

# Development of a DFT-Precoding Scheme for Spatially Multiplexed 4G Wireless Communication

Ale Daniel.T., Ogunti E.O. and Oyetunji S.A.

**Abstract**— The two major sources of interference experienced in spatially-multiplexed broadband multiple input multiple output (MIMO) communication system are multi-stream interference (MSI) and inter-symbol interference (ISI) as a result of the time-dispersive characteristics of the wireless channel. Both sources of interference have to be minimized appropriately at the receiver to ensure reliable communication. Orthogonal frequency division multiplexing (OFDM) is usually employed to combat ISI; hence the focus of this paper is to design an efficient precoding scheme to combat MSI while keeping ISI at the best possible minimum. The receiver architecture employs the minimum mean square error (MMSE) equalization technique while making use of minimal channel coding. QPSK, 16-QAM and 64-QAM modulation schemes were used to evaluate the performance of the system in line with the 4G wireless communication standard. The bit error rate performance of the developed system was found to be robust in the presence of a time-dispersive fading.

**Index Terms**— 4G, DFT-Precoding, MIMO, MMSE, OFDM

## I. INTRODUCTION

Wireless systems are pervading our everyday life more and more. The main advantage of wireless networking is its ubiquitous nature (anyone, anywhere, anytime), thus freeing users from the need of fixed settlements for accessing communication systems. Using wireless technologies is a natural choice in environments where cable is not feasible or economically convenient (e.g., rural areas), or where users are moving entities (e.g., nomads, vehicular). For this and other reasons, wireless technologies continuously evolve, affecting many facets of our society, offering services in a variety of contexts, like communication, education, entertainment, social networking, medicine, healthcare, location, commerce, security and defense, etc. Actually, this is much a feedback loop: the more the wireless technology grows, offering increasingly complex services, the more the new generation of applications and services asks for technology improvement, in order to meet their greater requests of resources.

In recent years several new techniques have been developed to increase the data transmission rate in wireless data communication. To achieve higher data rates, efficient

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use of the available radio spectrum is essential. The Vertical Bell Laboratories Layered Space-Time (V-BLAST) detection algorithm technique for Multiple Input Multiple Output (MIMO) wireless communication systems [1], increases spectral efficiency to near the theoretical Shannon bound. However its complex receiver makes it unsuitable for low-power VLSI implementation. Several alternative algorithms and architectures for V-BLAST detection has been proposed to reduce its complexity [2].

The spectral-efficiency benefits associated with MIMO processing hinge on the availability of a rich scattering environment. A MIMO channel with a high degree of scattering enables independent multipath links to be made from each transmit antenna to each receive antenna. As a result, the matrix of channel gains connecting each pair of transmit and receive antennas pairs will have a full rank and the resulting MIMO equation will be solvable. In a typical MIMO transmission, however, the assumption regarding a high level of scattering cannot be guaranteed [4]. As a result, in order to design a practical system, steps were taken to reduce the probability of channel matrices with reduced ranks occurring.

Multiple Input Multiple Output (MIMO) systems have attracted much attention because of high spectrum efficiency. Many different detection techniques are developed to get the diversity gain introduced by MIMO techniques [5]. The ML detector is able to provide optimal performance, but has a disadvantage of extremely high computational complexity [6].

Linear detectors for V-BLAST systems are ZF and Minimum Mean Square Error (MMSE) detectors, which are low in complexity and poor in performance, both methods uses the equalization matrices approach for detection. Ordered Successive Interference Cancellation (OSIC) detector, which detects the transmit symbols one by one according to the post-detection Signal-to-Noise Ratio (SNR) can achieve better performance at relatively high complexity [5].

Precoding is a generalization of beamforming to support multi-stream (or multi-layer) transmission in multi-antenna wireless communications [7]. In conventional single-stream beamforming, the same signal is emitted from each of the transmit antennas with appropriate weighting (phase and gain) such that the signal power is maximized at the receiver output. When the receiver has multiple antennas, single-stream beamforming cannot simultaneously maximize the signal level at all of the receive antennas. [6]

## II. SYSTEM MODEL

The developed spatially multiplexed MIMO-OFDM system developed is as shown in figure 1.

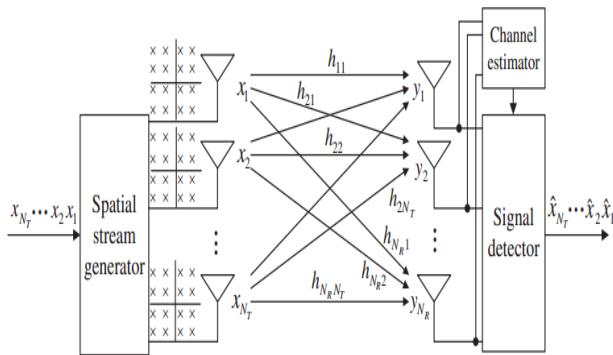


Figure 1. Spatially-Multiplexed (SM) MIMO systems

$$y = Hx + n \dots \dots \dots (1)$$

Where  $y$  is the received signal at the receive antenna,  $x$  is the transmitted signal,  $n$  is the additive white Gaussian noise (AWGN) and  $H$  is the channel characteristics matrix given in equation 2

$$H = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1m} \\ h_{21} & h_{22} & \dots & h_{2m} \\ \vdots & \ddots & \ddots & \vdots \\ h_{Nt1} & h_{Nt2} & \dots & h_{Nm} \end{bmatrix} \dots \dots (2)$$

Where  $h_{ij}$  is the channel gain between the receive antenna  $j$  and transmit antenna  $i$ .

Minimum Mean Square Error (MMSE) linear detection scheme was employed in the design of the receiver, this scheme was chosen because it does not lead to noise amplification as does Zero Forcing (ZF) detection scheme, and it is computationally efficient and easier to implement. In this research, a MIMO system with  $N_R \times N_T$  antennas was developed in which  $N_R = N_T$ . Let  $H$  denotes the channel matrix with its  $(j, i)^{\text{th}}$  entry and  $h_{ij}$  denotes the channel gain between the  $i^{\text{th}}$  transmit antenna and the  $j^{\text{th}}$  receive antenna,  $j = 1, 2, \dots, N_R$  and  $i = 1, 2, \dots, N_T$ .

The spatially multiplexed user data and the corresponding received signals are represented by

$x = [x_1, x_2, \dots, x_{N_T}]^T$  and  $y = [y_1, y_2, \dots, y_{N_R}]^T$  respectively, where  $x_i$  and  $y_j$  denote the transmit signal from the  $i^{\text{th}}$  transmit antenna and the received signal at the  $j^{\text{th}}$  receive antenna respectively. Also, the white Gaussian noise at the  $j^{\text{th}}$  receive antenna is represented by  $z_j$  and it has a variance  $\sigma_z^2$  while  $h_i$  denotes the  $i^{\text{th}}$  column vector of the channel matrix  $H$ .

Consequently, the  $N_R \times N_T$  MIMO system can be represented as

$$y = Hx + z \dots \dots \dots (3)$$

$$y = h_1 x_1 + h_2 x_2 + \dots + h_{N_T} x_{N_T} + z \dots \dots \dots (4)$$

Where  $z = [z_1, z_2, \dots, z_{N_R}]^T$

Linear signal detection (MMSE inclusive) method treats all transmitted signals as interferences except for the desired stream from the target transmit antenna. Therefore, interference signals from other transmit antennas are minimized or nullified in the course of detecting the desired signal from the target transmit antenna. To facilitate the detection of desired signals from each antenna, the effect of the channel is inverted by a weight matrix  $W$  such that the detection of each symbol is given by a linear combination of the received signals according to equation 3.9.

$$\tilde{x} = [\tilde{x}_1 \tilde{x}_2 \dots \tilde{x}_{N_T}]^T = Wy \dots \dots \dots (5)$$

In order to maximize the post-detection signal-to-interference plus noise ratio (SINR), the MMSE weight matrix given by equation 6 was employed.

$$W_{\text{mmse}} = (H^H H + \sigma_z^2 I)^{-1} H^H \dots \dots \dots (6)$$

Where  $H$  is the channel matrix,  $\sigma_z^2$  is the noise power,  $I$  is an identity matrix and  $(.)^H$  is the Hermittian transpose operation.

Substituting  $W_{\text{mmse}}$  into the expression for  $\tilde{x}$ , we obtain

$$\tilde{x}_{\text{mmse}} = W_{\text{mmse}} y \dots \dots \dots (7)$$

$$\tilde{x}_{\text{mmse}} = (H^H H + \sigma_z^2 I)^{-1} H^H y \dots \dots \dots (8)$$

$$\tilde{x}_{\text{mmse}} = \tilde{x} + (H^H H + \sigma_z^2 I)^{-1} H^H z \dots \dots \dots (9)$$

$$\tilde{x}_{\text{mmse}} = \tilde{x} + \tilde{z}_{\text{mmse}} \dots \dots \dots (10)$$

$$\text{Where } \tilde{z}_{\text{mmse}} = (H^H H + \sigma_z^2 I)^{-1} H^H z.$$

The practical challenge with the algebra of channel matrix is often heightened when the channel matrix is not full rank, the analysis of such system can be easily handled by employing the singular value decomposition which is more robust to numerical error. Employing the Singular Value Decomposition (SVD) technique, then  $H = V^H \Sigma U$ .

The post detection noise power is the norm of  $\tilde{z}_{\text{mmse}}$ , therefore,

$$\|\tilde{z}_{\text{mmse}}\|_2^2 = \|((V^H \Sigma U)^H (V^H \Sigma U) + \sigma_z^2 I)^{-1} (V^H \Sigma U)^H z\|^2 \dots \dots \dots (11)$$

Expanding this expression and making a note that

$$UU^H = 1, \text{ We have}$$

$$\|\tilde{z}_{\text{mmse}}\|_2^2 = \|(V\Sigma^2 V^H + \sigma_z^2 I)^{-1} V\Sigma U^H z\|^2 \dots \dots \dots (12)$$

Observe also that

$$(V\Sigma^2 V^H + \sigma_z^2 I)^{-1} V\Sigma = (V\Sigma^2 V^H + \sigma_z^2 I)^{-1} (\Sigma^{-1} V^H)^{-1} = (\Sigma V^H + \sigma_z^2 \Sigma^{-1} V^H)^{-1} \dots \dots \dots (13)$$

Therefore,

$$\|\tilde{z}_{\text{mmse}}\|_2^2 = \|V(\Sigma + \sigma_z^2 \Sigma^{-1})^{-1} U^H z\|^2 \dots \dots \dots (14)$$

Again by the fact that multiplication with a unitary matrix does not change the vector norm, that is  $\|Vx\|^2 = \|x\|^2$ .

The expected value of the noise is:

$$E\{\|\tilde{z}_{\text{mmse}}\|_2^2\} = E\{\|(\Sigma + \sigma_z^2 \Sigma^{-1})^{-1} U^H z\|^2\} \dots \dots \dots (15)$$

$$E\{\|\tilde{z}_{\text{mmse}}\|_2^2\} = E\{tr((\Sigma + \sigma_z^2 \Sigma^{-1})^{-1} U^H z z^H U (\Sigma + \sigma_z^2 \Sigma^{-1})^{-1})\} \dots \dots \dots (3.20)$$

$$E\{\|\tilde{z}_{\text{mmse}}\|_2^2\} = \text{tr}((\Sigma + \sigma_z^2 \Sigma^{-1})^{-2} E\{zz^H\}) \dots \dots \dots (16)$$

$$E\{\|\tilde{z}_{\text{mmse}}\|_2^2\} = \text{tr}(\sigma_z^2 (\Sigma + \sigma_z^2 \Sigma^{-1})^{-2}) \dots \dots \dots (17)$$

$$E\{\|\tilde{z}_{\text{mmse}}\|_2^2\} = \sum_{i=1}^{N_T} \sigma_z^2 \left( \sigma_i + \frac{\sigma_z^2}{\sigma_i} \right)^{-2} \dots \dots \dots (18)$$

$$E\{\|\tilde{z}_{\text{mmse}}\|_2^2\} = \sum_{i=1}^{N_T} \frac{\sigma_z^2 \sigma_i^2}{(\sigma_z^2 + \sigma_i^2)^2} \dots \dots \dots (19)$$

Noise enhancement effect in the course of linear filtering is significant when the condition number of the channel matrix is large, that is, the minimum singular value is very small. It is clear that the effect of noise enhancement in MMSE filtering is less critical than that in ZF filtering.

### III. PRE-CODING AND SPATIAL MULTIPLEXING

The pre-coder matrix  $W$  is employed and the equation for the received signal is modified as shown in equation 3.29

$$Y = WHX + Z \dots \dots \dots (20)$$

Where  $H$  is the channel matrix,  $X$  is the transmitted signal vector and  $Z$  is additive noise.

The two antenna port pre-coding consists of a combination of a  $2 \times 2$  identity matrix and a Discrete Fourier Transform (DFT) based pre-coding. The pre-coder was developed for  $N = 2$  and  $N = 4$  antenna configuration. In order to solve the rank deficiency problem of the MIMO channel matrix, a shift parameter,  $G$ , was introduced into the system to ensure that a phase shift is introduced into the MIMO links, hence making the rows of the MIMO channel matrix independent. The expression for  $G$  is given by equation 21 where  $g$  takes integer values from 0 to  $N - 1$ , and the resulting pre-coder matrix,  $W_g$ , which is a weighting factor is given by equation 22 .

$$G = \frac{g}{N} \dots\dots\dots (21)$$

$$W_g = \frac{1}{\sqrt{N}} e^{j \frac{2\pi m}{N} (n+G)} \quad m,n = 0,1,\dots,(N-1) \dots \dots (22)$$

The developed MIMO pre-coder,  $W_g$ , was then used to weight the modulated symbols.

$$W = \frac{1}{\sqrt{N}} e^{j2\pi mn/N} \quad m, n = 0, 1, \dots, (N-1) \quad \dots \dots \dots (23)$$

The set of four  $2 \times 2$  DFT matrices with  $g = 0, 1, 2$  and  $3$  are

$$W_0 = \frac{1}{\sqrt{2}} \begin{bmatrix} e^0 & e^0 \\ e^0 & e^{j\pi} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \dots \dots \dots (24)$$

$$W_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} e^0 & -e^0 \\ e^{j\pi/4} & -e^{j\pi/4} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1+j & -1-j \\ 1-j & -1+j \end{bmatrix} \dots \dots (25)$$

$$W_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{j\pi/2} & -e^{j\pi/2} \\ e^{j\pi/2} & -e^{j\pi/2} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix} \quad \dots \dots (26)$$

$$W_3 = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{j0} & -e^{j0} \\ e^{j3\pi/4} & -e^{j3\pi/4} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ \frac{-1+j}{\sqrt{2}} & \frac{1-j}{\sqrt{2}} \end{bmatrix} \dots (27)$$

The system was developed relative to the model shown in figure 2.

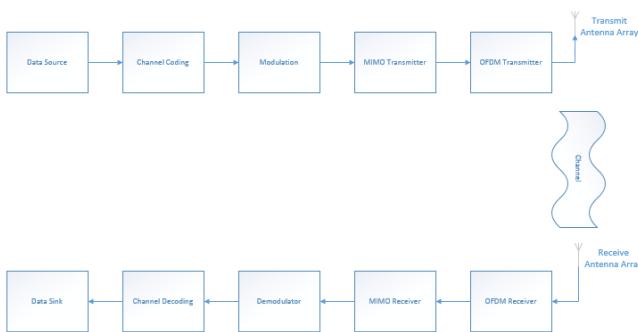


Figure 2. MIMO OFDM model

The MIMO Transmission block consists of the DFT-Precoding stage and the codeword to layer mapping employed for the spatial streams before OFDM was applied in the symbols, prior to transmission. While the MIMO receiver consists of the MMSE equalizer scheme.

#### IV. NUMERICAL RESULTS

The simulation was carried out in MATLAB 2014a, and the following were assumed:

- a. MIMO Channel Type : 3GPP MIMO Channel Model
  - b. Fading Distribution : Rayleigh, with Jake's spectrum
  - c. Additive White Gaussian Noise (AWGN) Channel
  - d. OFDM : 1024 subcarriers
  - e. Fading Profile: EVA 5Hz

- f. Antenna Configuration: 2x2, 4x4
  - g. Correlation Level: Medium
  - h. Modulation : QPSK, 16QAM, 64QAM
  - i. Channel Bandwidth : 10 MHz

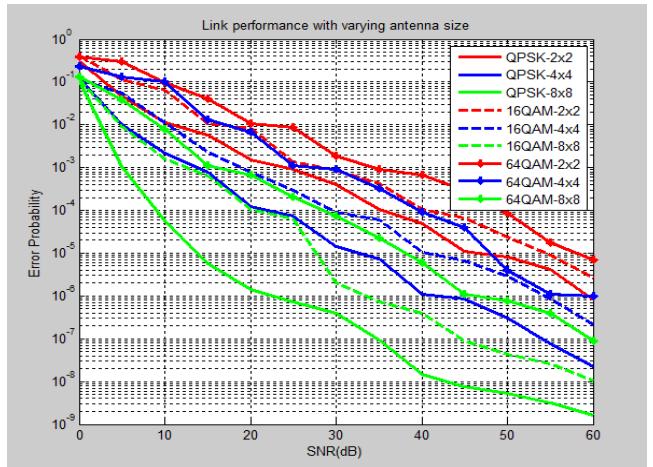


Fig 2: Simulation result

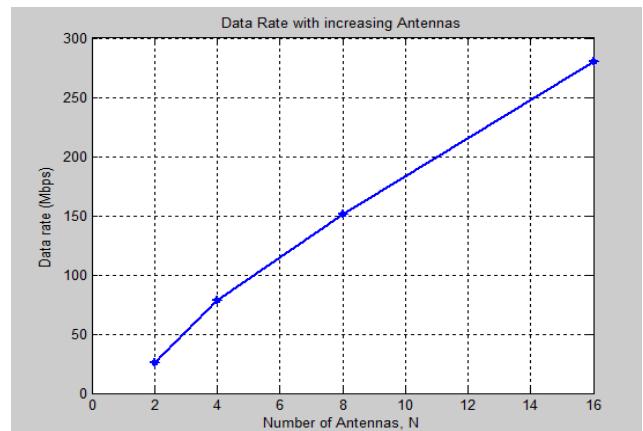


Fig 3: Data throughput with varying antenna size

Fig 2 and 3 shows that the developed DFT-precoded MIMO-OFDM meets the requirement for voice, data and video transmission and can be accommodated easily in 4G systems deployed with the model developed in this thesis.

## V. CONCLUSION

A spatial multiplexing MIMO-OFDM model was developed and the performance was analyzed in terms of bit error rates and, data throughputs using different modulations schemes, different MIMO configurations and different bandwidth requirements.

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