\theta'(0)$ at the surface. But Fig -11 describes that the temperature profile of particle phase increases on the increase of unsteady parameter.Fig-12 and Fig-13 represent the increase of Electrification parameter ‘M’ increase the velocity of both fluid phase and particle phase respectively.Fig-14 and Fig-15 demonstrate the temperature of both fluid and particle phase respectively. It is noticed that the temperature of fluid phase and temperature of particle phase increase on the increase of electrification parameter ‘M’ .Fig-16 and Fig-17 depict the variation of temperature of fluid phase and particle phase respectively. It is observed that the enhancing value of Radiation parameter ‘Ra’ enhances significantly the temperature of both fluid phase and particle phase. It means the thermal boundary layer thickness is increasing on the increase of radiation parameter.

TABLE-1 Showing initial values of wall velocity gradient$-f''(0)$ *and temperature gradient* $-\theta'(0)$

E_c	P_r	φ	β	A	M	Ra	$-f''(0)$	$u_p(0)$	$-v_p(0)$	H(0)	$-\theta'(0)$	$\theta_p(0)$
-	-						1.001397	-	-	-	1.082315	-
1.0	0.71	0.01	0.01	0.2	0.1	1.0	1.052208	0.130626	0.811903	0.123579	0.671283	0.009948
2.0							1.052206	0.130671	0.812219	0.129267	0.534532	0.019471
3.0							1.052189	0.130370	0.812175	0.129455	0.397702	0.027455
1.0	0.71	0.01	0.01	0.2	0.1	1.0	1.052208	0.130626	0.811903	0.123579	0.671283	0.009948
	1.5						1.051585	0.132219	0.811541	0.122889	0.993998	0.025936
	2.5						1.051365	0.130566	0.812247	0.129645	1.288708	0.042996
1.0	0.71	0.01	0.01	0.2	0.1	1.0	1.052208	0.130626	0.811903	0.123579	0.671283	0.009948
		0.03					1.050507	0.130558	0.812340	0.129592	0.671889	0.011453
		0.05					1.050291	0.130563	0.812472	0.129288	0.671744	0.011637
1.0	0.71	0.01	0.01	0.2	0.1	1.0	1.052208	0.130626	0.811903	0.123579	0.671283	0.009948
			0.03				1.053200	0.136088	0.804802	0.123819	0.671299	0.020095
			0.05				1.053253	0.140788	0.802073	0.121463	0.672194	0.021536
1.0	0.71	0.01	0.01	0.1	0.1	1.0	1.016463	0.154155	0.917946	0.127568	0.629145	0.008979
				0.2			1.052208	0.130626	0.811903	0.123579	0.671283	0.009948
				0.3			1.086868	0.117636	0.726370	0.119257	0.709159	0.013815
1.0	0.71	0.01	0.01	0.1	0.1	1.0	1.052208	0.130626	0.811903	0.123579	0.671283	0.009948
					0.2		1.039542	0.246553	0.830431	0.087001	0.679003	0.022943
					0.3		1.031135	0.343203	0.844636	0.062390	0.684428	0.040623
1.0	0.71	0.01	0.01	0.1	0.1	1.0	1.052208	0.130626	0.811903	0.123579	0.671283	0.009948
						2.0	1.051932	0.130302	0.811923	0.123559	0.544968	0.012198
						3.0	1.051898	0.130288	0.812013	0.123614	0.472075	0.016592

VII. FINAL COMMENTS:

In this investigation, the unsteady flow of viscous, incompressible dusty fluid in presence of radiation and electrification of particles is studied .The highly nonlinear momentum and energy equations (2.1) and (2.2) are converted into system of ordinary differential equations by using similarity transformations. The solutions are found numerically by Runge-Kutta 4-th order method with help of Shooting Technique using FORTRAN-77. We concluded by noting that , even though the particle laden flow over a stretching sheet is studied by other authors but the boundary layer flow and heat transfer characteristics have been discussed extensively by considering various physical parameters like unstedy parameter ‘A’,prandtl number ‘ Pr’,Eckert number ‘Ec’,Electrification parameter ‘M’,radiation parameter ‘Ra’,volume fraction ‘φ’ and particle interaction parameter ‘β’, ratio of specific heat γ ,diffusion parameter ‘ε’.

Finally, we observed the following results:

i. The increasing value of unsteady parameter A decreases the temperature profiles of fluid phase and increase the temperature of dust phase. Also increase value of A decreases velocity of both phases. The rate of cooling is much faster for higher values of unsteady parameter but it takes long times for cooling during the steady flow.

ii. The temperature profile of both phases increase with the increase of radiation parameter Ra. Thus the radiation should be at minimum in order to facilitate the cooling process.

iii. Increasing value of Ec is enhancing the temperature of both fluid phase as well as particle phase which indicates that the heat energy is generated in fluid due to frictional heating.

iv. The thermal boundary layer thickness decreases on the effect of Pr. The temperature decreases at a faster rate for higher values of Pr which implies the rate of cooling is faster in case of higher prandtl number.

v. The thermal boundary layer thickness and momentum boundary layer thickness increase for increase value of electrification parameter M.

vi. Increasing β increase the velocity and temperature of the particle phase.

vii. The increasing value of φ increases the temperature profile of particle phase.

viii. We have investigated the problem assuming the values γ=1200.0 ϵ=5.0

VIII. NOMENCLATURE:

Ra radiation parameter
M electrification parameter
m mass of particle
E electric field of force
e charge of particle
q_r radiative heat flux for fluid phase
q_{rp} radiative heat flux for particle phase
E_c eckert number

P_r prandtl number
T_∞ temperature at large distance from the wall.
T_p temperature of particle phase.
T_w wall temperature

$U_w(x)$ stretching sheet velocity

c_p specific heat of fluid

c_s specific heat of particles

k_s thermal conductivity of particle

u_p , v_p velocity component of the particle along x-axis and y-axis

A unsteady parameter

c stretching rate

k thermal conductivity of fluid

l characterstic length

T temperature of fluid phase.

u,v velocity component of fluid along x-axis and y-axis

x,y cartesian coordinate

Greek Symbols :

φ volume fraction

β fluid – particle interaction parameter

β^* volumetric coefficient of thermal expansion

σ^* Stefan-Boltzman constant

k^* is the mean absorption coefficient.

ρ density of the fluid

ρ_p density of the particle phase

ρ_s material density

η similarity variable

θ fluid phase temperature

θ_p dust phase temperature

μ dynamic viscosity of fluid

ν kinematic viscosity of fluid

γ ratio of specific heat

τ relaxation time of particle phase

τ_T thermal relaxation time i.e. the time required by the dust particle to adjust its temperature relative to the fluid.

τ_p velocity relaxation time i.e. the time required by the dust particle to adjust its velocity relative to the fluid.

ε diffusion parameter

ω density ratio

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