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New Method for Factor analysis

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***Abstract*— This paper was focused on the development of a new method base on block matrix to solve factor analysis problems. The factor analysis concept has been presented and a new method base on block matrix for exact factor analysis has been proposed. The performance of the proposed the new method has been measured on a numerical example.. The short time steps and the small number of steps involved in the process of the result showed that the new method is certainly a better choice for factor analysis problems. This is the first time a factor analysis problem has been resolved by using method base on block matrix and the results are very encouraging for further investigation**

***Index Terms*— Factor Analysis, Block matrix.**

I. INTRODUCTION

The idea of applying the block matrix for factor analysis emerges from application of neural network for the work conducted by Sanugi et. al. (1991) in the energy research, Abdelaziz Hamad Thesis(2002) in the job scheduling problem research and Abdelaziz Hammad et.al. (2014) in Factor analysis. There are similarities between factor analysis and job scheduling because both of them are considered very difficult to solve if the system is large and in both cases we have to do scheduling.

Factor analysis is a model for real –valued data in which correlations are explained by postulating the presence of one or more factors. These factors play the role of hidden variable, which are not directly observable, but allow the dependencies between the visible variable to be expressed in a convenient way. (Everitt, 1984) gives a good introduction to Latent or hidden variable models in general, and to factor analysis in particular. These models are widely used in psychology, social sciences and pure sciences such as Chemistry.

Many definitions are offered in the literature for factor analysis. Reymont and Joreskog (1993) provided a comprehensive definition:
" Factor analysis is a generic term that we use to describe a number of methods designed to analyze interrelationships within a set of variables or objects (resulting in) the construction of a few hypothetical variable (or objects) called factors, that are supposed to contain the essential information in larger set of observed variable or objects that reduces the overall complexity of the data by taking advantage of inherent interdependencies a small number of factors will usually account for approximately the same amount of information as do the much larger set of original observations."

The procedures for factor analysis were first developed early in the twentieth century by Spearman (1904). However, due to the complicated and time –consuming steps involved in the process, factor analysis was inaccessible to many researchers until both computers and user-friendly statistical software packages became widely available (Thompson & Dennings, 1993). For the above reason the new method base on block matrix approach is considered a good choice for factor analysis.
We organized this paper as follows. Section 2 described the purpose of the factor analysis. Section 3 outlines the steps of factor analysis and Section 4 provided an alternative solution of factor analysis using block matrix with a numerical example and Section 5 the conclusions of the paper

II. PURPOSE OF FACTOR ANALYSIS

The primary purpose of factor analysis is data reduction and summarization. Factor analysis has been widely used, especially in behavioral sciences, to assess the construct validity of a test or a scale. For example a psychologist developed a new battery of 15 subtests to measure three distinct psychological constructs and wanted to validate that battery. A sample of 300 subjects was drawn from the population and measured on the battery of 15 subtests. The 300 by 15 data matrix was submitted to a factor analysis procedure. The output from that procedure was a 15 by 3 factor loading matrix, which represented the relationships among the observed variables (the 15 subtests) and the 3 latent factors. The number of factors extracted and the pattern of relationships among the observed variables and the factors provided the researcher with information on the construct validity of the test battery.

III. OUTLINE OF STEP FOR FACTOR ANALYSIS

Normally, they are many step for doing factor analysis , The following are the steps used for factor analysis. Preparing data, selecting a factor model, Estimating communalities, Determining the Number of factors, The rotating of factor, and estimating factor scores.

IV. A NEW METHOD BASE ON BLOCK MATRIX FOR FACTOR ANALYSIS

In this Section a new method base on block matrix for exact factor analysis has been proposed. Suppose we have a matrix A of type $m \times n$ and we want to factorize it into two matrices w of type $m \times k$ and h of type $k \times n$, h is a matrix factor of A and is well known to us and we have to find w such that

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$$wh = A \qquad k \le n \qquad (1)$$

So we have to build another equation (we can call it a similar equation) of tow matrices $WH = A$ such that W is an $m \times n$ matrix and H is an square matrix of type $n \times n$, this construction of the similar equation can be done by adding $n-k$ zero columns to w so as to get W and adding $n-k$ rows to h in order to get H .

Now note that W is an $m \times n$ matrix and H is an square matrix of type $n \times n$, if we write the tow matrices in a block form we can write $W = [w \quad 0]$, o is an $m \times (n-k)$ zero matrix , and $H = \begin{bmatrix} h \\ c \end{bmatrix}$, c is any $(n-k) \times n$ which makes the square matrix H has an inverse matrix H^{-1}.

Note that we have the three following equations

$$WH = A \qquad (2)$$

$$W = [w \quad o] \qquad (3)$$

$$H = \begin{bmatrix} h \\ c \end{bmatrix} \qquad (4)$$

Now if we multiply the two sides of equation (2) by H^{-1} from the right ,we get

$$WHH^{-1} = AH^{-1} \qquad (5)$$

Which implies that

$$W = AH^{-1} \qquad (6)$$

From equation (3) we can deduce the matrix w

Numerical Examples:

In this Section we will give the following examples so as to measured our new method.

Example 1:

Consider the matrix

$$A = \begin{bmatrix} 4 & -1 & -3 \\ 7 & 14 & 33 \\ 11 & -1 & -4 \\ -1 & -5 & -12 \end{bmatrix}$$

If we want to analyses A into a product of two matrices w, h Such that

$$h = \begin{bmatrix} 1 & -2 & -5 \\ 2 & 3 & 7 \end{bmatrix} \qquad (7)$$

So it is obviously w is 4×2 matrix

Now add the row $c = [1 \quad 0 \quad 0]$ to **h** and the column $d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ to w so as to get W.

$$h = \begin{bmatrix} 1 & -2 & -5 \\ 2 & 3 & 7 \\ 1 & 0 & 0 \end{bmatrix} \qquad (8)$$

and

$$W = [w \quad d] \qquad (9)$$

$$H^{-1} = \begin{bmatrix} 0 & 7 & -3 \\ 0 & 5 & -2 \\ 1 & -17 & 7 \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & 1 \\ 7 & 5 & -17 \\ -3 & -2 & 7 \end{bmatrix}$$

Now $WH = A$

So

$$W = AH^{-1}$$

$$= \begin{bmatrix} 4 & -1 & -3 \\ 7 & 14 & 33 \\ 11 & -1 & -4 \\ -1 & -5 & -12 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 7 & 5 & -17 \\ -3 & -2 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 0 \\ -1 & 4 & 0 \\ 5 & 3 & 0 \\ 1 & -1 & 0 \end{bmatrix} \Rightarrow w = \begin{bmatrix} 2 & 1 \\ -1 & 4 \\ 5 & 3 \\ 1 & -1 \end{bmatrix}$$

Verification

$$wh = \begin{bmatrix} 2 & 1 \\ -1 & 4 \\ 5 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & -2 & -5 \\ 2 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 4 & -1 & -3 \\ 7 & 14 & 33 \\ 11 & -1 & -4 \\ -1 & -5 & -12 \end{bmatrix}$$

Example (2):

Consider the matrix

$$A=\begin{bmatrix}16 & 21 & 8\\ 28 & 37 & 14\\ 26 & 33 & 13\\ 16 & 22 & 8\end{bmatrix}\quad,\quad h=\begin{bmatrix}2 & 3 & 1\\ 4 & 5 & 2\end{bmatrix}$$

If we want to analyses A into a product of two matrices w,h Such that

$$wh=A \qquad (10)$$

By using the block matrix concept

$$H=\begin{bmatrix}2 & 3 & 1\\ 4 & 5 & 2\\ 1 & 0 & 0\end{bmatrix}\Rightarrow H^{-1}=\begin{bmatrix}0 & 2 & -5\\ 0 & -1 & 3\\ 1 & 0 & -2\end{bmatrix}^{T}=\begin{bmatrix}0 & 0 & 1\\ 2 & -1 & 0\\ -5 & 3 & -2\end{bmatrix}$$

So

$$W=AH^{-1}=\begin{bmatrix}16 & 21 & 8\\ 28 & 37 & 14\\ 26 & 33 & 13\\ 16 & 22 & 8\end{bmatrix}\begin{bmatrix}0 & 0 & 1\\ 2 & -1 & 0\\ -5 & 3 & -2\end{bmatrix}$$

$$=\begin{bmatrix}2 & 3 & 0\\ 4 & 5 & 0\\ 1 & 6 & 0\\ 4 & 2 & 0\end{bmatrix}\Rightarrow w=\begin{bmatrix}2 & 3\\ 4 & 5\\ 1 & 6\\ 4 & 2\end{bmatrix}$$

V. CONCLUSIONS

In this paper, The factor analysis concept has been presented and a new method base on block matrix has been proposed. The performance of the proposed method has been measured on a numerical examples. This is the first time a factor analysis problem is resolved by using block matrix method and the results are very encouraging for further investigation. The number of the steps and the short time s involved in the process of the result shows that the method base on block matrix is certainly a better choice for factor analysis problems

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REFERENCES

[1] Abdelaziz Hamad et.al. (2014) .A Neural Network Model for Factor Analysis . International Journal of Engineering and Technical Rsearch: 2,Issue 4 : 98-103
[2] Abdelaziz Hamad PhD Thesis (2002) "Application of neural network for solving job scheduling problems, UTM, Malaysia
[3] Abdelaziz Hamad and Bahrom Sanugi (2011) . Neural Network and Scheduling. Germany : Lap Lambert Academic Publishing.
[4] Everitt, B. S. (1984). An Introduction to Latent Variable Models. London: Chapman and Hall.
[5] Reymont, R., Joreskog, K.G. (1993). Applied Factor Analysis in the Natural Sciences. NewYork: Cambridge University Press.
[6] Sanugi, B. and Buang, S. (1991).” PPGs Modified Software Manual
[7] (Version1.0) for Power Station Siting Study for Tenaga Nasional Berhad (A Report on Sofware Development) UTM.
[8] Spearman, C. (1904). “ General Intelligence, “ Objectively Determined and Measured. American Journal of Psychology. 15, 201-293.