Mathematical modeling of heat transfer processes in energy-saving solar energy systems

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***Abstract*— Interest in renewable energy sources (RES) has been steadily increasing throughout the world each year. It is not so much due to possible depletion of fossil fuels (coal, oil, gas), but hopes for the environmentally sound and sustainable development of mankind in the future. Increasingly there is widespread use of solar radiation.**

***Index Terms*— solar energy, collector, temperature distribution, heat transfer**

I. Introduction

Solar energy system is the most promising and common energy source for heat [1,2]. According to [3] it is currently used in the world more than 180 million solar panels that provide heat to various customers. The most common of these technologies in China (59%), in second place - Europe (14%). 186 large firms produce solar collectors in 41 countries. Characterized by high values of the average daily radiation in the winter the amount of incoming solar energy is reduced by the latitudinal location of the installation several times [3,4]. The efficiency of solar collectors depends on the design features and climatic factors [5]. The most common solar air collectors are used for heating buildings and rooms for various purposes, as in agriculture. [2,3,6] Despite the large number of scientific publications on the use of solar energy, research local characteristics of reservoirs, taking into account the actual characteristics of their operation, such as a higher degree of turbulence of the coolant at the inlet, different wall temperature and so neglected..

II. Mathematical modelling

The role of mathematical modeling in research and development of this kind is very high. This is due to the need for deeper penetration into the essence of the objects and aims of reducing the cost and time of development. To create a simulation of the mathematical model the authors have considered a flat solar collector with active circulation of coolant (Figure 1). The main structural elements are the air duct without absorber, whose side facing the sun blackens and the other - Insulate. Collector frame is made of durable and lightweight metal, which reduce its weight and improve mobility. As assumptions considered further mathematical model will be assumed that the inner surface of the collector is a "black body" and does not reflect the solar radiation. The upper wall of the manifold is transparent, made of glass. The heating medium is passed through the collector due to the fan running on the photoelectric converter.

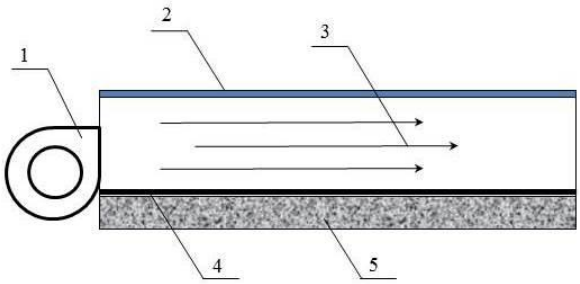

Fig. 1. Schematic diagram of the solar collector with active circulation of coolant: 1 - blowing fan; 2 -clear glass; 3 - the air flow within the reservoir; 4 - - black coating absorbing power metal coherency; 5 - insulating layer

One way to reduce convective heat loss through the transparent glass is the use of a cellular structure, located above the absorbing surface [7]. In this recirculation installation is not performed, the coolant (air) is forced through the manifold once, which simplifies the design of the apparatus.

For the formulation of the computational problem and heat transfer equation follows we assume that the air - it's viscous and incompressible medium. In order to construct a mathematical model primarily formulated boundary value problem and the boundary conditions. In addition discusses the main heat balance equation.

Complete the equation describing heat transfer in the reservoir with the free convection of the form:

$$\nabla \cdot \bar{v} = 0 \tag{1}$$

$$\frac{\partial \bar{v}}{\partial t} + (\bar{v} \cdot \nabla) \cdot \bar{v} = -\frac{1}{\rho} \nabla p + \beta \cdot \bar{g}(T - T_0) + \tau \Delta \bar{v} \tag{2}$$

$$\frac{\partial T}{\partial t} + (\bar{v} \cdot \nabla) \cdot T = \alpha \Delta T + \frac{1}{c\rho} \cdot q_v \tag{3}$$

$$\alpha = \frac{\lambda}{c\rho} \tag{4}$$

where (1) - the continuity equation ($\bar{v} \cdot \nabla = \operatorname{div} \bar{v} = 0$); (2) - equation of motion; (3) - the energy equation; $\bar{v}(u,v,w)$ - the air velocity vector in the channel; T - temperature in the duct; p - pressure caused by the movement of air; g - acceleration of

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gravity vector; β- expansion coefficient of air; ν- the kinematic viscosity; ρ- air density; q_v - heat flux density, c - specific heat of air (≈1.005 J/kg), λ - coefficient of thermal conductivity of air (≈0.241 W/m2), ($\bar{v}$•∇)- operator of the form:

$$(\bar{v}\cdot\nabla)=v\frac{\partial}{x\partial x}+v\frac{\partial}{y\partial y}+v\frac{\partial}{z\partial z};$$

Taking assumptions, consider a two-dimensional problem, and we believe the process to be stationary, hence, the time derivative is zero, the air velocity in the vertical v = 0. The initial portion of the channel is small and the velocity profile in the channel is known from the classical theory [8] (the initial distribution - parabola) (see Fig. 2).

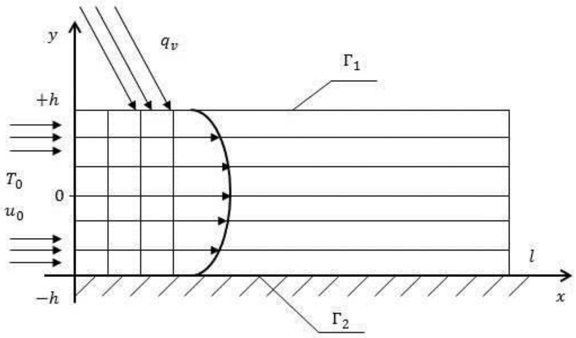

Fig. 2. The velocity profile in the air manifold

In this case, the initial system of equations is reduced to a boundary value problem:

$$u\cdot\frac{\partial T}{\partial x}=\alpha\frac{\partial^2 T}{\partial y^2}$$

$$T(0,y)=T_0$$

$$\lambda\frac{\partial T}{\partial y}=-\alpha(T-T_\infty)$$

When y = 0

$$-\lambda\cdot\frac{\partial T(x,0)}{\partial y}=q$$

When y = h, where

$$u=6u_0\left(\frac{y}{h}-\frac{y^2}{h^2}\right)$$

$q_1=q_2$

Where q is thermal flow directed from the top wall to the bottom.

This system is solved by one of the numerical methods. To simulate this process was chosen finite difference method [9, 10]. Differential equation as a result of transformations replaced by an equivalent value in the finite difference solution of which is to implement simple algebric operations. The final outcome of the decision given by the expression in which the value of the "future" of the potential (temperature) at a given point, (node) is determined by the "real" potential and the "real" potential adjacent nodal points. Repeatability of the same operations in the calculation of temperature fields a great convenience for the application of modern computer technology, so the efficiency of the many times increases. For calculated area take the inner surface of the collector for the approximation of the area and construct an orthogonal grid, instead of the area further consider the set of nodes formed by the intersection of the lines parallel to the coordinate axes. Fig. 3 shows an orthogonal grid and a plurality of nodes for calculating the temperature distribution in the reservoir.

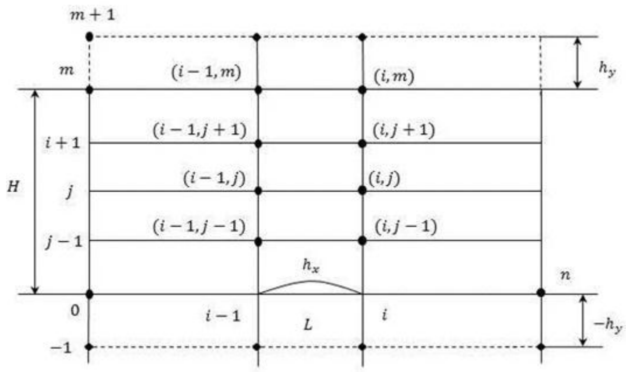

Fig. 3. Grid elements with dummy nodes

To set the boundary conditions G1 and G2 with the accuracy with which the original equation is approximated, we supplement the grid with dummy nodes, placing them at the top and bottom borders of the G1 and G2 at a distance h_y.

The desired function of the temperature distribution T (x, y) is approximated grid function that is a set of values of the function at the grid points. The magnitude of the size h_x and h_y respectively can be calculated from the following relations:

$$H_y=\frac{H}{m};\qquad h_x=\frac{L}{n};\qquad u_{i,j}=6u_0\left(\frac{y_j}{h_y}-\frac{y^2{}_j}{h_y}\right);$$

Approximation of derivatives on the four-point look at the template, the original equation can be rewritten in a Cartesian coordinate system. The original equation:

$$u\cdot\frac{\partial T}{\partial x}=\alpha\frac{\partial^2 T}{\partial y^2};$$

Approximated by the first and second derivatives with difference quotients can be carried out as follows:

$$u\frac{\partial T_i}{\partial x}\cong\frac{T_{i,j}-T_{i-1,j}}{h_x};$$

$$\frac{\partial^2 T_i}{\partial y^2}\approx\frac{1}{h_y}\left(\frac{T_{i+1}-T_i}{h_y}-\frac{T_i-T_{i-1}}{h_y}\right)=\frac{T_{i,j+1}-2T_{i,j}+T_{i-1}}{h_y{}^2}.$$

Replace the partial derivatives of $\partial T/\partial x$ and $\partial^2 T/\partial y^2$ at node 1 through the difference relations, as a result we obtain the equation in general form for i = 1. In the future, we form these equations for all nodes in which unknown values of the unknown function.

$$6u_0\left(\frac{y_j}{h_y}-\frac{y_j{}^2}{h_y{}^2}\right)\cdot\frac{T_{i,j}-T_{i-1,j}}{h_x}=a\frac{T_{i,j+1}-2T_{i,j}+T_{i,j-1}}{h_y{}^2};$$

i=1…n; j=1…m;

The boundary conditions can be represented as:

$$\lambda \frac{T_{i,1} - T_{i,-1}}{2h_y} = -\alpha (T_{i,0} - T_{окр});$$

- The node *(i,0)*

$$-\lambda \frac{T_{i,m} - T_{i,m-1}}{2h_y} = q_1;$$

- The node *(i,m).*

As a result, we obtain the following system of equations for i = 1:

$$-\lambda \frac{T_{i,m} - T_{i,m-1}}{2h_y} = q_1;$$

$$6u_0 (\frac{y_0}{h_y} - \frac{y_0^{\ 2}}{h_y^{\ 2}}) \cdot \frac{T_{1,0} - T_{0,0}}{h_x} = a \frac{T_{1,1} - 2T_{1,0} + T_{1,-1}}{h_y^{\ 2}};$$

$i = 0, j = 0;$

$$6u_0 (\frac{y_1}{h_y} - \frac{y_1^{\ 2}}{h_y^{\ 2}}) \cdot \frac{T_{1,1} - T_{0,1}}{h_x} = a \frac{T_{1,2} - 2T_{1,1} + T_{1,0}}{h_y^{\ 2}};$$

$i =1, j = 1;$

$$6u_0 (\frac{y_1}{h_y} - \frac{y_2^{\ 2}}{h_y^{\ 2}}) \cdot \frac{T_{1,2} - T_{0,2}}{h_x} = a \frac{T_{1,3} - 2T_{1,2} + T_{1,1}}{h_y^{\ 2}};$$

$i = 1, j = 2;$

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$$6u_0 (\frac{y_m}{h_y} - \frac{y_m^{\ 2}}{h_y^{\ 2}}) \cdot \frac{T_{1,m} - T_{0,m}}{h_x} = a \frac{T_{1,m-1} - 2T_{1,m} + T_{1,m-1}}{h_y^{\ 2}};$$

$i = 1, j =$ m;

$$\lambda \frac{T_{i,1} - T_{i,-1}}{2h_y} = -\alpha (T_{i,0} - T\infty);$$

The totality of the above equations written in the general form for the node (i, j) form a discrete mathematical model of the original problem.

A further step is to compile the simulation algorithm and writing software to solve the resulting lower system of linear algebraic equations On the basis of the above discrete mathematical model of the original problem to make the algorithm of formation of the coefficient matrix and right-hand side column.

For each i, starting with i=1, a system of linear algebraic equations consisting of m + 1 equation (j = 0, 1, 2 … m) of the form:

$$u_{i,j} \frac{T_{i,j} - T_{i-1,j}}{h_y} = a \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{h_y^{\ 2}};$$

- For the node (i, 0);

$$-\lambda \frac{T_{i,m} - T_{i,m-1}}{2h_y} = q_1;$$

- For the node (i, m).

The resulting system is solved by the Gauss method.To solve this system, a program that allows us to calculate the temperature distribution in the collector channel. To do this, the program stores the value of $T_{1,j}$ (j = 0,1,2 ..., m), then move to the next value of i, and so on until i = m. The result is an array of temperatures, which is the last column - is the temperature at the outlet of the reservoir. When i = 1, $T_{i-1,j}$= $T_{0,j}$ (j = 0,1,2 ... m), $T_{0,j}$ (j = 0,1,2 ... m), the values set in other i as $T_{i-1,j}$ (j = 0,1,2 ... m), value is the temperature obtained in the previous step.

In addition the program takes into account the influence of parameters such as the dimensions and layout of the reservoir capacity of the solar insolation, the slope of the collector, ambient air temperature, and air velocity in the channel.

Table 1. The results of calculation of temperatures for steps 0-3

Parameter grid x	parameter grid y			
	0	1	2	3
0	0.16	20.58053	21.67596	23.11328
1	20	20.56817	21.67956	23.10042
2	20	20.56582	21.67262	23.08755
3	20	20.56758	21.66568	23.08504
4	20	20.57717	21.66675	23.10209
5	20	20.59803	21.68298	23.14686
6	20	20.63356	21.72081	23.22661
7	20	20.6783	21.78618	23.34774
8	20	20.76316	21.88466	23.51596
9	20	20.86562	22.02168	23.73628
10	20	20.99992	22.20158	24.01301
11	20	21.17215	22.43265	24.34968
12	20	21.38929	22.71706	24.7489
13	20	21.65912	23.06067	25.21216
14	20	21.98997	23.46767	25.73958
15	20	22.39021	23.94119	26.3297
16	20	22.86737	24.48273	26.97926
17	20	23.42692	25.09163	27.68307
18	20	24.0705	25.76443	28.43392
19	20	24.79362	26.49455	29.22266
20	20	25.58268	27.27204	…

As seen from Table 1, the temperature is increased throughout the reservoir. With the help of the developed program can perform calculations for different initial conditions and obtain the values of the temperature at different points of the manifold. Varying the number of steps can increase the accuracy of calculations. For the most realistic data about the temperature in the collector must be set as the initial value of the measured quantity of solar radiation on the lower surface of the collector, after passing the radiation through a transparent surface of the upper wall.

The results of calculation of the temperature distribution along the length of the reservoir can also be presented graphically (see Fig. 4). Red line indicates the temperature in the first step of the program (on the inlet manifold), blue dotted line - in the middle collector orange dashed line - to the final step of the program, i.e. output from the manifold.

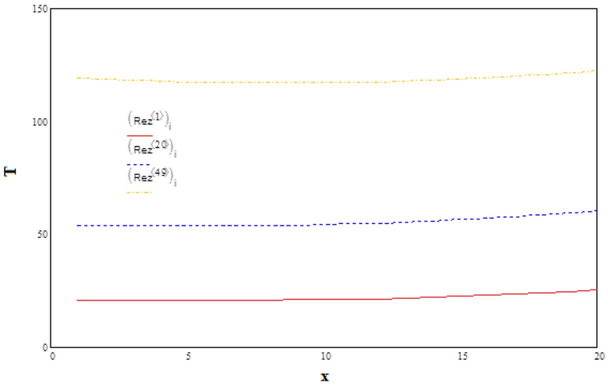

Fig. 4. The temperature distribution along the length of the reservoir under normal conditions

III. Conclusion

As a result of a simulation of a mathematical model obtained data on the temperature distribution within the reservoir, as well as to determine the temperature characteristics of the coolant at the outlet of the reservoir. In addition, the model allows us to obtain data on the influence of external conditions, such as outdoor temperature, the power of solar insolation, as well as the angle of the collector to the horizon and the percentage of heat loss through the walls of the reservoir.

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