# Improved Channel Matrix in Sphere Decoder for MIMO Wireless Systems

# Vandana Goyel, Garima Mathur, R.P.Yadav

Abstract— The recent demand for mobile communication systems with high data rates has increased in the recent years. New methods are essential in order to satisfy this huge communication demand, exploiting the limited resources like as the power and bandwidth as efficient as possible. Multiple Input and multiple Output (MIMO) systems with multiple antennas at the transmitter side and multiple antennas at the receiver side provides a better solution for the demand of high data rates by exploiting the spatial domain under the constraints of limited bandwidth and transmit power. Sphere decoding approach is one of the effective approach in MIMO system to identify the problem in multi user detection and ML detection problem. There are different approaches to improve the Sphere Decoding approach so that more accurate and optimized results will be obtained. One of such approach is suggested in this paper. In this work, instead of processing on complete state space, a K-best sphere decoder approach is suggested in this work. The presented paper is about to define an improve approach for the selection of K-best neighbors.

*Index Terms*— Intersymbol Interference (ISI), K-nearest neighbor (KNN), Multiple Input and Multiple Output (MIMO), Sphere Decoder (SD).

## I. INTRODUCTION

During the last few years, wireless communication has benefited with major advances and it is considered as the most unique technique. With the use of Multiple input and multiple output antennas at both the transmitter as well as the receiver side, which is popularly known as MIMO technique many advancements are done in the field of communication. Many problems most commonly Inter symbol Interference giving rise to the multipath fading are solved by the use of MIMO channels. MIMO systems are the advancement to the smart antennas systems. MIMO systems basically employ multiple number of antennas at the transmitter side and multiple number of antennas at the receiver side. MIMO systems are basically used to provide high data rates to achieve high data rate capacity. It provides high quality of service (QOS) while reducing the probability of error, Pe. Sphere decoding approach is one of the effecient approaches in MIMO system to identify the problem in multi user detection and ML detection problem. There are different approaches to improve the Sphere Decoding approach so that more accurate and optimized results will be obtained. One of such approach is

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suggested in this work. In this work, instead of processing on complete state space, a K-best sphere decoder approach is suggested here. The presented work is about to define an improve approach for the selection of K-best neighbors. To perform this selection, at first the distance analysis is been performed. To obtain the distance the dijkstra method is selected in this work. Once the distance analysis is done with all possible neighbors then the adaptive threshold will be implemented to identify the valid K-neighbor nodes. These K-Neighbors will be considered as the initial state space. The distance analysis between the nodes will be done using cosine distance method. The work is about to improve the performance of sphere decoding approach so that the noise estimation over the signal will be improved.

#### II. MIMO CHANNEL MODELING

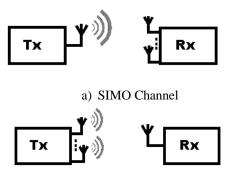
During the past few years, the intensive work of researchers on Multiple- Input Multiple-Output (MIMO) techniques has demonstrated their key role in increasing the channel capacity reliability and improving the spectral efficiency in wireless communication systems without the need to additional spectral resources [1].

In earlier times Single input Single Output antennas were used. A single-input and single output (SISO) system is a simple single variable control system with one input and one output. In some conditions, SISO systems are prone to problems caused by multipath effects. In a digital communications system, this may cause a reduction in data speed and an increase in the number of errors.



Fig. 1: SISO Channel

To overcome the drawbacks of SISO various smart antenna techniques are used. Single Input and multiple output (SIMO) and (MISO) systems came into the market.



## b) MISO Channel Fig.2 : SIMO and MISO Channel

MIMO system is an extension of smart antennas system. Traditional smart antennas system uses multiple antennas at the receiver, whereas in a MIMO system multiple antennas are used both at the transmitter as well as on the receiver side. Basic idea of MIMO is to improve quality BER and data rate (bits/sec) by using multiple Transmitter and multiple receiver antennas. It provides higher spectral efficiency (more bits per second per hertz of bandwidth). The main aim of the MIMO technology is to improve the capacity of the system [2]. In MIMO systems, a transmitter sends multiple streams by multiple transmit antennas. The transmit streams are passed through a channel matrix which consists of multiple paths between multiple transmit antennas at the transmitter side and multiple receiver antennas at the receiver side. Then, the receiver gets the received signal vectors by the use of multiple receiver antennas and decodes the received signal vectors into the original information.

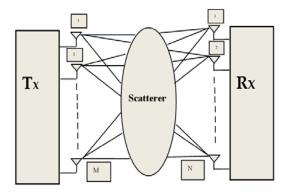


Fig.3: MIMO Channel Model

In MIMO, multiple antennas are present at both the transmitter and receiver side. The transmitted streams go through a matrix channel which consists of all  $N_t N_r$  paths between the  $N_t$  transmit antennas at the transmitter side and  $N_r$  receiver antennas at the receiver side. Then, the receiver gets the received signal vectors by the multiple receive antennas and decodes the received signal vectors into the original information.

The MIMO system model is given as,

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{G} \tag{1}$$

Where,

- Y=Receive vector
- X= Transmit vector
- H= Channel Matrix
- G= Noise Vector

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$
(2)

Inserting equation (2) in (1), the MIMO channel model equation can be given as,

$$Y = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} X + G$$
(3)

## III. MIMO SYSTEM CHANNEL CAPACITY

Channel capacity is the maximum information rate that can be transmitted and received with arbitrarily low probability of error at the receiver [3]. Considering the memory less channel with X and Y as input and output respectively, the channel capacity is explained as the maximum of the mutual information between X and Y:

$$C = max_{p(x)}I(X;Y)$$
<sup>(4)</sup>

A channel is said to memory less if the probability distribution of the output depends only on the input at that time and is conditionally independent of previous channel inputs and outputs. P(x) is the probability distribution function (pdf) of the input symbols X [4].

#### IV. SPHERE DECODER

The maximum likelihood decoder for a MIMO receiver operates by comparing the received signal vector with all possible noiseless received signals corresponding to all possible transmitted signals. The idea behind Sphere Decoder is very simple, here we search only those lattice points that lie within the sphere of radius r around the given vector x, thereby reducing the complexity as well as search space.

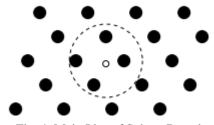


Fig. 4: Main Idea of Sphere Decoder

Now if we consider r to be too large than our search remains exponential in size, but if the value of r is too small no points lie within the sphere. Another question which arises is that how can we tell which lattice points are inside the sphere? Now if we consider to take the distance of each lattice point from x, than there is no point of considering sphere decoding approach as we still need an exhaustive search. Although it is tough to determine the lattice points inside a general n-dimensional sphere, it is trivial to do the same if n=1.The reason is that a one-dimensional sphere is simply an interval and so the desired lattice points will be the integer values that lie in this interval. We can consider this observation from dimension m to m+1. Suppose we have determined all m-dimensional lattice points that lie in a sphere of radius r. Then for any such m-dimensional point, the set of admissible values of the m + 1-th dimensional coordinate that lie in the higher dimensional sphere of the same radius r forms an interval [5].

By this we can say that we can determine all lattice points in a sphere of dimension n and radius r by successively determining all lattice points in a given sphere of lower dimensions 1,2,....,n and the same radius r. Such an algorithm for determining the lattice points in an n dimensional sphere essentially constructs a tree where the branches in the m-th level of the tree correspond to the lattice points inside the sphere of radius r and dimension m.

However, the complexity of such type of algorithm depends on the size of the tree. Normally, this algorithm is used as a depth first tree search, where each level in the search is expressed as one transmit antenna's signal.

#### V. QR DECOMPSITION OF CHANNEL MATRIX

In statistics, maximum-likelihood estimation (MLE) is a method of estimating the parameters of a statistical model. For example: If we are interested in knowing the height of the horses but we are unable to measure the height of every single horse due to the cost and the time limitation. Considering the heights are normally distributed we will calculate the mean and variance of the height by using MLE estimation by knowing the height of some sample of the overall population. MLE would implement this by taking the mean and variance as parameters and finding particular parametric values that make the observed results the most probable [6]. To reduce the complexity of Maximum Likelihood, Sphere Decoder restricts the searching space by radius r. So decision criteria of ML can be written:

$$\|y - H_{\rho}s\|^2 < r^2 \tag{5}$$

The main concept of SD is to find the nearest point from received vector in the lattice generated by the channel matrix  $\rm H_{\rm e}$  .

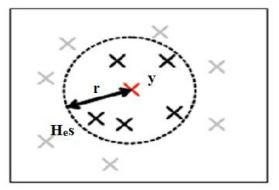


Fig. 5: Concept of Sphere Decoder

By using this complexity of the ML will be reduced as the candidates within the radius r will only be considered. The equation can further be reduced by the help of the QR decomposition of the channel matrix  $H_e$  as,

$$\|y - H_{e}s\|^{2} = \|y - QRs\|^{2} < r^{2}$$
(6)

$$= \|Q^H y - Rs\|^2 < r^2 \tag{7}$$

$$= \|y' - Rs\|^2 < r^2 \tag{8}$$

Where Q and R are the  $N_t x N_t$  matrix from QR decomposition of channel matrix and y' is the Q<sup>H</sup>y which is  $N_t x$  1 vector. By using the problem of upper triangular matrix R, the rule of Maximum likelihood detection can be mapped to tree.

## VI. K-BEST SPHERE DECODER

K-Best algorithm is the breadth first algorithm. It is almost same as M- algorithm in sequential decoding. After examining all the nodes at the first level precisely, then K-Best metric nodes are selected as survived nodes and those children are considered and evaluated at next step. As the number of child nodes of survived nodes is K x  $2^{Nc}$ , K-Best retains K history at each level of search [7].

K-Best algorithm keeps only K nodes at each level. Let p be the level number. The procedure of the algorithm is:

- a) Let p=N;
- b) Calculate PEDs of all nodes at level p expanded by the former level and choose min(K, M<sub>p</sub>) smallest ones, where M<sub>p</sub> is the number of nodes at level p;
- c) Expand the chosen nodes, and p=p-1;
- d) If  $p \neq 1$ , go to step 2; else go to step 5;
- e) Choose the smallest PED as the optimal solution.

## VII. DIJKSTRA'S ALGORITHM

The problem of finding shortest path from a source vertex v to all other vertices in the graph is solved by the help of dijkstra's algorithm. It is the solution to the single-source shortest path problem in graph theory. All edges must have nonnegative weights.

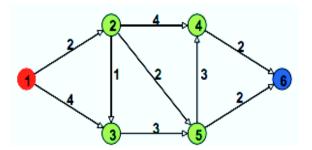


Fig.6: Example of Dijkstra's Algorithm

The shortest path length depends on its minimum weight. Here, path (1,2,5,6) is selected having a total weight of 6. So, this algorithm is the shortest path algorithm depending on the weight of the path [8], [9].

## VIII. SIMULATION RESULTS

The Simulation result for Bit error rate (BER) as a function of signal to noise ratio (SNR) is shown here for the BPSK modulation Scheme. The fig.7 shows a plot of BER and SNR of two receivers Rx1 and Rx2. Here, the value of BER at SNR value of 20 dB for Rx1 is  $10^{-2.8}$  dB and for the same value of SNR the BER of Rx2 is  $10^{-4.8}$  dB.So, from this graph it is clear that the performance of Rx2 is better than the Rx1 as for the same value of SNR the BER of Rx2 is much lower than the Rx1.

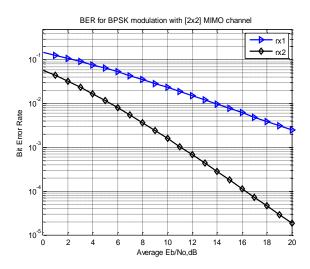


Fig.7: Graph of BER for BPSK modulation with [2x2] MIMO Channel

Fig.8 shows a plot of BER and SNR of three receivers Rx1 , Rx2 and Rx3. Here Rx1 has a BER of  $10^{-2.5}$  dB at SNR value of 18dB. For the same value of SNR the BER for Rx2 and RX3 are  $10^{-4.5}$ dB and  $10^{-4.9}$ dB respectively. So, the performance of Rx3 is much better than Rx1 and Rx2.

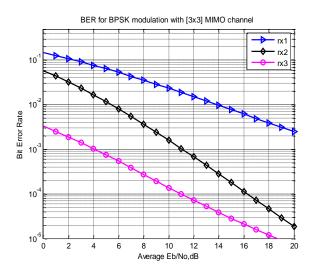


Fig.8: Graph of BER for BPSK modulation with [3x3] MIMO system

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