

# A Direct method for Combined Economic & Emission Dispatch (CEED)

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**Abstract**— Economic and Emission Dispatch (EED) represents one of the most important problem in Power system engineering. The major part of the power generation is due to fossil fired plants and their emission contribution cannot be neglected. This emission has to be reduced by minor change in the dispatch problem which results in increase in the fuel cost. There is a best compromise required in minimizing the emission and cost on pareto-optimum curve [6]. A direct method for cost optimization including losses was published by 2 authors in [8] & [7]. In this paper, direct method formulae taken from above 2 journals & described for emission and combined fuel cost and emission optimization for achieving the best compromise. This method needs the calculation of combined coefficients obtained by applying the concept of proposed total deviation approach. This paper presents derivation & best solution for combined optimization of emission and cost with and without transmission losses for optimal solutions. The effectiveness of the proposed method is demonstrated by considering IEEE 30 bus system. The results for direct methods have been compared. The results confirm the potential and effectiveness of direct method compare to conventional lambda iteration technique and also verified using load flow.

**Index Terms**— Combined Optimization, Cost & Emission co-efficient, Direct method, Economic and Emission Dispatch (EED).

## I. INTRODUCTION

Generally the coal used in thermal generation is of poor quality and high ash. Sulphur dioxide (SO<sub>2</sub>) and Oxides of Nitrogen (NO<sub>x</sub>) are the major emissions from thermal plants due to the combustion of coal which will cause ill effects on human beings as well as animals [3]. The NO<sub>x</sub> emission is required to be reduced by 2 million tons/year from 1980 level [4].

Minimizing operating cost can no longer be the only criterion for dispatching electric power due to increasing concern over the environmental consideration [5]. The quadratic fuel cost function is well accepted for optimization problem. The direct approach has been presented for optimum scheduling in terms of total system demand P<sub>D</sub>[1].  
Total fuel cost in \$/hr

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$$F_c = \sum_{i=1}^n F_n(P_n)$$

$F_n$ — Total fuel cost of n<sup>th</sup> generator in (\$/hr)

$P_n$ — generator power output of n<sup>th</sup> generator in (p.u)

$$X = \sum_{i=1}^n 1/2a_n$$

$$Y = \sum_{i=1}^n b_n/2a_n$$

$$Z = \sum_{i=1}^n (b_n^2/4a_n - c_n)$$

n- Corresponding generator (1,2,..N)

N- no. of generator.

$a_n, b_n, c_n$ — fuel cost coefficients

The closed form expression for calculation of optimum fuel cost has been given in terms of total system demand P<sub>D</sub>. All these closed form expression depends on the calculation of X, Y, Z calculated from the cost coefficients of the generation [2]. Optimum Generation Schedule

$$P_n^* = (P_D + Y - b_n X) / 2a_n \text{ (p.u)}$$

Optimum fuel cost

$$F_c^* = [(P_D + Y)^2 / 2X] - Z \text{ ($/hr)}$$

Co-ordination equation for determining the optimum generation schedule with transmission losses is

$$\frac{dF_n}{dP_n} + \lambda \frac{dP_L}{dP_n} = \lambda \tag{1}$$

Where,

$$P_L = \sum_m \sum_n P_m B_{mn} P_n$$

Subjected to,

$$\sum_{n=1}^N P_n - P_L - P_R = 0$$

$$\frac{dF_n}{dP_n} \cdot (1 + 2 \sum_{m=1}^N B_{mn} \cdot P_m)$$

& also Neglecting the off-diagonal elements of B<sub>mn</sub> assume the form,

$$A_n P_n^2 + B_n P_n + C_n = \lambda$$

Where,

$$A_n = 4a_n B_{nn}, \quad B_n = 2(a_n + b_n B_{nn}), \quad C_n = b_n$$

$A_n, B_n, C_n$  - functions of cost function coefficients and elements of '[B<sub>mn</sub>]', λ' obtained from,

$$\alpha \lambda^3 + \beta \lambda^2 + \gamma \lambda + \delta - P_R = 0$$

Where α,β,γ,δ are expressed as follows,

$$\alpha = \sum_{n=1}^N \left( \frac{2A_n^2}{B_n^2} + \frac{2A_n B_{nn}}{B_n^4} \right)$$

$$\beta = - \sum_{n=1}^N \left( \frac{6A_n^2 C_n}{B_n^5} + \frac{6A_n B_{nn} C_n}{B_n^4} + \frac{A_n}{B_n^3} + \frac{B_{nn}}{B_n^2} \right)$$

$$\gamma = \sum_{n=1}^N \left( \frac{6A_n^2 C_n^2}{B_n^5} + \frac{6A_n B_{nn} C_n^2}{B_n^4} + \frac{2A_n C_n}{B_n^3} + \frac{2B_{nn} C_n}{B_n^2} + \frac{1}{B_n} \right)$$

$$\delta = - \sum_{n=1}^N \left( \frac{2A_n^2 C_n^3}{B_n^5} + \frac{2A_n B_{nn} C_n^3}{B_n^4} + \frac{A_n C_n^2}{B_n^3} + \frac{B_{nn} C_n^2}{B_n^2} + \frac{C_n}{B_n} \right)$$

Optimal generation scheduling is calculated by

$$P_n = - \frac{B_n}{2A_n} + \frac{[B_n^2 + 4A_n(\lambda - C_n)]^{1/2}}{2A_n}$$

In [7], cubic should be solved.

In [8], there are no such assumptions to found the direct formulae. Ignoring the off-diagonal elements of „ $B_{mn}$ “ in equation (1) is equivalent to assuming „ $P_m=0$ “ for „ $m \neq n$ “. This introduces errors in computing „ $P_L$ “ and hence in „ $\lambda$ “. Also these errors can be minimized by assuming „ $P_m = P_n$ “, which modifies the values of [8]

$$B_{nn} = \sum_{m=1}^N B_{mnn}$$

For any values of „ $n$ “

$$\frac{dF_n}{dP_n} = 2a_n P_n + b_n$$

$$\frac{dP_L}{dP_n} = 2B_{nn} P_n$$

Substitute above equation in (1) & get,

$$\sum_{n=1}^N \frac{u(n) \cdot \lambda^2 + v(n) \cdot \lambda + w(n)}{(\lambda + Q(n))^2} - P_R = 0$$

Where

$$u(n) = \frac{1}{4 B_{nn}}$$

$$v(n) = \frac{a_n}{2 B_{nn}^2}$$

$$w(n) = - \frac{2 a_n b_n + B_{nn} b_n^2}{4 B_{nn}^2}$$

$$Q(n) = \frac{a_n}{B_{nn}}$$

Optimal generation scheduling is calculated by

$$P_n = \frac{\lambda - b_n}{2(a_n + B_{nn}\lambda)}$$

In [8], quadratic equation should be solved.

In this paper, [8] is taken to solve emission and combined fuel cost and emission optimization, since all are quadratic functions. The  $u, v, w, Q$  constants depends on the coefficients of respective quadratic functions and  $B_{nn}$ . The minimization of emission and cost optimization problem requires combined coefficients. These coefficients are obtained by applying the concept of proposed total deviation approach. This approach is the minimization of the sum of cost deviation from the optimum cost and emission deviation from the optimum emission. Transmission losses are included by using  $B_{mn}$  coefficients.

The effectiveness of the proposed direct approach in EED problem with & without transmission losses are demonstrated

in IEEE 30 bus system. The optimal solution results are compared with Direct method technique [7] and also with conventional lambda iteration technique.

## II. PROBLEM STATEMENT

Consider a system where ‘ $n$ ’ be the total number of generators. The EED problem is to minimize the two competitive function total fuel cost and emission while satisfying several equality and inequality constraints [4]. The problem is formulated as follows:

### A. Objective Function

*Minimization of cost:* For an ‘ $n$ ’ plant power system, the cost curves are expressed by a quadratic equation

$$F(P_n) = a_n P_n^2 + b_n P_n + c_n \text{ \$/hr ; } n=1, 2, \dots, N \quad (2)$$

Total fuel cost in \\$/hr

$$F_C = \sum_{i=1}^n F_n \quad (3)$$

Equation (3) To be minimized

Direct expression for optimum fuel cost [2]

$$F_C^* = [(P_D + Y)^2 / 2X] - Z \text{ (\$/hr)} \quad (4)$$

Optimum scheduling [1]

$$P_n^* = (P_D + Y - b_n X) / 2a_n \text{ (MW); } n=1, 2, \dots, N \quad (5)$$

*Minimization of emission:* Total emission of atmospheric pollutants such as Sulphur oxides (Sox) and Nitrogen oxides (NOx) caused by the fossil fired thermal generation expressed by a quadratic equation

$$F(P_n) = A_e P_n^2 + B_e P_n + C_e \text{ ton/hr ; } n=1, 2, \dots, N \quad (6)$$

Total fuel cost in ton/hr

$$F_e = \sum_{i=1}^n E_n \quad (7)$$

Equation (7) To be minimized

Direct expression for optimum emission

$$F_e^* = [(P_D + Y)^2 / 2X] - Z \text{ (ton/hr)} \quad (8)$$

Where X, Y, Z are calculated using A, B, C

Optimum scheduling [1]

$$P_{Gi}^* = (P_D + Y - B_e X) / 2A_e \text{ (MW); } i=1, 2, \dots, n \quad (9)$$

### B. Objective constraints

*Generation capacity constraint:* Real power output of each generator with in lower and upper limits expressed as

$$P_n^{\min} \leq P_n \leq P_n^{\max} ; n=1, 2, \dots, N$$

*Power balance constraint:* the sum of total system demand  $P_D$  and transmission losses  $P_L$  equals to total real power generation. Hence,

$$P_D = \sum_{n=1}^n P_n - P_L$$

### C. Problem Formulation(EED)

Multi-objective problem is formulated into single objective problem as

$$\begin{aligned} & \text{Minimize } [F_c, F_e] \\ & \text{Subject to: } P_n^{\min} \leq P_n \leq P_n^{\max}; n=1, 2, \dots, N \\ & P_D = \sum_{n=1}^N P_n - P_L \end{aligned}$$

$$\frac{dF_n}{dP_n} = \lambda$$

In this equation incremental transmission loss is changed at the equal incremental production cost value.

### III. PRINCIPLE OF COMBINED OPTIMIZATION

Multi objective problem is formulated into single objective problem in terms of percentage deviation, obtained from the individual optimum solution  $F_c^*$ ,  $F_e^*$ .

- Cost deviation function obtained from the individual optimum cost in percentage :  

$$D_c = \frac{F_c - F_c^*}{F_c^*} * 100 = \left( \frac{F_c}{F_c^*} - 1 \right) * 100 \quad (10)$$

- Emission deviation function obtained from the individual optimum emission in percentage:

$$D_e = \frac{F_e - F_e^*}{F_e^*} * 100 = \left( \frac{F_e}{F_e^*} - 1 \right) * 100 \quad (11)$$

Total deviation function:

$$T_d = D_c + D_e \quad (12)$$

From the equations (1), (5), (9) & (10) in (11).

$$\begin{aligned} & = \frac{\sum_{n=1}^N F(P_n) - F(P_n)^*}{F(P_n)^*} + \frac{\sum_{n=1}^N E(P_n) - E(P_n)^*}{E(P_n)^*} \\ & = \left\{ \frac{\sum_{n=1}^N F(P_n)}{F(P_n)^*} - 1 \right\} + \left\{ \frac{\sum_{n=1}^N E(P_n)}{E(P_n)^*} - 1 \right\} \\ & = \left\{ \frac{\sum_{n=1}^N F(P_n)}{F(P_n)^*} \right\} + \left\{ \frac{\sum_{n=1}^N E(P_n)}{E(P_n)^*} \right\} - 2 \\ & = \left\{ \frac{\sum_{n=1}^N (a_n P_n^2 + b_n P_n + c_n)}{F(P_n)^*} \right\} + \left\{ \frac{\sum_{n=1}^N (A_n P_n^2 + B_n P_n + C_n)}{E(P_n)^*} \right\} - 2 \\ & = \sum_{n=1}^N \left\{ \frac{a_n P_n^2}{F(P_n)^*} + \frac{b_n P_n}{F(P_n)^*} + \frac{c_n}{F(P_n)^*} \right\} + \sum_{n=1}^N \left\{ \frac{A_n P_n^2}{E(P_n)^*} + \frac{B_n P_n}{E(P_n)^*} + \frac{C_n}{E(P_n)^*} \right\} - 2 \\ & = \sum_{n=1}^N \left\{ P_n^2 \left[ \frac{a_n}{F(P_n)^*} + \frac{A_n}{E(P_n)^*} \right] + P_n \left[ \frac{b_n}{F(P_n)^*} + \frac{B_n}{E(P_n)^*} \right] + \left[ \frac{c_n}{F(P_n)^*} + \frac{C_n}{E(P_n)^*} \right] - 2 \right\} \\ & = \alpha_n P_n^2 + \beta_n P_n + \gamma_n \quad (13) \end{aligned}$$

Total deviation function coefficients are,

$$\alpha_n = \frac{a_n}{F_c^*} + \frac{A_n}{F_e^*} \quad (14)$$

$$\beta_n = \frac{b_n}{F_c^*} + \frac{B_n}{F_e^*} \quad (15)$$

$$\gamma_n = \frac{c_n}{F_c^*} + \frac{C_n}{F_e^*} - 2 \quad (16)$$

The transmission losses are included by 'Bmn' coefficients.

### IV. PROPOSED DIRECT METHOD

The closed form expressions depends on the total system demand  $P_D$  are used in all the three cases of optimization (i)Cost (ii)Emission and (iii)Combined cost and emission. The approach is described in a common flowchart fig.1.

i. Suboptimal solution:

Co-ordination equation is taken as

### V. RESULTS

#### A. Direct method results

In order to validate the proposed method, the emission and economic dispatch problem was solved for IEEE 30 bus system shown in fig.3 and results are presented in this section. There are 6-Generators. The total system demand is 283.4MW. The values of fuel cost coefficients and emission coefficients are given in Table I. MATLAB program was developed to perform economic and emission dispatch (EED) problem.

Results are obtained for Cost, Emission and Combined fuel cost and emission optimization for both with and without losses for optimal and suboptimal solutions. Results obtained demonstrate its effectiveness in finding optimal solution and compared with conventional lambda iteration technique. The computational result of the proposed method for the EED problem, where as total fuel cost is minimized as shown in Table II. From Table.II it is clear that fuel cost is minimized in EED problem and the corresponding emission is higher. Results of best emission are shown in Table III. Here the emission is minimized and the total fuel cost is higher.

The fuel cost and emission are compromised in the optimization of total deviation function. Results of best combined fuel cost and emission is shown in Table IV.

Graph 1. Clearly shows that, when fuel cost is minimized and emission is higher. When emission is minimized and fuel cost production is higher. In the combined optimization both fuel cost and emission are compromised. It gives a best solution in the pareto-optimum curve.

From the best solution, if the fuel cost deviation is reduced, the deviation of emission will increase and vice versa. This optimal solution is rechecked by substituting generator power in load flow and from the result, it is found that same transmission loss is obtained.

#### B. Comparison of Optimal solutions

##### Case 1: EED by lambda iteration technique

The total generation costs and emission of case1 are shown in Table V. The results show that the lambda iteration technique consumes least computing time when compared to other methods in terms no. of iteration is high and time taken also high. The total fuel cost calculated by lambda iteration method for demand of 2.834p.u is 616.64\$/hr.

##### Case 2: EED by Direct method [7]

The total generation costs and emission of case 2 are shown in Table V. in this method there several approximations made in co-ordination equation and non-diagonal elements get eliminated. So the total fuel cost gets deviated. The total fuel cost calculated by direct method [7] method for demand of 2.834p.u is 628.819\$/hr.

##### Case 3: EED by Direct method [8]

The total generation costs and emission of case 3 are shown in Table V. in this method there is no such approximation made. Non diagonal terms also included. The total fuel cost obtained by this method is very nearly equal to lambda



iterative technique. The total fuel cost calculated by direct method [8] method for demand of 2.834p.u is 615.186 \$/hr

This method results is verified by the optimum generator power ( $P_G$ ) is substitute into load flow get the same loss which is equal to loss obtained by this proposed method.

The results obtained by the three methods for a 30 bus system are compared and it is shown as Graph 2. It shows the merit of direct method. The fuel cost of direct method [7] is equal to lambda iteration technique. Computation time is also very less in the direct method. The Graph 3 shows the comparison of optimal and suboptimal solutions for combined fuel cost and emission optimization. It clearly shows the merits of optimal solutions. The sub optimal fuel cost is nearly equal to optimal solutions. The percentage deviation between optimal & sub optimal is only 0.06899%.

## VI. CONCLUSION

This paper presents the direct approach for combined emission and cost optimization for thermal power plants. The proposed method derived from basic work presented for obtaining closed form expression (in term of  $P_D$ ) [8]. In order to convert the multi-objective (EED problem) into single objective problem, Total deviation approach is applied. The total deviation is the sum of cost deviation and emission deviation. This approach helps for arriving the best balanced compromised optimization solution for any multi-objective problem. Here optimal and suboptimal solutions are compared. Transmission losses are included. In optimal solutions losses are also included by direct approach. In order to include the transmission losses in sub optimal solutions, Two-Fold method is applied for feasible operating point of the system in EED. Combined optimization gives best solution by the proposed approach. The following conclusions are drawn from the proposed work,

1. Shouted for all Real time application.
2. Optimal and suboptimal solutions are not having much difference (ie., 5% when network loss is less than 25%).
3. No iterative technique.
4. Using this method Multi objective is converted into single objective problem.
5. Easiest technique to find the best feasible global optimum.
6. Transmission losses are included.
7. The solution of combined cost and emission optimization is the best compromised. The solution is arrived by using the closed form expressions.
8. Computational time is very less, since the closed form expressions are used for obtaining the solution.

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