Improvement of Voltage Stability by Optimal Capacitor placement using Sensitivity Matrix

Gogu Rajendar, Basavaraja Banakara

***Abstract*— This paper presents a method to improve the Voltage Stability of the radial distribution system by Optimal Capacitor placement using L-Index Sensitivity Matrix. The proposed methodology calculates a sensitivity matrix, which gives the capacitor location; it is effective than the conventional optimization or meta-heuristic optimization usage. The objective of the capacitor placement is to enhance the voltage stability. The proposed sensitivity matrix approach requires calculation of the matrix once and using it for multiple load conditions, against the iterative solution approach of the optimization methods. The characteristics of the radial distribution system is considered and have been used for modifications in the present approach.**

***Index Terms*— Voltage stability, Sensitivity matrix, Capacitor Placement.**

I. Introduction

According to voltage and power levels, an electric power system as classified into a generating system, a transmission system and a distribution system. In many utilities, even though new generators have been added to augment the existing capacity for meeting the increase in demand, but the transmission capacity has not changed much due to constraints like rights of way and cost of tower etc. Hence, transmission systems are forced to carry increasingly more and more power over long distances using the FACTS devices. Voltage stability analysis is important to assess the criticality of the existing networks and voltage Instability is a very challenging problem being faced by power systems all over the world.

The transmission system which is quite different, in both its characteristics and operation, from the distribution system. We cannot use same analysis approach that used in transmission system to distributions system. The analysis of a distribution system is an important area of activity, as distribution systems provide the vital link between the bulk power system and the consumers. A distribution circuit normally has a primary or main feeders which further rooted into lateral distributors and sub lateral distributors. Radial systems are popular because of their simple design and generally low cost.

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Gogu Rajendar, Research Scholar,JNTUH, Electrical & Electronics Engineering Department.
Basavaraja Banakara, Professor & Chairman (HOD), Electrical & Electronics Engineering Department, University of BDT Engineering college Davanagere.

It has been observed that the distribution system experiences voltage collapse under certain critical loading conditions, Brownell and Clarke [1] have reported the actual system phenomenon in which system voltage collapses periodically and reactive compensation is to be supplied to avoid repeated voltage collapse. In distribution networks, voltage instability is a local phenomenon and occurs near the buses which have high variation in loading and low-voltage profiles. So voltage stability analysis is very much important in distribution system as in transmission system for reliable and secured operation of the system.

In Literature most of the work has been done on transmission system, voltage stability analysis [2-10], but very less work has been done on the voltage stability analysis in the radial distribution system. G.B.Jasmon and L.H.C.C Lee, [11-13] discussed about voltage stability analysis using various load flow analysis and network reduction technique and maximizing voltage stability by loss reduction. In [14] M.Moghavvemi, M.O.Faruque proposed a technique to study the voltage collapse situation by reducing the network to a single line equivalent circuit and then represented through a π-network. A stability index is developed to indicate the severity of the loading situation of the system through which voltage collapse is determined. An artificial Neural Network based technique [15] is used to predict the voltage stability and the learning is done is based on a new voltage stability index for assessment of radial distribution systems taking loading values P and Q as the inputs. Catastrophic theory [16] is proposed to identify how the equilibrium voltage point of the system is changed as the system parameters change during sudden variations in system operation. This paper explores the applicability of the catastrophe theory to identify a new VSI (voltage stability index) and finding the critical voltage and maximum loading for each bus in the system in terms of one of the elementary catastrophe. In this paper different indices which are used in voltage stability assessment of the radial distribution system is discussed.

II. Voltage Stability Indices

Voltage Instability is a very challenging problem being faced by power systems all over the world. In many utilities, even though new generators have been added to augment the existing capacity for meeting the increasing load, the transmission capacity has not increased correspondingly due to problems like rights of way, environmental concerns etc. Hence, transmission systems are forced to carry increasingly more and more power over long distances.

An Index has to be calculated for assessing the voltage stability and by ranking the buses, the most prone bus for voltage instability is chosen. This bus becomes the location for the Capacitor to be placed in the system and the sensitivity indices for the buses remain same as long as the system configuration remains the same.

A. Line Flow Indices:

The four line flow indices are calculated as follows. Line flow index for the sending end real power:

$$LFISP : 4\frac{r_i}{V_i^2}\left(P_r + \frac{r_i}{V_i^2}Q_i^2\right) \qquad (1)$$

Line flow index for receiving end real power:

$$LFIRP : 4\frac{r_i}{V_{i+1}^2}\left(-P_i + \frac{r_i}{V_{i+1}^2}Q_r^2\right) \qquad (2)$$

Line flow index for sending end reactive power:

$$LFISQ : 4\frac{X_i}{V_i^2}\left(Q_r + \frac{X_i}{V_i^2}P_i^2\right) \qquad (3)$$

Line flow index for receiving end reactive power:

$$LFIRQ : 4\frac{X_i}{V_{i+1}^2}\left(-Q_i + \frac{X_i}{V_{i+2}^2}P_r^2\right) \qquad (4)$$

where
V_i :- voltages at the sending end node i
V_{i+1}:-voltages at the receiving end node $i+1$
ri:-resistance of the line joining the nodes i *and* $i+1$
Xi:-reactance of the line joining the nodes
Pi :- real power flowing from node i
Qi :- reactive power flowing from node i
P_r :- real power entering from node $i+1$
Q_r :- real power entering from node $i+1$
These line flow indices are calculated for all the lines in the system and the lines with a high value (close to 1) are considered as critical lines. The receiving end bus of the critical line/lines is identified as the weakest bus from the voltage stability perspective.

Issues with Line flow indices:

Consider the expressions for LFISP and LFIRP. If i, = 0 then *LFISP* = 0 and *LFIRP* = 0 indicating that the lines are stable in the case of real power flow which could be misleading. This is because, the power transferred over a line is given by

$$P_r = \frac{V_i V_{i+1}}{Xi}\sin\delta \qquad (5)$$

where δ:- difference in the angles of the sending and receiving end voltages.

Say the resistance of a line feeding a PQ bus 'B' of a system is zero. Let the maximum P_{load} at the bus B at a fixed value of Q_{load} be P_{max}. If the indices LFISP and LFIRP are calculated for the line 'L' under these conditions, the values turn out to be zeros, since the resistance of the line is zero. Hence the line L will be classified as stable as per the real power loading based on this index whereas the system is in the verge of collapse.

TABLE I. LINE FLOW INDICES OF 15- BUS SYSTEM

From	To	LFISP	LFISQ	LFIRP	LFIRQ
1	2	0.1075	-0.1111	0.1048	-0.1082
2	3	0.0495	-0.0505	0.0483	-0.0493
3	4	0.0185	-0.0186	0.0180	-0.0182
4	5	0.0025	-0.0025	0.0017	-0.0017
2	8	0.0081	-0.0081	0.0054	-0.0055
8	9	0.0026	-0.0026	0.0018	-0.0018
2	6	0.0445	-0.0454	0.0299	-0.0305
6	15	0.0055	-0.0055	0.0037	-0.0037
6	7	0.0032	-0.0032	0.0021	-0.0021
3	10	0.0166	-0.0168	0.0112	-0.0113
10	11	0.0103	-0.0103	0.0069	-0.0070
11	12	0.0033	-0.0033	0.0022	-0.0022
4	13	0.0057	-0.0057	0.0039	-0.0039
4	14	0.0061	-0.0062	0.0041	-0.0042

B. L –index:

In this method the Y-bus matrix of the system is split into rows and columns of generators and load buses.

$$\begin{bmatrix} I_G \\ I_L \end{bmatrix} = \begin{pmatrix} Y_{GG} & Y_{GL} \\ Y_{LG} & Y_{LL} \end{pmatrix}\begin{bmatrix} V_G \\ V_L \end{bmatrix} \qquad (6)$$

$$\begin{bmatrix} V_L \\ I_G \end{bmatrix} = \begin{pmatrix} Z_{LL} & F_{LG} \\ K_{GL} & Y_{GG} \end{pmatrix}\begin{bmatrix} I_L \\ V_G \end{bmatrix} \qquad (7)$$

FLG = [YLL]-1 [YLG] (8)

$$L_j = \left|1 - \sum_{i=1}^{i=g} F_{ji}\frac{V_i}{V_j}\right| \qquad (9)$$

where the subscript,
"G":- refers to the generator buses in the system.
"L":- refers to the load buses in the system.

An L-index value away from 1 and close to 0 indicates a large voltage stability margin. The maximum of L-indices (L_{max}) of the buses to which it corresponds is the most critical bus. Also the summation of the squares of the L indices of individual buses (∑L2) is used as a relative indicator of the overall voltage stability of the system at different operating conditions.

TABLE II. L- INDICES OF 15- BUS SYSTEM

Bus no	L-Index
2	0.0292
3	0.0448
4	0.0511
5	0.0521
6	0.0431
7	0.0445
8	0.0327
9	0.0339
10	0.0521
11	0.0567
12	0.0581
13	0.0536
14	0.0538
15	0.0455

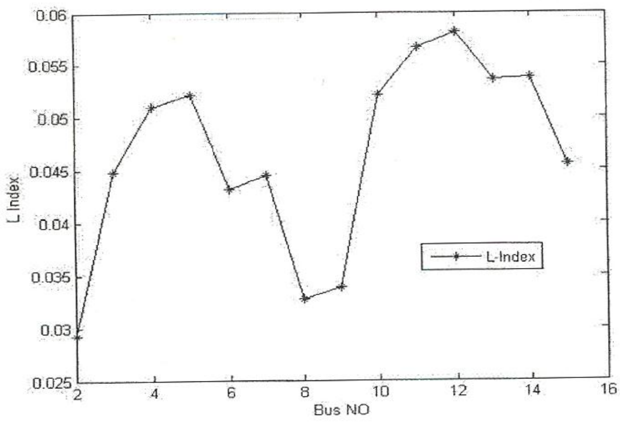

Fig. 1. L- index of 15- bus system.

C. Participation factor:

It is developed as follows. Consider the load flow Jacobian.

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}=\begin{pmatrix}H & N\\ M & L\end{pmatrix}\begin{bmatrix}\Delta\theta\\ \Delta V\end{bmatrix} \qquad (10)$$

Although both P and Q changes affect system conditions, it is possible to study the effect of reactive power injections on the voltage stability by setting ΔP (P constant) and deriving the Q – V sensitivities at different loads. Thus

$$\Delta Q=[L-MH^{-1}N]\Delta V=J_R\Delta V \qquad (11)$$

$$J_R=\xi\Lambda\eta \qquad (12)$$

where

Λ : Left Eigen matrix of J_R

η : Right Eigen matrix of J_R

ξ - Eigenvalues J_R

The participation factors for the bus '*k*' and the critical mode '*i*' are defined as $\xi_{ki}\eta_{ik}$

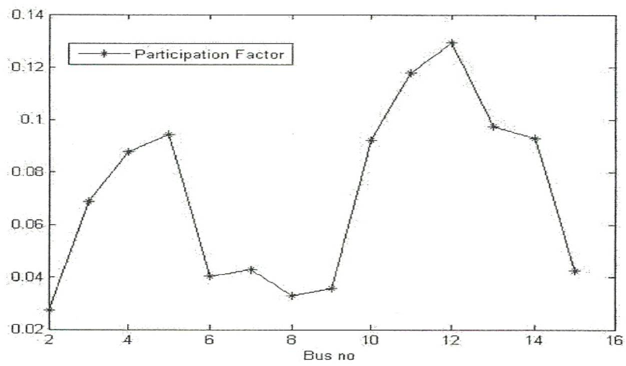

Fig. 2. Participation factor of 15- bus system

III. SENSITIVITY MATRIX

L- Index as introduced in section. 2. would predict voltage stability.

To derive L-Index sensitivities Let {1,2,3…….g} be no of generator nodes{g+1, g+2, g+3……n} be no of load nodes The L-index is,

$$L_j=\left|1-\sum_{i=1}^{i=g}F_{ji}\frac{V_i}{V_j}\right| \qquad (13)$$

where the subscript

"G" :- refers to the generator buses in the system

"L" :- refers to the load buses in the system

Squaring the equation then,

$$L_j^2=\left(\left|1-\sum_{i=1}^{i=g}F_{ji}\frac{V_i}{V_j}\right|\right)^2=\left(1-\sum_{i=1}^{i=g}F_{ji}\frac{V_i}{V_j}\right)^2 \qquad (14)$$

Put,

$$K_j=L_j^2 \qquad (15)$$

$$K_j=\left(1-\sum_{i=1}^{i=g}F_{ji}\frac{V_i}{V_j}\right)^2 \qquad (16)$$

Differentiating the above equation, the new equation is:

$$\frac{\Delta K_j}{\Delta V_j}=-2\left(1-\sum_{i=1}^{i=g}F_{ji}\frac{V_i}{V_j}\right)\left(\sum_{i=1}^{i=g}F_{ji}\frac{V_i}{V_j^2}\right) \qquad (17)$$

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}=\begin{pmatrix}H & N\\ M & L\end{pmatrix}\begin{bmatrix}\Delta\theta\\ \Delta V\end{bmatrix} \qquad (18)$$

As the inclusion of an extra capacitor would change only the reactive power, we would make ΔP in to be zero. From this,

$$\Delta Q=\left[L-MH^{-1}N\right]\Delta V=J_R\Delta V\quad \Delta V=\left[J_R\right]^{-1}\Delta Q \qquad (19)$$

where (JR) is the reduced Jacobian.

where ΔQ is a vector of change in reactive power injections

$$\Delta Q=\left[\Delta Q_{g+1}\Delta Q_{g+2}\Delta Q_{g+3}\text{………}\right] \qquad (20)$$

where ΔV is a vector of change in voltages

$$\Delta V = [\Delta V_{g+1} \Delta V_{g+2} \Delta V_{g+3} \ldots\ldots\ldots] \quad (21)$$

$$J_R^{-1} = \begin{pmatrix} a_{g+1,g+1} & a_{g+1,g+2} & \cdots & a_{g+1,n} \\ a_{g+2,g+1} & a_{g+2,g+2} & \cdots & a_{g+2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,g+1} & a_{n,g+2} & \cdots & a_{n,n} \end{pmatrix} \quad (22)$$

$$\Delta V_j = \sum_{i=g+1}^{i=n} a_{ji} \Delta Q_i \quad (23)$$

$$\Delta V_j = \frac{\Delta Kj}{2\left(1 - \sum_{i=1}^{i=g} F_{ji} \frac{V_i}{V_j}\right)\left(\sum_{i=1}^{i=g} F_{ji} \frac{V_i}{V_j^2}\right)} \quad (24)$$

$$\Delta K_j = -2\left(1 - \sum_{i=1}^{i=g} F_{ji} \frac{V_i}{V_j}\right)\left(1 - \sum_{i=1}^{i=g} F_{ji} \frac{V_i}{V_j^2}\right) * \sum_{i=1}^{i=g} a_{ji} \Delta Q_i \quad (25)$$

As $\Delta K_j = 2L_j \Delta L_j$

$$\Delta L_j = -\frac{1}{L_j}\left|\left(1 - \sum_{i=1}^{i=g} F_{ji} \frac{V_i}{V_j}\right)\left(1 - \sum_{i=1}^{i=g} F_{ji} \frac{V_i}{V_j^2}\right)\sum_{i=1}^{i=g} a_{ji}\right| * \Delta Q_i \quad (26)$$

$$L_q = -\left\|\left\{\frac{1}{L_j}\left(1 - \sum_{i=1}^{i=g} F_{ji} \frac{V_i}{V_j}\right)\left(\sum_{i=1}^{i=g} F_{ji} \frac{V_i}{V_j^2}\right)\sum_{i=1}^{i=g} a_{ji}\right\}\right\| \quad (27)$$

This is called L- Index sensitivity matrix.

TABLE III. COMPARISON OF ΔL OBTAINED BOTH IN CONVENTIONAL AND L-INDEX SENSITIVITY

Bus no	ΔL from L-Index sensitivities	ΔL from conventional method
2	0.001561375	0.00153235
3	0.002982529	0.002926712
4	0.003036517	0.002977799
5	0.003045882	0.002986667
6	0.001625685	0.00159306
7	0.001632134	0.001599146
8	0.001577317	0.001547467
9	0.001582524	0.001552402
10	0.004477482	0.004394593
11	0.006484893	0.006368077
12	0.008109174	0.007966572
13	0.003058382	0.002998492
14	0.003059996	0.003000017
15	0.001636937	0.001603675

IV. ALGORITHM FOR CAPACITOR PLACEMENT

- Calculate L- Index of all the load buses as given in section 2.
- Calculate the sensitivity matrix as given in section 3.
- Choose a limit for the L- Index, let it be L-limit
- Find the buses which is having L- Index more than L-limit, let the buses be [B2 B5 B7].
- Find ΔLr_{ed}, ΔLr_{ed} is taken for the buses which are exceeding L-limit.
- ΔLr_{ed} = [ΔL2 ΔL5 ΔL7].
- Find the reduced sensitivity matrix (L_{rq}), reduced sensitivity matrix would relate ΔL_{rq} and ΔQr_{ed} where ΔQ_{red} = [Q2 Q5 Q7].
- $\Delta Lr_{ed} = \Delta Lr_d \Delta Qr_{ed}$.
- Now perform optimization minimizing objective function satisfying the constraints

  $$\Delta L_{\min} \le \Delta L \le L_{\max}$$
- where
- $$\Delta L_{\min} = L_{actual} - L_{\lim it}$$
- $$\Delta L_{\max} = [1] - L_{actual}$$
- The optimal location of capacitor along with the Q value is found out.

V. CASE STUDY AND RESULTS

The method is carried out for finding the capacitor location, in 15 bus system with four times the normal load and the results are presented below.

Buses exceeding L_{limi} are [B_4 B_5 B_{10} B_{11} B_{12} B_{13} B_{14}].

ΔL_{red} = [$\Delta L4$ $\Delta L5$ $\Delta L10$ $\Delta L11$ $\Delta L12$ $\Delta L13$ $\Delta L14$].

TABLE IV. REDUCED SENSITIVITY MATRIX OF 15 BUS EXPERIMENT SYSTEM

	4	5	10	11	12	13	14
4	0.18 241	0.186 57	0.160 42	0.178 13	0.184 33	0.192 5	0.193 3
5	0.18 861	0.224 97	0.165 87	0.184 18	0.190 59	0.199 03	0.199 86
10	0.16 299	0.166 71	0.209 07	0.233 26	0.241 74	0.172	0.172 72
11	0.19 035	0.194 69	0.244 16	0.335 98	0.348 64	0.200 87	0.201 71
12	0.20 023	0.204 79	0.256 83	0.353 42	0.419 22	0.211 3	0.212 18
13	0.19 758	0.202 08	0.173 77	0.192 95	0.199 66	0.258 67	0.209 37
14	0.19 88	0.203 33	0.174 84	0.194 14	0.200 9	0.209 8	0.237 82

ΔL_{max} = [0.53262 0.54971 0.55208 0.63254 0.66014 0.57359 0.57676]

ΔL_{min} = [0.032616 0.049709 0.052083 0.13254 0.16014 0.073588 0.076765]

Imposing the condition ΔLmin ≤ ΔL ≤ ΔLmax and running optimization program we get values of capacitors as given below:

TABLE V. CAPACITOR PLACEMENT FOR VOLTAGE STABILITY IMPROVEMENT

Bus no	Value of Capacitor
4	2.3036e-011
5	1.8078e-011
10	1.6882e-011
11	1.5086e-010
12	0.38191
13	1.2864e-011
14	0.00016339

The values of capacitor to be placed at bus 12 is 0.38191MVAR and at bus 14 is 0.00016339MVAR. This would reduce the values of L- Index to maximum of 0.5. This would make the system stable.

VI. CONCLUSION

This paper emphasis on the analyzing and improvement of voltage stability of radial power systems. The various indices which have been proposed for assessing voltage stability are studied. Patterns of voltage stability indices are been observed and their dependence on the electrical distance has been studied. An algorithm is proposed for optimal placement of the capacitors to improve the voltage stability to the required level based on the new sensitive index proposed. The effect of reactive power injections at a bus on the entire power system in consideration is studied. The algorithms which have been proposed for radial power systems can also be suitably modified and can be applied to meshed systems.

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