International Journal of Engineering and Technical Research (IJETR)
ISSN: 2321-0869, Volume-2, Issue-9, September 2014

Size Effect on Flexural Behaviour of Reinforced High Strength Concrete Beams

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Abstract— **The mechanical properties of concrete, which is a quasi brittle material, are size dependent. This paper investigates the size effect on flexural behaviour of reinforced HSC beams under one dimensional similarity with varying depths. In the experimental investigation, flexural behaviour of 3 series of beams with size 100x100x1000 mm, 100x150x1000 mm and 100x200x1000 mm with 1.5% tensile reinforcement were investigated under two point loading. The load deflection characteristics and the cracking behaviour were investigated. The study revealed a significant size effect on the flexural behaviour of reinforced HSC beams.**

Index Terms— **High strength concrete, Size effect, Ductility, Bazant law**

I. INTRODUCTION

Construction industry demands concretes with excellent rheological characteristics, improved hardened properties, enhanced durability and reduced consumption of raw materials. The attempts for achieving these goals resulted in a variety of special concretes such as High Strength Concrete (HSC), High Performance Concrete (HPC) and Fibre Reinforced Concrete (FRC) [1]. HSC is required in engineering projects having concrete components (such as columns, shear walls, and foundations) that must resist high compressive loads. HSC permits reinforced or prestressed concrete girders to span greater lengths than normal strength concrete girders. HSC is also applied to bridges and offshore structures and a variety of other applications such as harbour and coastal structures, hydraulic structures, underground construction, pavement and industrial floors, water treatment plants, storage facilities for aggressive waste and chemicals etc.

The mechanical properties of cement-based materials, which are quasibrittle in nature, are size dependent [2, 3, and 4]. This phenomenon is called size effect and stems from the nonlinear fracture in concrete. Since the mechanical properties of large structural members might be different with those obtained using the small laboratory specimens, the reported higher strength performance of HSC cannot be directly used in the design of structural members. Understanding the size dependent strength performance of HSCs would thus greatly help in successful application of HSCs in structural members.

The size effect phenomenon in quasi-brittle structures is related to a transition from a ductile behaviour of small specimens to a totally brittle response of large ones. Thus, the nominal strength at failure (σ_N) decreases with increasing characteristic specimen dimension, D [2]. The proper explanation of this size effect has been shown to lie in the release of strain energy due to fracture growth, producing damage localization instabilities. Prior to failure, distributed damage, consisting principally of microcracking, localizes into a narrow fracture process zone, which ultimately becomes the final, major crack. The localization is driven by the release of stored strain energy from the structure. In a larger structure, the strain energy is released from a larger zone, and so the total amount that would be released for a unit crack advance would be larger if the nominal stress were the same. However, because the energy required to produce unit fracture extension is approximately independent of the structure size, the nominal stress at failure of a larger structure must be lower, so that the energy release would exactly match the energy required for the fracture formation [3].

Size effect in reinforced concrete beams could be described by the analytical deterministic (energetic) size effect law (SEL) of Type II by Bazant [2], being valid for structures of a positive similar geometry possessing large stress-free cracks that grow in a stable manner up to the maximum load.

$$\sigma_N = Bf_t\,(1 + D/D_0)^{-1/2} \tag{1}$$

Where σ_N is the nominal strength, D is the characteristic dimension of the structural element and B and D_0 are the empirical parameters depending on material properties, structure geometry and structure shape. The parameter D_0 separates the ductile failure ($D_0 \gg D$) from the brittle one ($D_0 \ll D$).

Reported studies are available on the size effect on bending failure of notched concrete beams, with and without steel fibres [5] (Sener et al. (2002)), reinforced concrete members [6] (Yi et al. (2007)), overreinforced concrete beams [7] (Belgin et al. (2008)) and Ultra High Performance Fibre Reinforced Concrete Beams [8 and 9] (Mahmud et al. (2013)) and Nguyen et al. (2013). These studies revealed a significant size effect on the flexural behaviour of beams made up of different types of concretes.

While the size effect on normal and special concretes in various modes of failures has been intensively investigated by many researchers, only a few studies are reported regarding the size effect on flexural behaviour of reinforced high strength concrete beams. It is worth investigating the size effect on the flexural behaviour of high strength concrete beams.

Manuscript received September 20, 2014.

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Although the superior properties of HSC are very attractive to structural engineers, the material properties of concrete used in various sizes might be different due to size effect. Hence, an understanding of the size dependent mechanical performance of HSC is expected to help the prediction of their performance in large structural member. Very few studies were reported on the size effect on flexural properties of reinforced concrete beams using high strength concrete members. The objective of the present study is to investigate the effect of specimen size on the flexural behaviour of reinforced high strength concrete beams with varying depths.

In this study, based on the theoretical aspects of size effect reported in the literature, a comprehensive experimental program was conducted to investigate the size effects on flexural failure of reinforced high strength concrete beam in one dimensional similarity with varying depths, keeping width and length constant.

II. MATERIALS AND METHODS

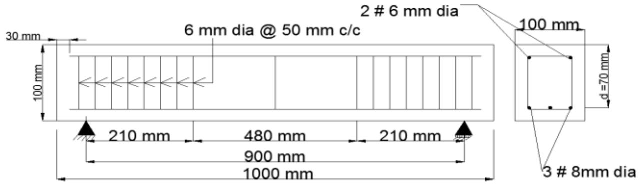
High strength concrete with compressive strength of 60 MPa was used for the study. The maximum aggregate size used was 20 mm. The proportion of mix used is given in Table I. High Yield Strength Deformed bars were used as reinforcing steel.

Table I Mix proportion

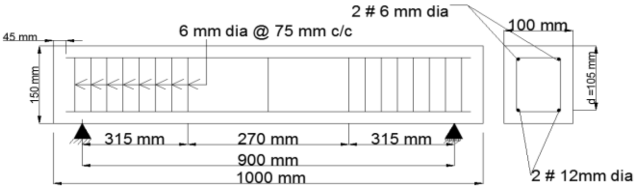
Particulars	Quantity (kg/m^3)
Cement	480
Fine Aggregate	745
Coarse Aggregate	1163
Water	135
Superplasticizer	3.84

A. *Specimen Details*

The depth of the beams were varied (100 mm, 150 mm and 200 mm) keeping the width and length as constant. The shear span to effective depth ratio was kept constant (a/d=3). Shear reinforcement is used only at the two ends of the specimen to eliminate shear failure at the two end sections. The effective cover (30 mm, 45 mm and 60 mm), diameter of tensile reinforcement (8 mm, 12 mm and16 mm) and spacing of vertical stirrups (50 mm, 75 mm and 100 mm) were all proportioned with respect to the total depth of the beams. Beams were reinforced with 1.5% steel. The dimensional and reinforcement detailing of the three sizes of beam specimens (D100, D150 and D200) are shown in Fig. 1(a-c).



(a)



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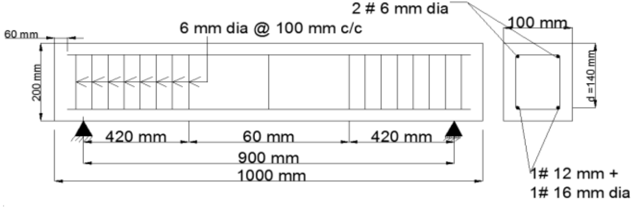

(c)

Fig. 1 Details of Specimens (a) D100, (b) D150 and (c) D200

The beams were tested under two point bending. The loads were applied using a hydraulic jack of capacity 50 tonnes. The applied loads were measured using a load cell, with digital indicator, of capacity 50 tonnes. Prior to testing, all specimens were carefully aligned to minimize the effect of eccentricity on the obtained flexural response of the specimens. Vertical deflections at mid span length of the beams were measured using mechanical dial gauges (Fig. 2). The curvature of the beams was measured using two LVDTs attached at a specific distance apart to the side face of the beams at mid span (Fig. 3). Strain in tensile reinforcement was measured using embedded strain gauges of gauge length 5 mm attached to the reinforcing bars at mid span. 10 mm strain gauges were attached to the concrete surface at top most compression fibre at mid span region to measure the maximum compressive strain in concrete.

Fig. 2 Typical test setup – Beam specimen with surface strain gauges

Fig. 3 Attachment of LVDTs to the top and bottom of beam specimen

III. RESULTS AND DISCUSSION

A. *Load Deflection Behaviour*

From the study on load deflection behaviour it is seen that the beam D100 showed a ductile post-peak behaviour before failure compared to D150 and D200. It is also noticed that the deflection of beams at failure decreased as the depth of the beams increased. The ultimate deflection decreased to 41.46% and 53.79% when the depth is increased by 50% and 100%. This indicates that rotational capacity and ductility of decreased with increase in depth. Furthermore the nature of failure changed from ductile to brittle as the depth of the beams increased.

The slope of the load deflection graph, an indication of the stiffness of the member, increases with increase in depth. From Fig. 4 it is seen that beam D200 was stiffer than D150 and D100 and the member stiffness decreased with decrease in depth. A similar behaviour was reported by Belgin et al., 2008 while studying the size effect of overreinforced beams [7].

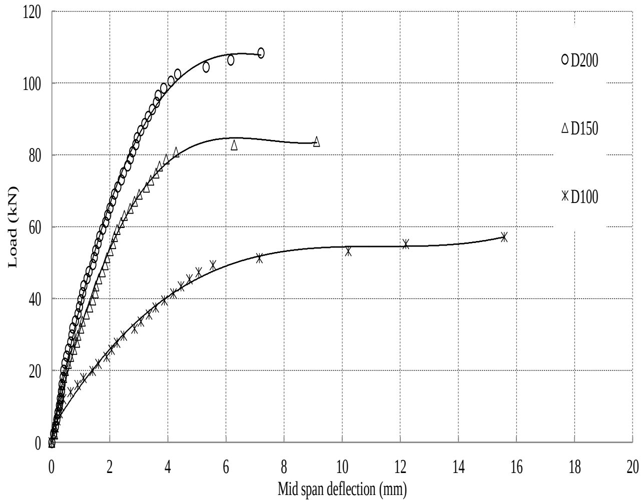

Fig. 4 Load-deflection behaviour

The average values of first crack load, yield load and ultimate load for the three series of beams are tabulated in Table II. The deflection ductility of the beams were calculated as the ratio between mid span deflections corresponding to ultimate loading and steel yielding and the obtained values were 2.807, 2.315 and 1.663 for D100, D150 and D200 respectively. This also infers decreased ductility with increase in depth of beams.

To investigate the service load behaviour with respect to deflection, maximum (mid span) deflections δs at service load (experimental ultimate load Pu divided by a factor of 1.7) was also considered. δs is 3.07 mm for D100, 1.8 mm for D150 and 1.92 mm for D200. The deflection at service load is much higher for D100 when compared to that for D150 and D200. Further, it is also found that the energy absorption capacity decreases with increase in depth of members.

Table II Variation of ductility with depth

Beam		D100	D150	D200
Load (kN)	Cracking	12.022	17.658	22.082
	Yielding	49.300	78.855	102.524
	Ultimate	57.148	83.760	108.410
Deflection (mm)	Cracking	0.390	0.380	0.450
	Yielding	5.550	3.940	4.330
	Ultimate	15.58	9.120	7.200
Deflection ductility		2.807	2.315	1.663
Percentage decrease in deflection ductility		-	17.544	40.766
Energy absorption capacity (kNm)		0.706	0.619	0.579
Percentage decrease in energy absorption capacity		-	12.299	17.989

B. *Moment Curvature Relation*

The moment curvature diagram for the beams are shown in Fig. 5. The gradient of moment curvature curve is an indication of beam stiffness and from Fig. 5 it is seen that stiffness of beam increases with size. It is also observed that the ultimate curvature decreased with increase in beam size. Curvature ductility which is defined as the ratio between ultimate curvature and curvature at yielding also showed a decreasing trend with depth. The beams D150 and D200 exhibited 21.8% and 38.8% reduction in curvature ductility compared to Beam D100.

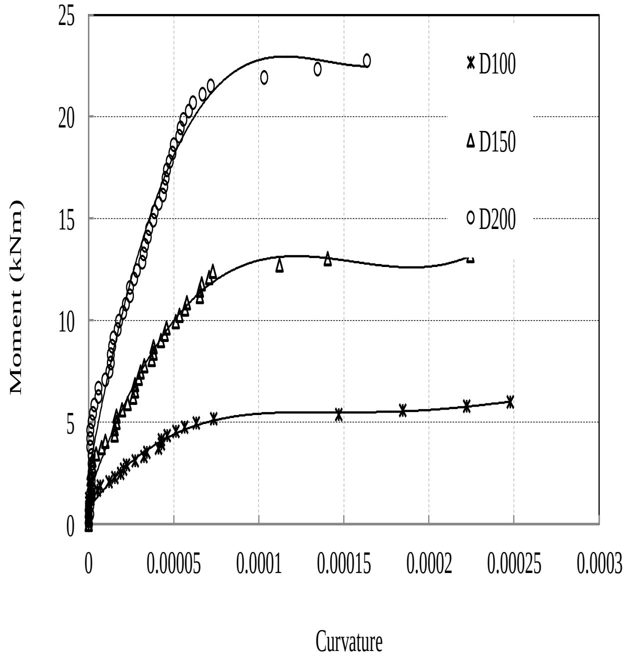

Fig. 5 Moment-curvature diagram

C. *Stress Strain Behaviour*

Fig. 6 shows the variation of strain in tensile reinforcement with the stress developed.

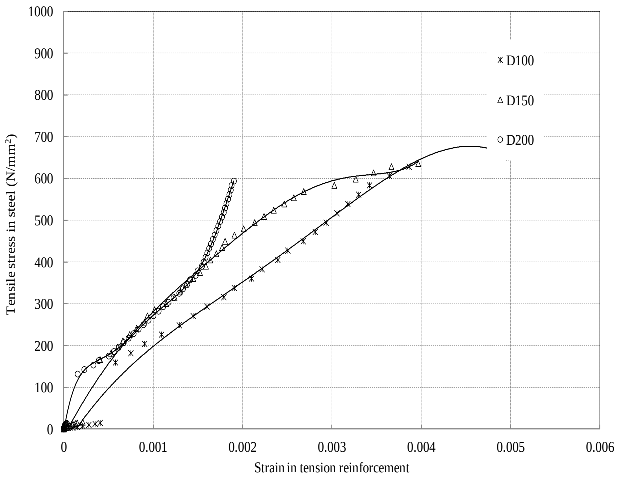

Fig. 6 Stress in steel reinforcement versus strain in them

Up to the moment of cracking, the tension developed in the beams was carried by both the concrete and steel. But after cracking, all the tension was carried by the reinforcement alone resulting in substantial increase in stress accompanied by a relatively small increase in strain. The stresses and strains at ultimate loads were more for beam D100 and it decreased with increase in depth of beams. This also indicates that the strength and ductility of the beam specimens decreased with increase in depth. It is observed that there was no yielding of tensile reinforcement in beam D200 before failure. That is, the failure could be interpreted as a purely brittle one. This behaviour also confirmed the quasibrittle nature of concrete and increase in failure brittleness with increasing size.

The variation of strain in extreme compression fibre with the stress developed due to applied loading is shown in Fig. 7. It was observed that the ultimate strain in extreme compression fibre is more for beam D200 and decreased with decrease in depth.

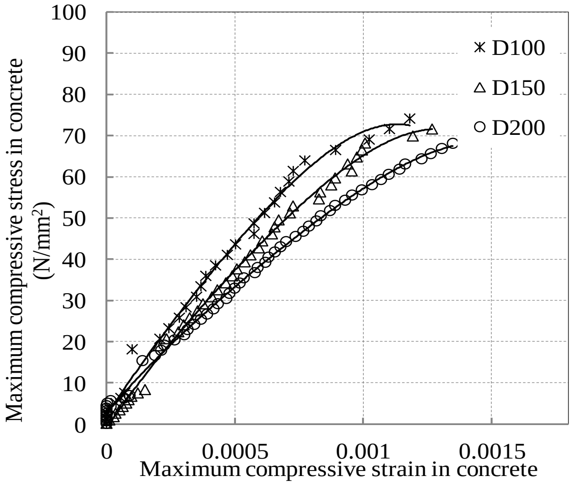

Fig. 7 Maximum compressive stress in concrete versus strain in extreme compression fibre

D. *Failure Pattern*

The average values of ultimate load, corresponding nominal flexural strength, ultimate curvature and maximum tensile and compressive strains of the three beams were as given in Table III.

Table III Behaviour of beams under ultimate load

Beam Identification	D100	D150	D200
Ultimate Load (kN)	57.148	83.760	108.410
Nominal flexural compressive strength (N/mm^2)	73.476	71.794	69.692
Ultimate curvature (Radians)	0.00024	0.00022	0.00016
Maximum tensile strain	0.00498	0.00396	0.00190
Maximum compressive strain	0.00118	0.00127	0.00135

While the load resistance of larger specimen was higher than that of smaller ones, the nominal flexural strength of smaller specimen was higher than that of larger one. The decrease in ultimate curvature and ultimate strain in tension reinforcement with increasing depth of beams indicates an increase in failure brittleness as the size of beams increases. However it is also noticed that the ultimate strain in extreme compression fibres increase with increase in beam depth. It may be due to the higher compressive force acting on the beam specimen.

All beams failed due to the formation of flexural crack originated from the extreme tension fibres in the flexure region as shown in Figs. 8-10.

Fig. 8 Typical failure pattern of beam specimen D100

Fig. 9 Typical failure pattern of beam specimen D150

Fig. 10 Typical failure pattern of beam specimen D200

During testing, concrete showed more brittle failure mechanism as the specimen size increased. This is related to quasibrittle characteristics of concrete materials, energy release rate accumulated in the experimental device, and stiffness of the testing device. The relative number of cracks increases with a decreasing specimen size. For the small size specimen, since the formation of micro-cracks is evenly distributed in a comparatively wide region, it can be noted that size effect is not apparent [6].

E. Bazant size effect parameters B and D_0

The parameters of Bazant's theory, B and D_0, indicate the sensitivity of the failure strength to the size of the specimen. In this study, the characteristic dimension D of Bazant law was taken as the effective depth d of the beam specimens. To estimate the values of B and D_0, a procedure analogous to that described in the RILEM recommendation was adopted. A linear regression can be achieved by rearranging (1) as;

$$Y=AX+C \qquad (2)$$

in which

$$Y=(ft/\sigma_N)^2$$

$$X=d$$

$$A=C/D_0$$

$$C=B^{-2}$$

Since σ_N and f_t are known for various values of d, a plot of $(ft/\sigma_N)^2$ against d allows A and C to be determined from a regression analysis of the test results. Fig. 11 shows the plot of $(ft/\sigma_N)^2$ against d for the results obtained in this study.

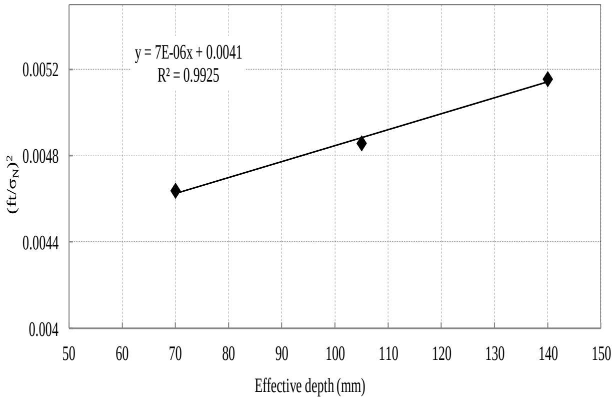

Fig. 11 Determination of size effect parameters

From Fig. 11 the value of B and D_0 were obtained as 15.62 and 585.71 respectively and the slope of the curve (α) between log (σ_N/Bf_t) and log (d/D_0) was obtained as 4.54. In a study by Belgin et al (2008), B, and α values of 12.4 and 4.2 were obtained for over reinforced normal strength concrete beams in one dimensional similarity and the D/D_0 value varied between 0.16 and 1.2. The size effect curve for under reinforced high strength concrete beams obtained in this study is shown in Fig. 12. The dotted horizontal line in this figure represents the strength criterion in which there is no size effect. The inclined line is the size effect curve for reinforced HSC beams which is a transition between strength criterion and LEFM. The size effect achieves a LEFM trend when the effective depth is more than 585.714 mm (D_0).

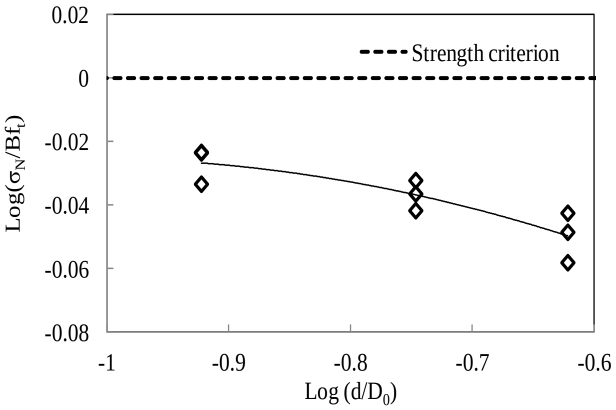

Fig. 12 General graph of size effect law of reinforced HSC beams

Using the values of B and D_0, (1) can be rewritten as:

$$\sigma_N = 15.6174 f_t (1+ D/585.71)^{-1/2} \qquad (3)$$

The ultimate flexural compressive strength values for the beams were calculated using (3) and is given in Table IV. The average value for the ratio between predicted and experimental results was obtained as 1.0048.

Table 4 Actual and predicted values for ultimate flexural compressive strength of reinforced HSC beams

Beam Identification	Actual Flexural Strength (N/mm²)	Flexural strength by size effect equation (N/mm²)	Predicted strength/ actual strength
B1	73.476	73.860	1.0052
B6	71.794	71.964	1.0024
B7	69.692	70.207	1.0074

IV. CONCLUSION

Reinforced High Strength Concrete beams with varying depths were tested under two point loading. From the test results the following conclusions were derived.

1. Size effect is evident in flexural behaviour of reinforced high strength concrete beams with varying depths subjected to two point loading.
2. Size effect is evident in the load-deflection relationship. The energy absorption capacity is more for the smallest beams and is decreased with increase in size of members. The energy absorption capacity decreased to 12.3% when the effective depth is increased by 50% and it is decreased to 18% when the effective depth is increased by 100%.

3. Ultimate tensile strain decreased with increase in specimen size conforming an increase in failure brittleness with size.
4. The moment-curvature relationship exhibited a significant size effect. The curvature ductility and the ratio between the actual and theoretical moment carrying capacity decreased with increase in size. The curvature ductility decreased 22% to when the effective depth is increased by 50% and it is decreased to 39% when the effective depth is increased by 100%.
5. The transitional size between strength criterion and linear elastic fracture mechanics is obtained as 585.74 mm.
6. The values for the Bazant size effect parameters were proposed for the size effect in flexural failure of under reinforced HSC beams with varying depths.
7. The size effect existing in reinforced high strength concrete beams with one dimensional similarity is consistent with Bazant size effect law.

ACKNOWLEDGMENT

This research was financially supported by the Centre for Engineering Research and Development (CERD), CET campus, Thiruvananthapuram. The authors are grateful to CERD and Dr. K. Balan, Director, CERD for the financial support.

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