

# Mathematical model formulations for block adjustment in Aerial Triangulation from high resolution satellite data

Mr. SS.Manugula, Dr.Saroj Patel, Dr.Veerana Bommakanti

**Abstract**— Mathematics plays an important role with stereo data which is collected from satellite. Presently, satellite images can be used for various applications in civil engineering. Satellite stereo data plays an important role in water resources application by automatic generation of Digital Elevation Model (DEM) from aerial triangulation.

The 3D rational function models build a correlation between the pixels (2D image space) and their ground locations (3D object space). This correlation is based on ratios of polynomials.

Early in the research for the epipolar geometric relationship of linear pushbroom images, the exact epipolar curve was not easy to derive because of the complexity of the relationship and the difficulty in expressing it in mathematical form.

Conventional aerial triangulation is reviewed. This review encompasses various mathematical models, self calibration technique, additional parameters, and the associated mathematical models. Mission planned satellites like IKONOS, QB, GeoEye and Cartosat provide the metric quality data. In this research work, it is proposed to use high resolution satellite stereo data i.e. GeoEye-1 for creating the block setup and AT.

**Index Terms**—Aerial Triangulation, Quick Bird, Digital Elevation Model (DEM)

## I. INTRODUCTION

During the last two decades the space age has made a remarkable progress in utilizing GIS (Geographic Information System), Remote Sensing and Photogrammetry data to describe, study, monitor, and model the earth's surface. Improvements in sensor technology, especially in the spatial, spectral, radiometric and temporal resolution, have enabled the scientific community to operational the methodology.

Early in the research for the epipolar geometric relationship of linear pushbroom images, the exact epipolar curve was not easy to derive because of the complexity of the relationship and the difficulty in expressing it in mathematical form [Otto, 1998]. Subsequently, Kim [2000] derived an exact mathematical equation for the epipolar curve of linear pushbroom images using a collinearity-based sensor model.. However, collinearity equations can be used for linear images

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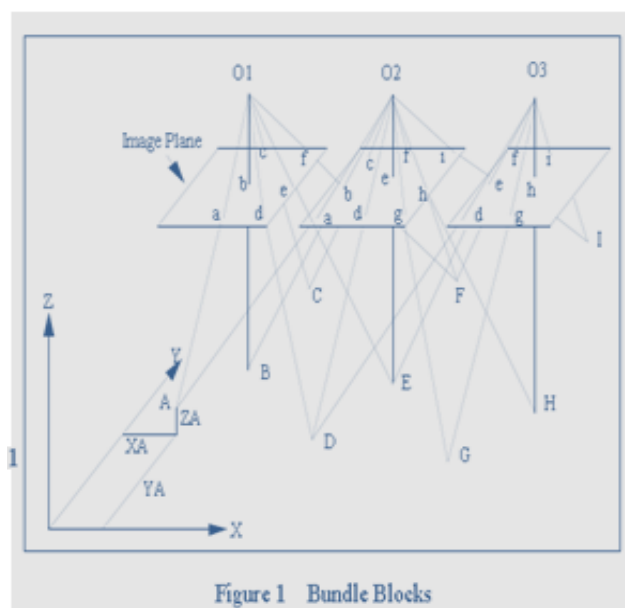
**SS.Manugula**, Department of Civil Engineering, Gurunak Institutions Technical Campus, Hyderabad, India, Mobile No. +91 8096541592.

**Dr. Saroj Patel**, Department of Mathematics, Jodhpur National University, Jodhpur, Rajasthan, India, Mobile No. +91 9461117200.

**Dr Veeranna.B.** Department of Civil Engg, Gurunak Institutions Technical Campus, Hyderabad, India, Mobile No. +91 9618558123.

by modifying the equations for perspective images in the scanning direction [Kim, 2000].

The end of development of remote sensing is being from panchromatic, multi-spectral and hyper-spectral to ultra-spectral with the increase in spectral resolution. On the other hand, present spatial resolution is impressed to sub-meter for example 0.5m from GEO- Eye



- Triangulation is the process of contiguous densifying and extending ground control through computational means.
- This operation includes establishing ground control points; performing interior orientation, Relative orientation; measuring and transferring all tie, check, and control points appearing on all photographs; and performing a least squares block adjustment.
- This process ultimately provides exterior orientation parameters for photographs and three-dimensional co-ordinates for measured object points.

The fundamental equation of aerial triangulation is the collinearity equation which states that an object point, its homologous image point, and the perspective centre are collinear

## II. CONTROL REQUIREMENTS FOR PHOTOGRAMMETRIC BLOCKS

Any block consisting of two or more overlapping photographs requires that it be absolutely oriented to the ground coordinate system. The 3D spatial similarity transformation with 7 parameters (3 rotations, 3 translations, and 1 scale) is usually employed for absolute orientation and requires at least 2 horizontal and 3 vertical control points. Due to some influences caused by transfer errors (e.g., image coordinate measurements of conjugate points), and extrapolation beyond the mapping area, the theoretical minimum control requirement is unrealistic.

Controls & Block properties: Various control points are placed in the stereo image (GeoEye 1) can be seen visually the block properties are also with X, Y, Z.

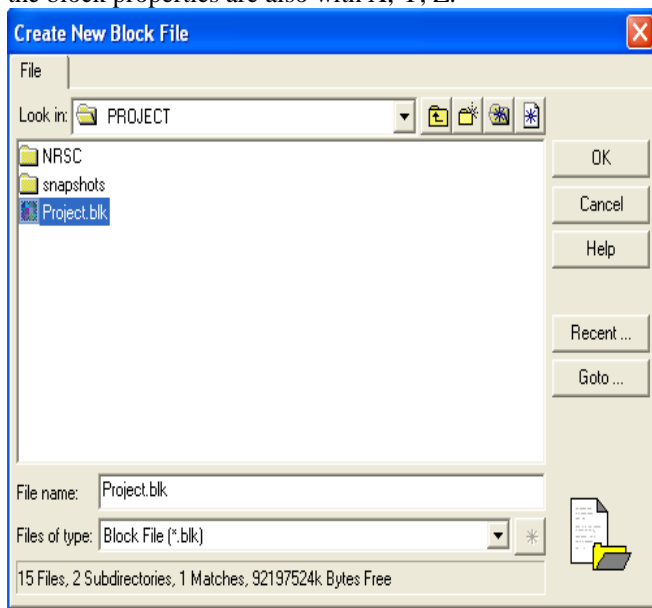


Fig-2 Creation of block

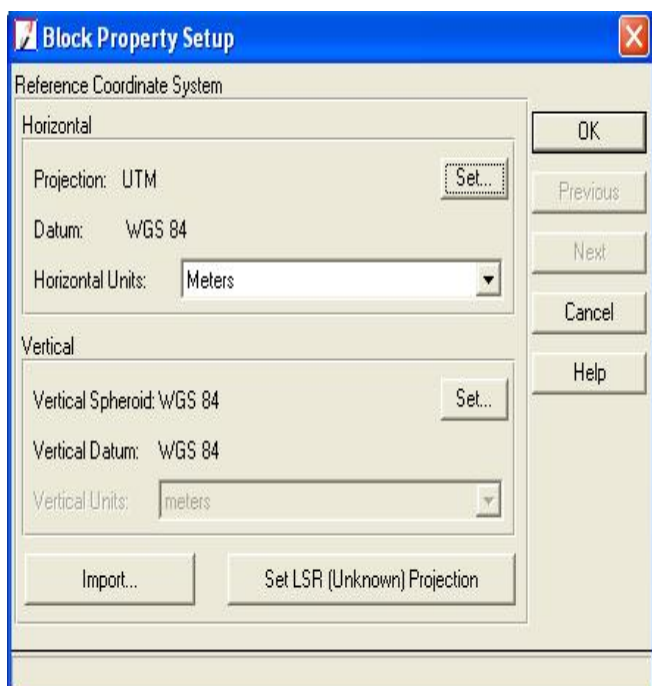


Fig-3 Block properties setup

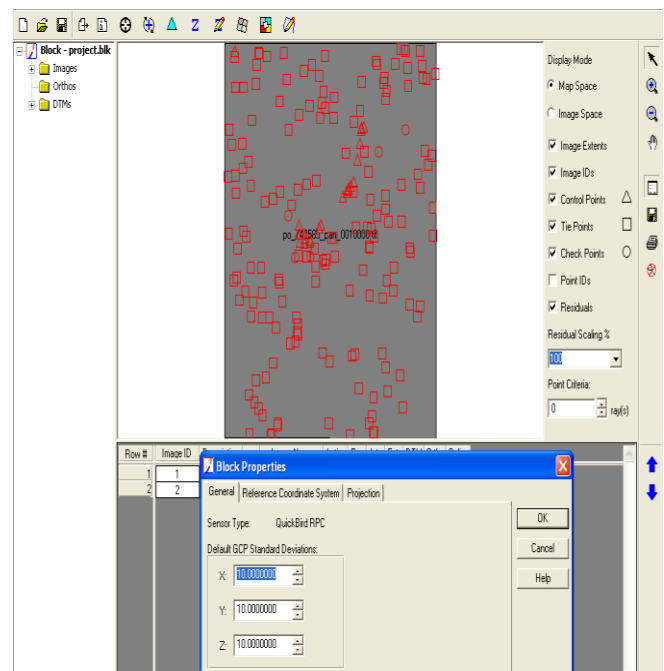


Fig-4 Controls & Block properties

## III. CONDITION AND EQUATIONS

### A. Geometric Correction Models

In satellite imagery, a sensor model or geometric correction model relates object point positions (X, Y, Z) to their corresponding 2D image positions (x, y). Several sensor models can be used to correct satellite imagery: 2D polynomial functions, 3D polynomial functions, affine model, 3D rational functions, and 3D physical models (Tao and Hu, 2001; Fraser et al., 2002; Fraser and Yamakawa, 2004; Toutin, 2004a). However, for the 3D geometric correction of VHR satellite images, two sensor models are commonly used:

(a) 3D rational functions using vendor image support data, and (b) 3D physical models. These two models are used in this work to calculate the sensor orientation for Ikonos and Quick Bird images respectively. Both are supported by OrthoEngine® from PCI Geomatica v.9.1.7 (PCI Geomatics, Richmond Hill, Ontario, Canada), which is the commercial software used here for the bundle adjustment.

### B. Collinearity condition

The collinearity condition is illustrated in the figure below. The exposure station of a photograph, an object point and its photo image all lie along a straight line. Based on this condition we can develop complex mathematical relationships

### C. Collinearity Condition Equations

In Fig -5 Coordinates of exposure station be XL, YL, ZL wrt object (ground) coordinate system XYZ  
Coordinates of object point A be XA, YA, ZA wrt ground coordinate system XYZ

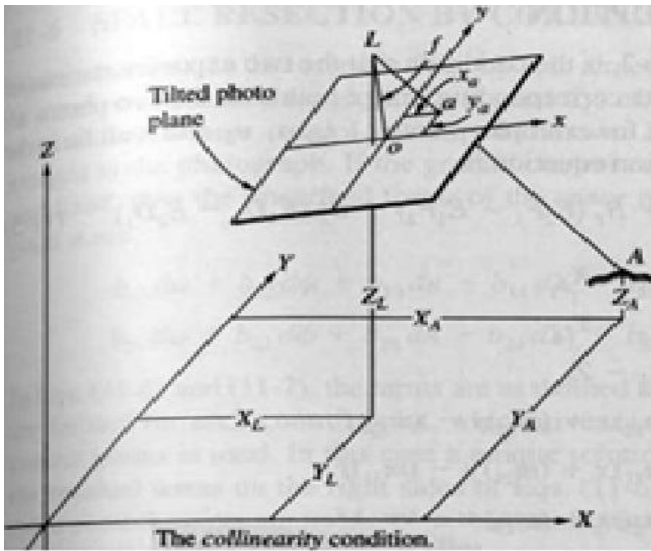


Fig -5

Coordinates of image point a of object point A be  $x_a, y_a, z_a$  wrt  $xy$  photo coordinate system (of which the principal point  $o$  is the origin; correction compensation for it is applied later) Coordinates of image point a be  $x_a', y_a', z_a'$  in a rotated image plane  $x'y'z'$  which is parallel to the object coordinate system  
Transformation of  $(x_a', y_a', z_a')$  to  $(x_a, y_a, z_a)$  is accomplished using rotation equations, which we derive next.

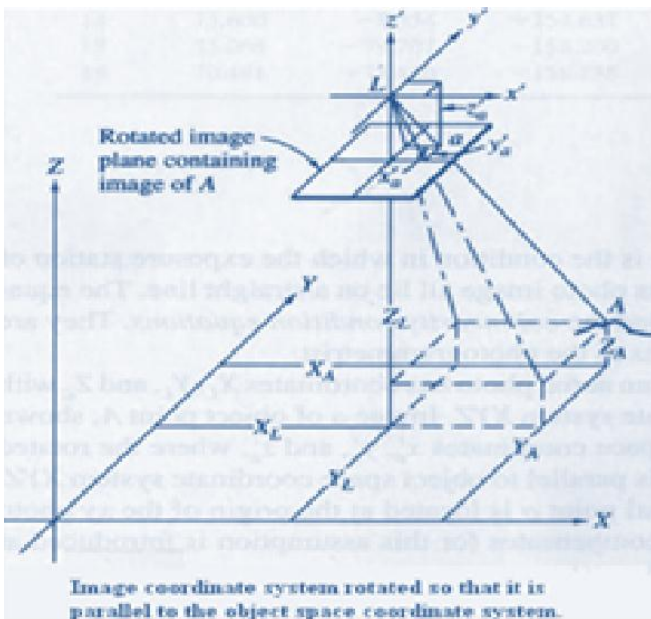


Fig-6

#### D. Rotation Equations

Omega rotation about  $x'$  axis:  
New coordinates  $(x_1, y_1, z_1)$  of a point  $(x', y', z')$  after rotation of the original coordinate reference frame about the  $x$  axis by angle  $\omega$  are given by:

$$\begin{aligned} x_1 &= x' \\ y_1 &= y' \cos \omega + z' \sin \omega \\ z_1 &= -y' \sin \omega + z' \cos \omega \end{aligned}$$

Similarly, we obtain equations for phi rotation about  $y$  axis:

$$x_2 = -z_1 \sin \Phi + x_1 \cos \Phi$$

$$y_2 = y_1$$

$$z_2 = z_1 \cos \Phi + x_1 \sin \Phi$$

And equations for kappa rotation about  $z$  axis:

$$x = x_2 \cos \kappa + y_2 \sin \kappa$$

$$y = -x_2 \sin \kappa + y_2 \cos \kappa$$

$$z = z_2$$

#### E. Final Rotation Equations

We substitute the equations at each stage to get the following:

$$x = m_{11} x' + m_{12} y' + m_{13} z'$$

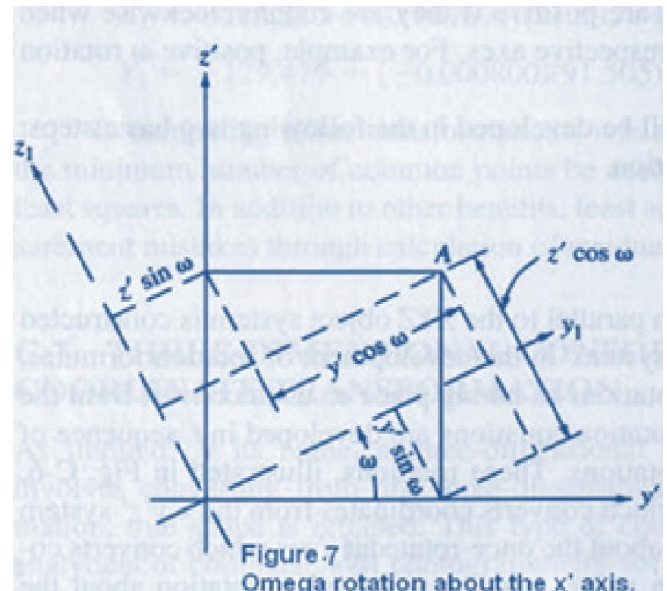


Figure-7  
Omega rotation about the  $x'$  axis.

Fig-7

$$y = m_{21} x' + m_{22} y' + m_{23} z'$$

$$z = m_{31} x' + m_{32} y' + m_{33} z'$$

Where  $m$ 's are function of rotation angles  $\omega, \Phi$  and  $\kappa$

In matrix form:  $X = M X'$

$$\text{where } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \quad X' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

Sum of squares of the 3 direction cosines (elements of  $M$ ) in any row or column is unity.  $M$  is orthogonal

#### IV. TRIANGULATION OF GEO EYE STEREO DATA:

In analytical photogrammetry, the collinearity condition is the most useful and fundamental relationship. The condition is expressed by the collinearity of three points: exposure station, object, and its photo image, in a 3D space.

The image vector and ground vector are only collinear if one is a scalar multiple of the other [10], [11]

Therefore the following statement can be made:

$$a = KA$$

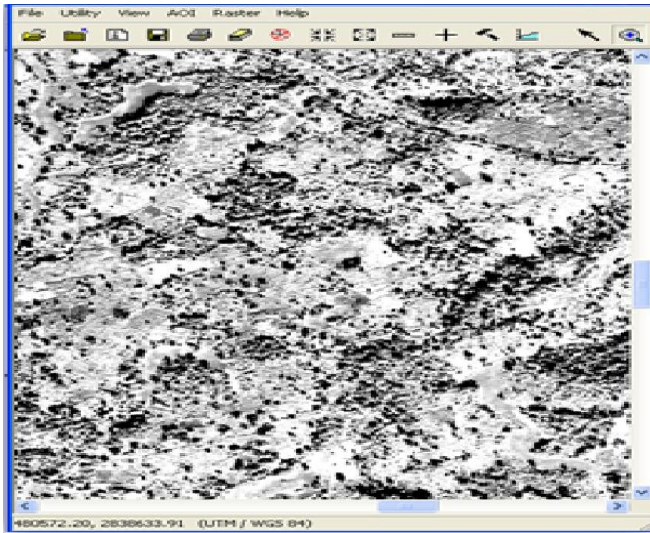


Fig 8: Geo Eye stereo data:

In the equation, K is a scalar multiple. The image and ground vectors must be within the same coordinate system. Therefore, image vector ‘a’ is comprised of the following components:

$$a = \begin{bmatrix} xp - x0 \\ yp - y0 \\ -f \end{bmatrix}$$

Where  $x0$  &  $y0$  represent the image coordinates of the principal point. Similarly the ground vector can be formulated as follows:

$$A = \begin{bmatrix} Xp - X0 \\ Yp - Y0 \\ Zp - Z0 \end{bmatrix}$$

Thus the following equation can be formulated as:  $a = KMA$

where,

$$\begin{bmatrix} xp - x0 \\ yp - y0 \\ -f \end{bmatrix} = KM \begin{bmatrix} Xp - X0 \\ Yp - Y0 \\ Zp - Z0 \end{bmatrix}$$

The above condition defines the relationship between the perspective center of the sensor/camera, exposure station and ground point. Thus the collinearity specifies that the exposure station, ground point and its corresponding image point location must all lie along a straight line in a 3-dimensional space, therefore this condition is defined by two equations (equations 1 and 2): one equation for the x coordinate, and another for the y coordinate, in the image plane, Wolf [12].

$$x_a = x_o - f \left[ \frac{m_{11}(X_A - X_C) + m_{12}(Y_A - Y_C) + m_{13}(Z_A - Z_C)}{m_{31}(X_A - X_C) + m_{32}(Y_A - Y_C) + m_{33}(Z_A - Z_C)} \right] \quad (1)$$

$$y_a = y_o - f \left[ \frac{m_{21}(X_A - X_C) + m_{22}(Y_A - Y_C) + m_{23}(Z_A - Z_C)}{m_{31}(X_A - X_C) + m_{32}(Y_A - Y_C) + m_{33}(Z_A - Z_C)} \right] \quad (2)$$

Where m's are the functions of the rotation angles omega, phi, and kappa to the object space coordinate system; and f is the focal length of the camera.  $x_a, y_a$  are the photo coordinates of the image point; and  $x0$  and  $y0$  are the photo coordinates of the principal point that usually is known from camera calibration;  $X_A, Y_A, Z_A$  are object space coordinates of the object point; and  $X_C, Y_C, Z_C$  are object space coordinates of the exposure station.  $m_{11}-m_{33}$  are the coefficients of the orthogonal transformation between the image plane orientation and object space orientation. The rotation angles omega, phi, and kappa define a sequence of three elementary rotations around the x, y, and z axes, respectively, where the xyz coordinate system is parallel to the XYZ object system and is constructed with its origin at the exposure station.

The coplanarity condition holds that the two exposure stations, the object, and its image points in two image planes are in the epipolar plane. Because the expression of the coplanarity equation is difficult to calculate, it is not used as widely as the collinearity condition in analytical photogrammetry.

## V. CONCLUSION

Theoretical and practical studies (Ackermann, 1966, 1974 and Brown, 1979) showed that only planimetric points along the perimeter of the block and relatively dense chains of vertical points across the block are necessary to relate the image coordinate system to the object coordinate system. These measures also ensure the geometric stability of the block as well as control the error propagation.

The mathematical concepts covered in this paper are:

- Collinearity condition equations (derivation and linearization)
- Ground control for Aerial photogrammetry
- Bundle adjustment (adjusting all photogrammetric measurements to ground control values in a single solution)- conventional and RPC based

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### Authors:



**Prof Santhi Swarup Manugula** has B.Tech Civil Engineering., M.Tech Remote Sensing and pursuing Ph.D. in Civil Engineering, Integrating Remote Sensing Applications in Civil Engineering. He has 16 years of experience (Civil Engg, GIS-Photogrammetry- Remote Sensing) worked with National & International Clients in various MNC. He worked as a Dy. General Manager & Head of GIS department and holds the credit of gaining global exposure by visiting Abu-Dhabi (UAE) as a part to support client international project work.



**Dr. Saroj Patel** has more than 10 years of rich experience in teaching and research field. She is currently working as a Associate Professor in Jodhpur National University, Jodhpur, Rajasthan in the department of Mathematics



**Dr. Veeranna Bommakanti** has B.Tech Civil Engineering., M.Tech Remote Sensing and Ph.D. in Spatial Information Technology. He presented more than five papers in International Conferences/ journals out of which one paper was presented in 29th Annual ESRI International User Conference, San Diego, CA, USA, and July 13 – 17, 2009. With more than 25 years of experience, currently he is working as a Professor & Dean in the department of Civil Engineering in GNITC Hyderabad involving in various academic and research activities