

RADIATION AND MASS TRANSFER EFFECTS ON UNSTEADY MHD FREE CONVECTIVE FLOW PAST A VERTICAL POROUS PLATE EMBEDDED IN A POROUS MEDIUM IN A SLIP FLOW REGIME WITH HEAT SOURCE/SINK AND SORET EFFECT

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Abstract—In the present analysis, we study the two-dimensional unsteady MHD free convection laminar heat and mass transfer flow past a semi-infinite vertical porous plate embedded in a porous medium in a slip flow regime in presence of thermal radiation, heat source/sink and Soret effects. A magnetic field of uniform strength is assumed to be applied transversely to the direction of the main flow. Perturbation technique is applied to transform the non-linear coupled governing partial differential equations in dimensionless form into a system of ordinary differential equations. The equations are solved analytically and the solutions for the velocity, temperature and concentration fields are obtained. The effects of various flow parameters on velocity, temperature and concentration fields are presented graphically. For different values of the flow parameters involved in the problem, the numerical calculations for the skin-friction coefficient, Nusselt number and Sherwood number at the plate are performed in tabulated form.

Index Terms— MHD, skin-friction coefficient Introduction, Magnetohydrodynamic.

I. INTRODUCTION

Natural convection flow over vertical surfaces immersed in porous media has paramount importance because of its potential applications in soil physics, geohydrology, and filtration of solids from liquids, chemical engineering and biological systems. Ostrach [1], the initiator of the study of convection flow, made a technical note on the similarity solution of transient free convection flow past a semi-infinite vertical plate by an integral method. Study of fluid flow in porous medium is based upon the empirically determined Darcy's law. Such flows are considered to be useful in diminishing the free convection, which would otherwise occur intensely on a vertical heated surface. Study of flow problems through porous medium is heavily based on Darcy's experimental law [2]. Wooding [3] and Brinkman [4,5] have modified Darcy's law, which are used by many authors on study of convective flow in porous media. Many transport processes exist in nature and in industrial applications in which the simultaneous heat and mass transfer occur as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species.

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A few representative fields of interest in which combined heat and mass transfer plays an important role are designing of chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and groves of fruit trees, crop damage due to freezing, and environmental pollution. In this context, Soundalgekar [7] extended his own problem of Soundalgekar [6] to mass transfer effects. Callahan and Marner [8] considered the transient free convection flow past a semi-infinite vertical plate with mass transfer. Unsteady free convective flow on taking into account the mass transfer phenomenon past an infinite vertical plate was studied by Soundalgekar and Wavre [9].

Magnetohydrodynamic flows have many applications in solar physics, cosmic fluid dynamics, geophysics and in the motion of earth's core as well as in chemical engineering and electronics. Verma and Srivastava [10] proposed the effect of magnetic field on unsteady blood flow through a narrow tube. Huges and Young [11] gave an excellent summary of applications. MHD free convective flow of an electrically conducting fluid between two heated parallel plates in the presence of induced magnetic field by using analytical solution was carried out by Sharma [12]. Chakrabarti and Gupta [13] explained the heat transfer effect on hydromagnetic flow over a stretching sheet. Convective heat transfer effect on MHD flow past a continuously moving plate embedded in a non-Darcian porous medium has been proposed by Abo-Eldahab and El-Gendy [14].

Radiation is a process of heat transfer through electromagnetic waves. Radiative convective flows are encountered in countless industrial and environmental process. For example, heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows etc. Radiative heat and mass transfer play an important role in space related technology. The effect of radiation on various convective flows under different conditions have been studied by many researchers including Hussain and Thakar [15], Ahmed and Sarmah [16], Rajesh and Varma [17], Pal and Mondal [18], Samad and Rahman [19], Karthikeyan *et al.* [20], Das *et al.* [21] and Pal *et al.* [22].

The study of heat source/sink effects on heat transfer is very important because its effects are crucial in controlling the heat transfer. Radiation effects on unsteady MHD flow through a porous medium with variable temperature in presence of heat source/sink is studied by Vijaya Kumar *et al.*

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[23]. Vijaya sekhar and Viswanadh reddy [24] have obtained the analytical solution for the effects of heat sink and chemical reaction on MHD free convective oscillatory flow past a porous plate with viscous dissipation. Gireesh Kumar and Satyanarayana [25] studied the heat and mass transfer effects on unsteady MHD free convective walter’s memory flow with constant suction in presence of heat sink. Choudhury and Paban Dhar [26] investigated the effects of MHD Visco-elastic fluid past a moving plate with double diffusive convection in presence of heat generation/absorption.

At the macroscopic level, it is accepted that the boundary condition for a viscous fluid at a solid wall is one of “no slip”. While no-slip boundary condition has been proven experimentally to be accurate for a number of macroscopic flows, it remains an assumption that is based on physical principles. In fact nearly two hundred years ago, Navier’s [27] proposes a general boundary condition that incorporates the possibility of fluid slip at a solid boundary. Navier’s proposed condition assumes that the fluid slip velocity at a solid surface is proportional to the shear stress at the surface. The mathematical form of the Navier’s proposed condition on slip

$$\text{velocity as emphasized by Goldstein [28] is } v = \gamma \left(\frac{dv}{dy} \right)$$

where γ being the slip coefficient, v the slip velocity and y the normal coordinate. The fluid slippage phenomena at solid boundaries appear in many applications such as in micro-channels or nano channels and in applications where a thin film of light oil is attached to the moving plates or when the surface is coated with special coating such as thick monolayer of hydrophobic octadecyl- trichorosilane. Due to practical applications of the fluid slippage phenomenon at solid boundaries, several scholar have carried out their research work in that literature, the names of whom (Yu & Ahem [29], Waltanebe *et al.* [30], Jain and Sharma [31], Khaled and Vafai [32], Saxena and Dubey [33] and Poonia and Chaudhary [34]) are worth meaning.

Due to the importance of Soret (thermal-diffusion) and Dufour (diffusion-thermal) effects for the fluids with very light molecular weight as well as medium molecular weight. The Soret effect arises when the mass flux contains a term that depends on the temperature gradient. Bhavana et al [35] proposed the Soret effect on unsteady MHD free convective flow over a vertical plate in presence of the heat source. Anand Rao et al. [36] explained the Soret and Radiation effects on unsteady MHD free convective flow past a vertical porous plate. Soret effect on MHD flow of heat and mass transfer over a vertical stretching plate in a porous medium in presences of heat source was proposed by Mohammad Ali and Mohammad Shah Alam [37]. Dufour and Soret effects on unsteady MHD free convective flow past a vertical porous plate embedded in a porous medium with mass transfer was studied by Alam et al [38]. Mohammed Ibrahim [39] studied the Soret and Dufour effects on unsteady MHD convective heat and mass transfer flow past an infinite vertical plate embedded in a porous medium in presence of radiation.

The present work is concerned with the effect of mass transfer and thermal radiation on magnetohydrodynamic

convection flow of an unsteady viscous incompressible electrically conducting fluid past a semi-infinite vertical porous plate immersed in a porous medium in slip flow regime in presence of heat sink and Soret effects. The classical model for radiation effect introduced by Cogley et al. [40] is used. Perturbation technique is applied to convert the governing non-linear partial differential equations into a system of ordinary differential equations, which are solved analytically.

II. MATHEMATICAL ANALYSIS

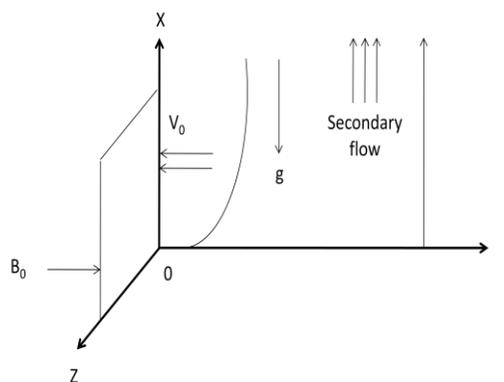


Figure 1. Physical model of the problem.

We consider a two-dimensional unsteady flow of a laminar, incompressible, electrical conducting and heat absorbing fluid past a semi-infinite vertical porous plate embedded in a uniform porous medium. We introduce the coordinate system $(\bar{x}, \bar{y}, \bar{z})$ with X axis is chosen along the plate, Y axis perpendicular to it and directed in the fluid region and Z axis is along the width of the plate as shown in the Figure 1. A uniform magnetic field of strength B_0 in the presence of radiation is imposed transversely in the direction of Y axis. The induced magnetic field is neglected under the assumption that the magnetic Reynolds number is small. It is assumed that there is no applied voltage which implies the absence of any electrical field. The radiative heat flux in the X direction is considered negligible in comparison to that in Y direction. The governing equations for this study are based on the conservation of mass, linear momentum, energy and species concentration. Taking in to consideration the assumptions made above, these equations in Cartesian frame of reference are given by equation of

Continuity:

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

Momentum equation:

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{x}} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial \bar{x}} + g\beta(\bar{T} - \bar{T}_\infty) + g\beta^*(\bar{C} - \bar{C}_\infty) + \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma B_0^2}{\rho} \bar{u} - \frac{\nu}{K^*} \bar{u} \quad (2)$$

Energy equation:

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{x}} = \frac{k}{\rho C_p} \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} - \frac{1}{\rho C_p} \frac{\partial q_r^*}{\partial \bar{y}} - \frac{Q_0(\bar{T} - \bar{T}_\infty)}{\rho C_p} \quad (3)$$

Species equation:

$$\frac{\partial \bar{C}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{C}}{\partial \bar{x}} = D_M \frac{\partial^2 \bar{C}}{\partial \bar{y}^2} + D_T \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \quad (4)$$

where \bar{x} , \bar{y} and \bar{t} are the dimensional distances along and perpendicular to the plate and dimensional time, respectively. \bar{u} and \bar{v} are the components of the dimensional velocities along \bar{x} and \bar{y} respectively. ρ is the density of the medium, g is the acceleration due to gravity, ν is the kinematic viscosity, σ is the fluid electrical conductivity, B_0 is the magnetic induction, K^* is the permeability of the porous medium, β is the coefficient of thermal expansion, β^* is the coefficient of mass expansion, \bar{T} is the dimensional temperature of the fluid near the plate, \bar{T}_∞ is the dimensional free stream temperature, \bar{C} is the dimensional concentration of the fluid near the plate, \bar{C}_∞ is the dimensional free stream concentration, k is the thermal conductivity of the fluid, q_r^* is the radiative heat flux, the term $Q_0(\bar{T} - \bar{T}_\infty)$ is assumed to be amount of heat generated or absorbed per unit volume, Q_0 is constant, which may take on either positive or negative values. When plate temperature \bar{T} exceeds the free stream temperature \bar{T}_∞ , the source term $Q_0 > 0$ and heat sink when $Q_0 < 0$, μ is the fluid viscosity, C_p is the specific heat at constant pressure, D_M is the coefficient of chemical molecular diffusivity and D_T is the coefficient of thermal diffusivity.

Cogley et al. [40] showed that, in the optically thin limit for a non-gray gas near equilibrium, the radiative heat flux is represented by the following form:

$$\frac{\partial q_r^*}{\partial \bar{y}} = 4(\bar{T} - \bar{T}_\infty) I^* \quad (5)$$

where $I^* = \int K_{\lambda w} \frac{\partial e_{b\lambda}}{\partial \bar{T}} d\lambda$, $K_{\lambda w}$ is the absorption coefficient at the wall and $e_{b\lambda}$ is the Planck's function.

Under the assumption, the appropriate boundary conditions for velocity involving slip flow, temperature and concentration fields are given by

$$\bar{u} = \bar{u}_{slip} = \bar{h} \frac{\partial \bar{u}}{\partial \bar{y}}, \quad \bar{T} = \bar{T}_w + \varepsilon(\bar{T}_w - \bar{T}_\infty) e^{n^* t^*},$$

$$\bar{C} = \bar{C}_w + \varepsilon(\bar{C}_w - \bar{C}_\infty) e^{n^* t^*} \quad \text{at } \bar{y} = 0 \quad (6)$$

$$\bar{u} \rightarrow \bar{U}_\infty = U_0(1 + \varepsilon e^{n^* t^*}), \quad \bar{T} \rightarrow \bar{T}_\infty, \quad \bar{C} \rightarrow \bar{C}_\infty \quad \text{as } \bar{y} \rightarrow \infty \quad (7)$$

where \bar{T}_w and \bar{C}_w are the dimensional temperature and species concentration at the wall respectively and \bar{h} is the characteristic dimension of the flow fluid. The suction velocity normal to the plate is a function of time only, it can be taken in the exponential form as

$$\bar{v} = -V_0(1 + \varepsilon A e^{n^* t^*}) \quad (8)$$

where A is a real positive constant, ε and εA are small quantities less than unity and V_0 is a scale of suction velocity which is a non-zero positive constant.

In the free stream, from equation (2) we get

$$-\frac{1}{\rho} \frac{\partial \bar{p}}{\partial \bar{x}} = \frac{d\bar{U}_\infty}{d\bar{t}} + \frac{\sigma B_0^2}{\rho} \bar{U}_\infty + \frac{\nu}{K^*} \bar{U}_\infty \quad (9)$$

Now we introduce the dimensionless variables as follows

$$u = \frac{\bar{u}}{U_0}, \quad v = \frac{\bar{v}}{V_0}, \quad y = \frac{\bar{y} V_0}{\nu}, \quad U_\infty = \frac{\bar{U}_\infty}{U_0}, \quad t = \frac{\bar{t} V_0^2}{\nu},$$

$$\theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty},$$

$$\phi = \frac{\bar{C} - \bar{C}_\infty}{\bar{C}_w - \bar{C}_\infty}, \quad n = \frac{n^* \nu}{V_0^2}, \quad K = \frac{K^* V_0^2}{\nu^2}, \quad \text{Pr} = \frac{\mu C_p}{k},$$

$$M = \frac{\sigma B_0^2 \nu}{\rho V_0^2}, \quad (10)$$

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$$Gr = \frac{\nu\beta g(\bar{T}_w - \bar{T}_\infty)}{U_0 V_0^2}, \quad Gm = \frac{\nu\beta^* g(\bar{C}_w - \bar{C}_\infty)}{U_0 V_0^2},$$

$$Q = \frac{Q_0 \nu}{\rho C_p V_0^2},$$

$$R = \frac{4\nu I^*}{\rho C_p V_0^2}, \quad h = \frac{V_0 \bar{h}}{\nu}, \quad So = \frac{D_T}{\nu} \left(\frac{\bar{T}_w - \bar{T}_\infty}{\bar{C}_w - \bar{C}_\infty} \right),$$

$$Sc = \frac{\nu}{D_M},$$

where Pr is the Prandtl number, M is the magnetic field parameter, Gr is the Grashof number for heat transfer, Gm is the Grashof number for mass transfer, Q is the heat sink parameter, α is the permeability parameter, θ is the non dimensional temperature, ϕ is the non dimensional concentration, R is the radiation parameter, h is the rarefaction parameter, So is the Soret number and Sc is the Schmidt number.

In view of equations (8) to (10) the governing equations (2), (3) and (4) reduce the following non-dimensional form:

$$\frac{\partial u}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial u}{\partial y} = \frac{dU_\infty}{dt} + \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gm\phi + N(U_\infty - u) \quad (11)$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} - R\theta - Q\theta \quad (12)$$

$$\frac{\partial \phi}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial \phi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} + So \frac{\partial^2 \theta}{\partial y^2} \quad (13)$$

where $N = M + \frac{1}{K}$

The boundary conditions (6) and (7) in the dimensionless form can be written as

$$u = u_{slip} = h \frac{\partial u}{\partial y}, \quad \theta = 1 + \varepsilon e^{nt}, \quad \phi = 1 + \varepsilon e^{nt} \quad \text{at } y = 0 \quad (14)$$

$$u \rightarrow U_\infty = 1 + \varepsilon e^{nt}, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \quad \text{as } y \rightarrow \infty \quad (15)$$

III. SOLUTION OF THE PROBLEM

Equations (11) to (13) are coupled non-linear partial differential equations and these can be solved in closed form.

However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. These can be done by representing the velocity, temperature and concentration of the fluid in the neighborhood of the plate as

$$u = u_0(y) + \varepsilon e^{nt} u_1(y) + O(\varepsilon^2) \quad (16)$$

$$\theta = \theta_0(y) + \varepsilon e^{nt} \theta_1(y) + O(\varepsilon^2) \quad (17)$$

$$\phi = \phi_0(y) + \varepsilon e^{nt} \phi_1(y) + O(\varepsilon^2) \quad (18)$$

Substituting (16) to (18) in equations (11) to (13) and equating the harmonic and non harmonic terms and neglecting the coefficient of $O(\varepsilon^2)$ we get the following pairs of equations for (u_0, θ_0, ϕ_0) and (u_1, θ_1, ϕ_1) .

$$u_0'' + u_0' - Nu_0 = -N - Gr\theta_0 - Gm\phi_0 \quad (19)$$

$$u_1'' + u_1' - (N + n)u_1 = -Au_0' - Gr\theta_1 - Gm\phi_1 - (N + n) \quad (20)$$

$$\theta_0'' + Pr\theta_0' - Pr(R + Q)\theta_0 = 0 \quad (21)$$

$$\theta_1'' + Pr\theta_1' - Pr(R + Q + n)\theta_1 = -APr\theta_0' \quad (22)$$

$$\phi_0'' + Sc\phi_0' = -ScSo\theta_0'' \quad (23)$$

$$\phi_1'' + Sc\phi_1' - nSc\phi_1 = -ASc\phi_0' - ScSo\theta_1'' \quad (24)$$

where the primes denote the differentiation with respect to y .

The corresponding boundary conditions can be written as

$$u_0 = hu_0', \quad u_1 = hu_1', \quad \theta_0 = 1, \quad \theta_1 = 1, \quad \phi_0 = 1, \quad \phi_1 = 1 \quad \text{at } y = 0 \quad (25)$$

$$u_0 = 1, \quad u_1 = 1, \quad \theta_0 \rightarrow 0, \quad \theta_1 \rightarrow 0, \quad \phi_0 \rightarrow 0, \quad \phi_1 \rightarrow 0 \quad \text{as } y \rightarrow \infty \quad (26)$$

The solutions of equations (19) to (24) which satisfy the boundary conditions (25) and (26) are given by

$$u_0 = 1 + A_{10}e^{-m_5 y} - A_7 e^{-m_1 y} - A_8 e^{-m_3 y} + A_9 e^{-m_4 y} \quad (27)$$

$$u_1 = 1 + A_{23}e^{-m_6y} + A_{11}e^{-m_5y} - A_{12}e^{-m_4y} - A_{13}e^{-m_3y} + A_{14}e^{-m_2y} - A_{15}e^{-m_1y} - A_{16}e^{-m_4y} - A_{17}e^{-m_4y} - A_{18}e^{-m_3y} - A_{19}e^{-m_2y} - A_{20}e^{-m_1y}$$

$$(28)$$

$$\theta_0 = e^{-m_1y} \quad (29)$$

$$\theta_1 = A_0e^{-m_2y} + A_1e^{-m_1y} \quad (30)$$

$$\phi_0 = (1 + A_2)e^{-m_3y} - A_2e^{-m_1y} \quad (31)$$

$$\phi_1 = A_6e^{-m_4y} + A_3e^{-m_3y} + A_4e^{-m_2y} + A_5e^{-m_1y} \quad (32)$$

In view of the above solutions, the velocity, temperature and concentration distributions in the boundary layer become

$$u(y,t) = 1 + A_{10}e^{-m_5y} - A_7e^{-m_4y} - A_8e^{-m_3y} + A_9e^{-m_2y} + \varepsilon e^{nt} (1 + A_{23}e^{-m_6y} + A_{11}e^{-m_5y} - A_{12}e^{-m_4y} - A_{13}e^{-m_3y} + A_{14}e^{-m_2y} - A_{15}e^{-m_1y} - A_{16}e^{-m_4y} - A_{17}e^{-m_4y} - A_{18}e^{-m_3y} - A_{19}e^{-m_2y} - A_{20}e^{-m_1y})$$

$$\theta(y,t) = e^{-m_2y} + \varepsilon e^{nt} (A_0e^{-m_2y} + A_1e^{-m_1y})$$

$$\phi(y,t) = (1 + A_2)e^{-m_3y} - A_2e^{-m_1y} + \varepsilon e^{nt} (A_6e^{-m_4y} + A_3e^{-m_3y} + A_4e^{-m_2y} + A_5e^{-m_1y})$$

It is now important to calculate the physical quantities of primary interest, which are the local wall shear stress, the local surface heat and mass flux. Given the velocity field in the boundary layer, we can now calculate the local wall shear stress (i.e., skin-friction) is given by

$$\tau_w^* = \mu \left(\frac{\partial \bar{u}}{\partial y} \right)_{\bar{y}=0}$$

and in dimensionless form, we obtain

$$C_f = \frac{\tau_w^*}{\rho U_0 V_0} = \left(\frac{\partial u}{\partial y} \right)_{y=0} = u'(0) = \left(\frac{\partial u_0}{\partial y} + \varepsilon e^{nt} \frac{\partial u_1}{\partial y} \right)_{y=0}$$

$$= (-m_5A_{10} + m_1A_7 + m_3A_8 - m_1A_9) + \varepsilon e^{nt} (-m_6A_{23} - m_5A_{11} + m_1A_{12} + m_3A_{13} - m_1A_{14} + m_2A_{15} + m_1A_{16} + m_4A_{17} + m_3A_{18} + m_2A_{19} + m_1A_{20})$$

Knowing the temperature field, it is interesting to study the non-dimensional form of the rate of heat transfer in terms of Nusselt number at the plate is given by:

$$N_u = - \left(\frac{\partial \theta}{\partial y} \right)_{y=0} = - \left(\frac{\partial \theta_0}{\partial y} + \varepsilon e^{nt} \frac{\partial \theta_1}{\partial y} \right)_{y=0} = -(-m_1 + \varepsilon e^{nt} (-m_2A_0 - m_1A_1))$$

Knowing the concentration field, it is interesting to study the non-dimensional form of the rate of mass transfer in terms of Sherwood number at the plate is given by:

$$S_h = - \left(\frac{\partial \phi}{\partial y} \right)_{y=0} = - \left(\frac{\partial \phi_0}{\partial y} + \varepsilon e^{nt} \frac{\partial \phi_1}{\partial y} \right)_{y=0} = -((1 + A_2)m_3 + m_1A_2 + \varepsilon e^{nt} (-m_4A_6 - m_3A_3 - m_2A_4 - m_1A_5))$$

IV. RESULTS AND DISCUSSION

In order to get physical insight into the problem, we have carried out numerical calculations for non-dimensional velocity field, temperature field, concentration field, coefficient of skin-friction C_f at the plate, the rate of heat transfer in terms of Nusselt number N_u and the rate of mass transfer in terms of Sherwood number S_h by assigning specific values to the different values to the parameters involved in the problem, viz., Magnetic field parameter M , Grashof number for heat transfer Gr , Grashof number for mass transfer Gm , permeability parameter K , heat sink parameter Q , rarefaction parameter h , Prandtl number Pr , Radiation parameter R , Schmidt number Sc and Soret number So , time t . In the present study, the following default parametric values are adopted. $Gr = 6.0$, $Gm = 4.0$, $M = 3.0$, $K = 1.0$, $n = 0.1$, $A = 1.0$, $t = 1.0$, $Pr = 0.71$, $R = 1.0$, $Q = 1.0$, $Sc = 0.6$, $So = 1.0$, $h = 0.3$ and $\varepsilon = 0.2$. All graphs therefore correspond to these unless specifically indicated on the appropriate graph. The numerical results are demonstrated through different graphs and table and their results are interpreted physically.

It is observed from Figure 2 that an increase in Grashof number for heat transfer Gr leads to a rise in the values of velocity u due to enhancement in buoyancy force.

The plot of velocity profile for different values of Grashof number for mass transfer Gm is given in Figure 3. It is observed that velocity increase for the increasing values of Grashof number for mass transfer Gm .

Figure 4 plots the velocity profiles against the span-wise coordinate y for different magnetic field parameter M . this illustrates that velocity decreases as the existence of magnetic field becomes stronger. This conclusion agrees with

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the fact that magnetic field exerts retarding force on the free-convection flow.

The changes in velocity profile due to different permeability of the porous medium are plotted in Figure 5. This figure shows that the velocity profiles increases rapidly as increases permeability of porous medium K .

Figure 6. illustrates the dimensionless velocity u for different values of the Prandtl number Pr . The analytical results show that the effect of increasing values of Prandtl number results in a decreasing velocity.

The effect of the Radiation parameter R on the dimensionless velocity u is shown in Figure. 7 Figure7 shows that velocity component decreases with an increase in the radiation parameter R .

Figure 8 depict the dimensionless velocity component u profiles for different values of heat sink parameter Q . It is noticed that an increase in the heat sink parameter Q results in decrease in the dimensionless velocity component u within the boundary layer.

The influence of the Schmidt number Sc on velocity profiles are plotted in Figure 9. The Schmidt number Sc embodies the ratio of the momentum to the mass diffusivity. It is noticed that

as the Schmidt number Sc increases the velocity decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity.

Figure 10 depicts the velocity profiles for different values of the Soret number So . The Soret number defines the effect of the temperature gradients inducing significant mass diffusion effects. It is noticed that an increase in the Soret number results in an increase in the velocity within the boundary layer.

Figure 11 indicated that the velocity component u for different values of rarefaction parameter h . The velocity u increases as the rarefaction parameter h is increased indicating the fact that slips at the surface accelerates the fluid motion.

The horizontal velocity profile in the boundary layer for different values of time t is depicted in Figure 12. It is observed that the horizontal velocity slowly attains the peak value close to the porous boundary and then it decreases till it reaches the minimum value at the end of the boundary layer

for all the values of time. It is noticed that the velocity increases as the time increases.

From Figure 13. it is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated plate more rapidly than for higher values of Pr . Hence in the case of smaller Prandtl numbers as the boundary layer is thicker and the rate of heat transfer is reduced.

Figure 14 shows the variation of temperature profiles with respect to the radiation parameter R . from this figure, it is observed that as temperature increases for the increasing values of radiation parameter R . This result qualitatively agrees with expectations, since the effect of radiation is to decrease the rate of energy transport to the fluid, thereby decreasing the temperature of the fluid.

The influence of the parameter heat sink on dimensionless temperature profiles θ is plotted in Figure 15. It is noticed that dimensionless temperature decreases with an increase in heat sink parameter Q .

Moreover Figures 16 displays that the temperature profiles θ increases as time t is increased.

The concentration profiles for different values of Schmidt number Sc are plotted in Figure 17. For Schmidt number Sc , the value 0.6 corresponds to water vapor and represents a diffusing chemical species of most common interest in air. The analytical results show that the effect of increasing Schmidt number in air and water results in a decreasing concentration distribution across the boundary layer. The results also indicate that the effect on Schmidt number on the velocity and concentration is more in water as compared to that in air.

For various values of Soret parameter So , the concentration profiles is plotted in Figure 18. Clearly as So increases, the dimensionless concentration increases.

Concentration profiles ϕ for different values of ε and time t are shown in Figures 19. From this figure, it is observed that dimensionless concentration profiles increases as time t is increased.

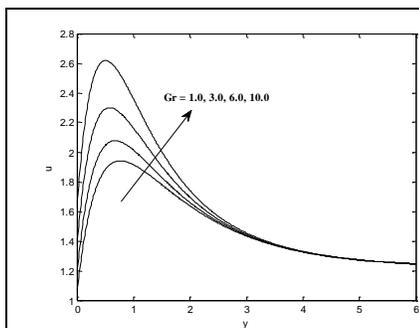


Figure 2. Velocity u versus y , under the effect of Gr

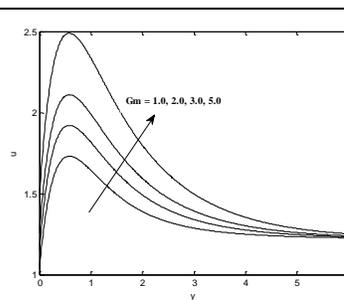


Figure 3. Velocity u versus y , under the effect of Gm

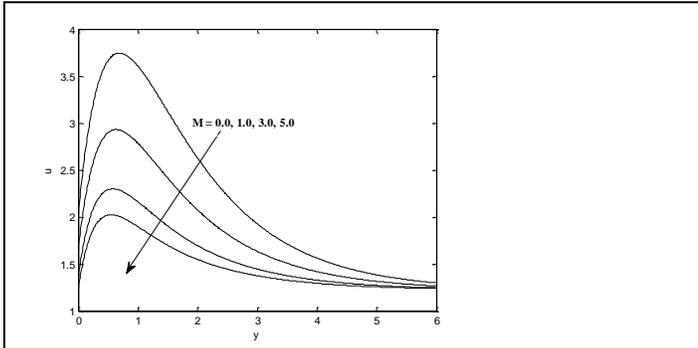


Figure 4. Velocity u versus y , under the effect of M

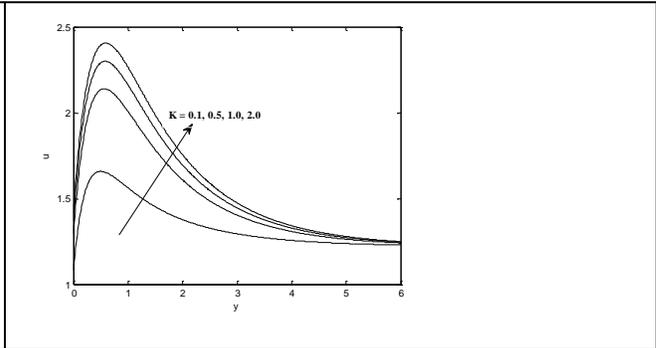


Figure 5. Velocity u versus y , under the effect of K

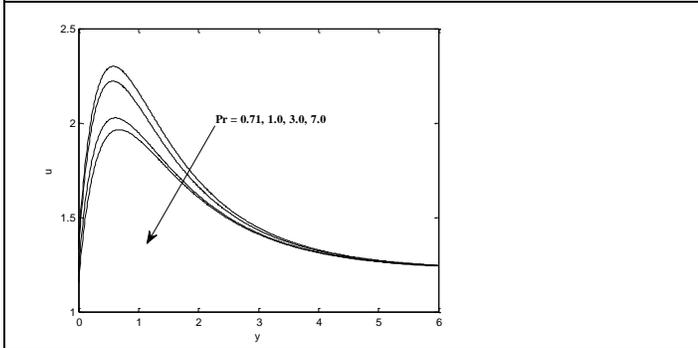


Figure 6. Velocity u versus y , under the effect of Pr

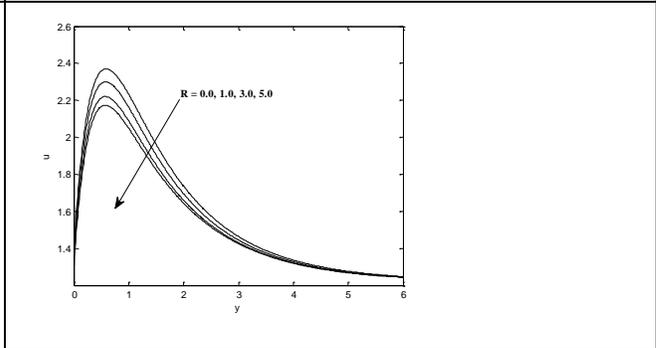


Figure 7. Velocity u versus y , under the effect of R

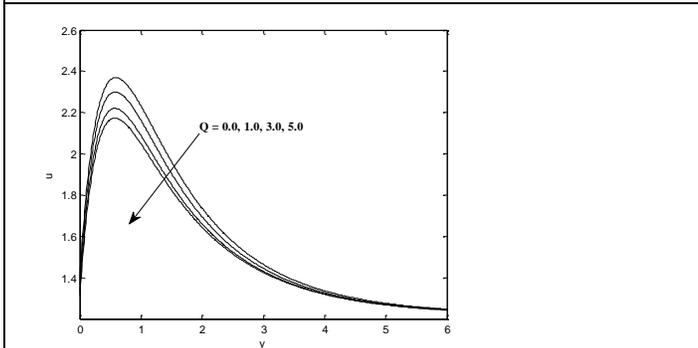


Figure 8. Velocity u versus y , under the effect of Q

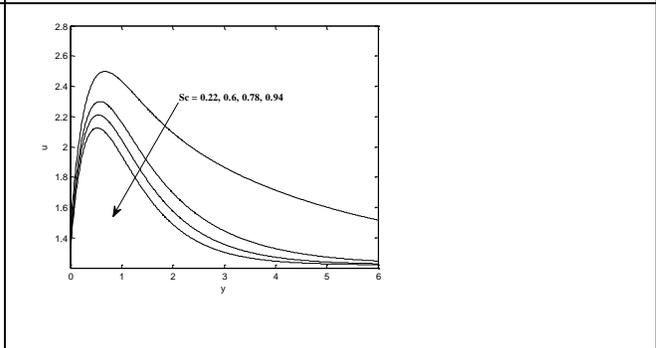


Figure 9. Velocity u versus y , under the effect of Sc

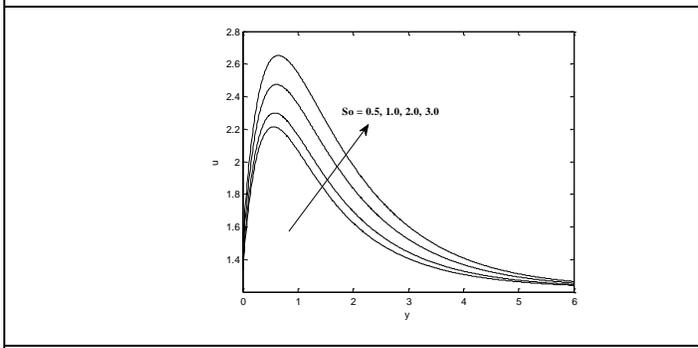


Figure 10. Velocity u versus y , under the effect of So

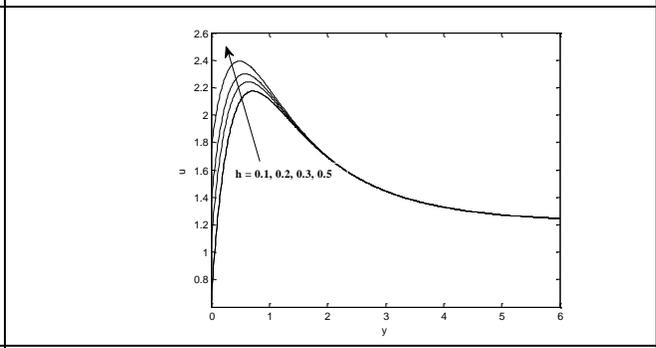


Figure 11. Velocity u versus y , under the effect of h

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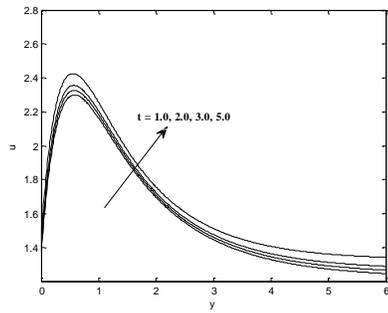


Figure 12. Velocity u versus y , under the effect of t

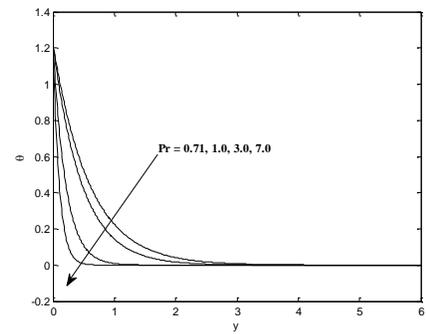


Figure 13. Temperature θ versus y , under the effect of Pr

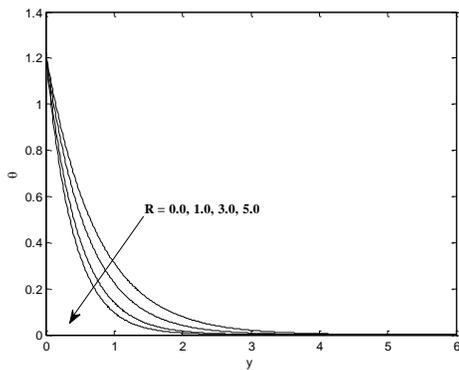


Figure 14. Temperature θ versus y , under the effect of R

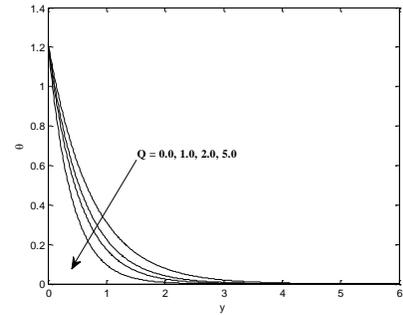


Figure 15. Temperature θ versus y , under the effect of Q

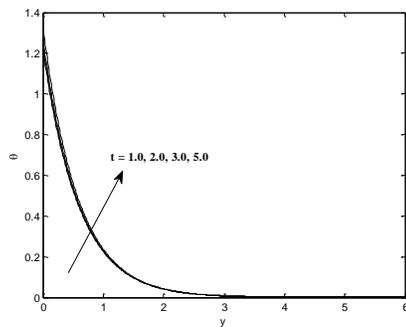


Figure 16. Temperature θ versus y , under the effect of t

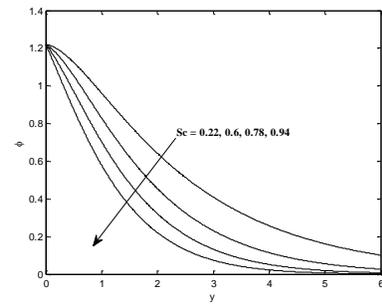


Figure 17. Concentration ϕ versus y , under the effect of Sc

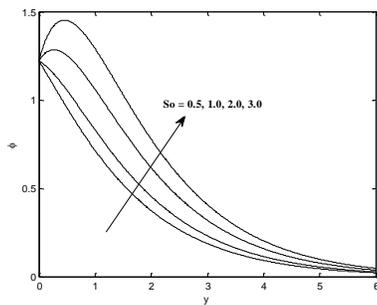


Figure 18. Concentration ϕ versus y , under the effect of So

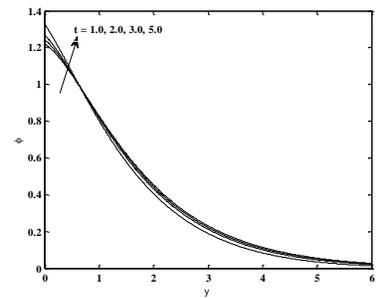


Figure 19. Concentration ϕ versus y , under the effect of t

Table 1. Skin friction coefficient, Nusselt number and Sherwood number for various values of Gr , Gm , M , K , h with $Pr = 0.71$, $R = 1.0$, $Q = 1.0$, $Sc = 0.6$, $So = 1.0$, $\varepsilon = 0.2$, $n = 0.1$, $A = 1.0$, $t = 1.0$.

Gr	Gm	M	K	h	C_f	N_u	S_h
6.0	4.0	3.0	1.0	0.3	4.5564	2.0577	0.1785
2.0	4.0	3.0	1.0	0.3	3.7052	2.0577	0.1785
4.0	4.0	3.0	1.0	0.3	4.1308	2.0577	0.1785
6.0	1.0	3.0	1.0	0.3	3.4571	2.0577	0.1785
6.0	2.0	3.0	1.0	0.3	3.8235	2.0577	0.1785
6.0	4.0	1.0	1.0	0.3	5.5261	2.0577	0.1785
6.0	4.0	2.0	1.0	0.3	4.9226	2.0577	0.1785
6.0	4.0	3.0	2.0	0.3	4.7198	2.0577	0.1785
6.0	4.0	3.0	3.0	0.3	4.7823	2.0577	0.1785
6.0	4.0	3.0	1.0	0.5	3.5121	2.0577	0.1785
6.0	4.0	3.0	1.0	0.7	2.8571	2.0577	0.1785

Table 2. Skin friction coefficient, Nusselt number and Sherwood number for various values of Pr , R , Q , Sc , So with $Gr = 6.0$, $Gm = 4.0$, $M = 3.0$, $K = 1.0$, $h = 0.3$, $\varepsilon = 0.2$, $n = 0.1$, $A = 1.0$, $t = 1.0$.

Pr	R	Q	Sc	So	C_f	N_u	S_h
0.71	1.0	1.0	0.6	1.0	4.5564	2.0577	0.1785
1.0	1.0	1.0	0.6	1.0	4.4364	2.5951	- 0.0256
3.0	1.0	1.0	0.6	1.0	4.0611	5.8503	- 1.1499
0.71	2.0	1.0	0.6	1.0	4.4873	2.3689	0.0337
0.71	3.0	1.0	0.6	1.0	4.4350	2.6350	- 0.0869
0.71	1.0	2.0	0.6	1.0	4.4873	2.3689	0.0337
0.71	1.0	3.0	0.6	1.0	4.4350	2.6350	- 0.0869
0.71	1.0	1.0	0.78	1.0	4.4478	2.0577	0.3835
0.71	1.0	1.0	0.94	1.0	4.3351	2.0577	0.6758
0.71	1.0	1.0	0.6	2.0	4.7931	2.0577	- 0.5107
0.71	1.0	1.0	0.6	3.0	5.0298	2.0577	- 1.1998

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APPENDIX

$$m_1 = \frac{\text{Pr} + \sqrt{\text{Pr}^2 + 4 \text{Pr}(R + Q)}}{2}$$

$$m_2 = \frac{\text{Pr} + \sqrt{\text{Pr}^2 + 4 \text{Pr}(R + Q + n)}}{2}$$

$$m_3 = Sc$$

$$m_4 = \frac{Sc + \sqrt{Sc^2 + 4nSc}}{2}$$

$$m_5 = \frac{1 + \sqrt{1 + 4N}}{2}$$

$$m_6 = \frac{1 + \sqrt{1 + 4(N+n)}}{2}$$

$$A_1 = \frac{A \Pr m_1}{m_1^2 - \Pr m_1 - \Pr(R+Q+n)}$$

$$A_0 = 1 - A_1$$

$$A_2 = \frac{Sc Som_1}{m_1 - Sc}$$

$$A_3 = \frac{AScm_3(1+A_2)}{m_3^2 - Scm_3 - nSc}$$

$$A_4 = \frac{ASc^2 Som_2^2(1-A_1)}{m_2^2 - Scm_2 - nSc}$$

$$A_5 = \frac{ASc^2 Som_1^2 A_1}{m_1^2 - Scm_1 - nSc}$$

$$A_6 = 1 - A_3 - A_4 - A_5$$

$$A_7 = \frac{Gr}{m_1^2 - m_1 - N}$$

$$A_8 = \frac{Gm(1+A_2)}{m_3^2 - m_3 - N}$$

$$A_9 = \frac{GmA_2}{m_1^2 - m_1 - N}$$

$$A_{10} = \frac{(-1 + A_7 + A_8 - A_9 + hA_7m_1 + hA_8m_3 - hA_9m_1)}{1 + hm_5}$$

$$A_{11} = \frac{Am_5 A_{10}}{m_5^2 - m_5 - (N+n)}$$

$$A_{12} = \frac{Am_1 A_7}{m_1^2 - m_1 - (N+n)}$$

$$A_{13} = \frac{Am_3 A_8}{m_3^2 - m_3 - (N+n)}$$

$$A_{14} = \frac{Am_1 A_9}{m_1^2 - m_1 - (N+n)}$$

$$A_{15} = \frac{GrA_0}{m_2^2 - m_2 - (N+n)}$$

$$A_{16} = \frac{GrA_1}{m_1^2 - m_1 - (N+n)}$$

$$A_{17} = \frac{GmA_6}{m_4^2 - m_4 - (N+n)}$$

$$A_{18} = \frac{GmA_3}{m_3^2 - m_3 - (N+n)}$$

$$A_{19} = \frac{GmA_4}{m_2^2 - m_2 - (N+n)}$$

$$A_{20} = \frac{GmA_5}{m_1^2 - m_1 - (N+n)}$$

$$A_{21} = -1 - A_{11} + A_{12} + A_{13} - A_{14} + A_{15} + A_{16} + A_{17} + A_{18} + A_{19} + A_{20}$$

$$A_{22} = -hm_5 A_{11} + hm_1 A_{12} + hm_3 A_{13} - hm_1 A_{14} + hm_2 A_{15} + hm_1 A_{16} + hm_4 A_{17} + hm_3 A_{18} + hm_2 A_{19} + hm_1 A_{20}$$

$$A_{23} = \frac{(A_{21} + A_{22})}{1 + hm_6}$$

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