Five-axis Machining Cutter Axis Vector Optimization Based on Quaternion

Kai Zhang, Liqiang Zhang

***Abstract*—Quaternion has many excellent features to describe rotational motion of rigid body, so it can be used to describe the process of tool axis vector. By making unit tool axis attitude points correspond with the quaternion spatial points, the optimization of two axis vector can be converted into the optimization of the quaternion spots in quaternion space. In quaternion space, through the use of B-spline curve and global interpolation method, the quaternion spots can be interpolated and optimized. In this way, it is able to achieve that fit tool axis vector, optimize machining process of tool axis trajectory, and make the cutter path smoother.**

***Index Terms*— Quaternion, B-spline curve, global interpolation method, cutter axis vector, smoother**

I. INTRODUCTION

Be different from three axis NC machining, cutter axis vector can be changed flexibly in five axis machining, this difference is caused by the two axes of rotation. Because of the two axis of rotation, the five axis CNC machine can control tools to finish works at arbitrary attitude, especially in the complex curved surface machining at high speed and precision, have demonstrated irreplaceable advantages. However, if two rotating shaft became more flexibility, it also increase the complexity of the structure of the machine tool, make it difficult to visualize the actual processing of the machine tool movement , and increase the difficulty to control the tool posture vector. In addition, same with the feeding speed, abilities of tool axis rotation also have limitation.

The sudden changes of the tool axis vector are easy to cause the vibration of the machine, result in the declining in the quality of processing. Therefore, in five axis NC machining process, the tool axis vector should be avoided having sudden changes, which include its rotation direction and rotation speed.

In process of manufacturing, feed speed and rotational speed are fitted on each other. When tool axis vector rotation is large between two adjacent cutter contacts, in order to meet the requirement of the rotating speed, the feed speed should be reduced to relatively lower. It likes that the cutter is staying in one position for a long time, and leaving cut pit more obvious in the part surface, affecting the quality of parts. So, it is necessary to smooth the tool axis vector.

Manuscript received Aug 17, 2014.

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Literature [2] was first used C-space to search the interference free tool axis vector of each cutter location, and in each cutter axis vector C-space range, calculate the global optimal cutter axis vector according to the principle of minimum rotation of adjacent cutter. But the problem is that, on the one hand, the method have too much iterative, result in serious decline of efficiency, and on the other hand, the method still cannot guarantee a smooth transition between adjacent cutter axis vectors. The literature on [3] and [4] respectively used the method of quaternion linear interpolation to smooth cutter axis vector between two key cutter shafts. All these methods are just applicable to a smooth transition between the two key tool axis vectors, and cannot handle the transition between multiple key tool axis vectors. The literature on [5] research literature only one cutter axis vector, without considering the problem of smooth transition of global tool axis vector.

In the actual machining, CNC system always based on CAM system to generate the tool posture vector, and the tool vector points can be regarded as unit vector points. Fig.1 shows the process of tool axis vector optimization.

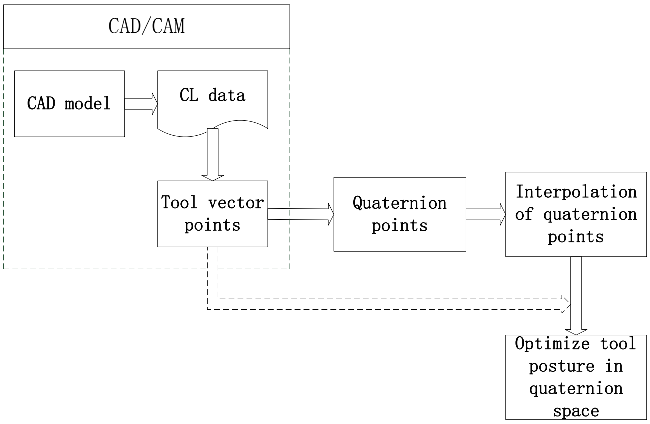


Fig.1 Flowchart of the tool axis vector optimization based on quaternion

II. QUATERNING BACKGROUND

The so-called quaternion is defined by a number of unit 1 and three imaginary unit i, j and k. The general form is $\Lambda = (\lambda_0, \lambda_1, \lambda_2, \lambda_3) = \lambda_0 + \lambda_1 i + \lambda_2 j + \lambda_3 k$, and the unit 1, i, j, k can be regarded as the unit total in four-dimensional space vector. Any of the quaternion numbers can be represented by points or vectors in space. Quaternion could be well used to express rotation, and provides a very convenient way for the study of rigid body motion. Quaternion operation have many properties, and the properties of the operations were used in this article are:① When a quaternion number is multiplied by a scalar, all elements should be multiplied by this scalar, in particular, the negative of quaternion number is $-\Lambda = -\lambda_0 - \lambda_1 i - \lambda_2 j - \lambda_3 k$.② The quaternion element of product is still quaternion number, the algorithm is that $i \circ i = -1$, $j \circ j = -1$, $k \circ k = -1$, $i \circ j = -j \circ i = k$, $k \circ i = -i \circ k = j$, $j \circ k = -k \circ j = i$.

III. QUATERNION OPTIMIZATION TOOL POSTURE

Quaternion provides a very convenient way for the study of rigid body motion, because of the quaternion, the rigid body movements can be described conveniently and intuitively. For a series of tool posture vector are generated by CAM such as $p_1, p_2, \ldots, p_m$,and they can be expressed by quaternion as $p_i = Q_i p_1 Q_i^{-1}$ （ i =1,2 ， ... ， m ） .... (1)

Here, $Q_i = \lambda_0 + \lambda_1 i + \lambda_2 j + \lambda_3 k$, p_1 =i, and Q_i =E.

Given $p_i = p_{i1} i + p_{i2} j + p_{i3} k$, and by compared with (1), the equations can be obtained as:

$$\begin{cases} \lambda_0^2 + \lambda_1^2 - \lambda_2^2 - \lambda_3^2 = p_{i1} \\ 2(\lambda_0\lambda_3 + \lambda_1\lambda_2) = p_{i2} \\ 2(\lambda_1\lambda_3 + \lambda_0\lambda_2) = p_{i3} \end{cases} \quad \text{.. (2)}$$

By the formula (2), the expression of quaternion points Q_i will be expressed as follows:

$$Q_i = \pm\frac{1}{\sqrt{2}}(1+\omega_i)(\cos 2\theta_i + \sin 2\theta_i) \times (i + \frac{p_{i2}}{1+p_{i1}} j + \frac{p_{i3}}{1+p_{i1}} k)$$

.... (3)

Here θ_i are angles around the axis of rotation. Since formula (3) it can obtain a series of quaternion points: Q_1 , Q_2 , Q_3 ,...., Q_m . As shown in Fig.2, the rotation of tool posture points is described in quaternion space as follows:

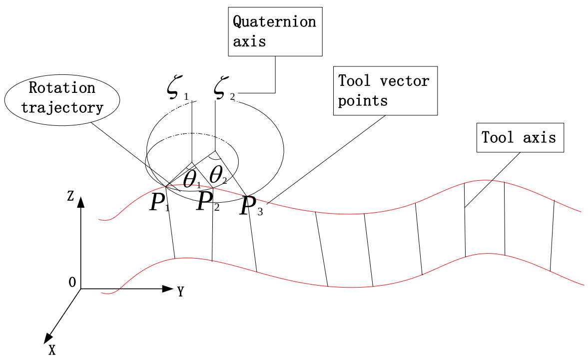


Fig. 2 The tool posture points rotation in quaternion space

IV. INTERPOLATE THE QUATERNION POINT BY B-SPLINE

Now it is going to use the global interpolation method to interpolate quaternion points Q_1 , Q_2 , Q_3 ,...., Q_m and get the quaternion curve $Q_i = \sum_{i=1}^{m} B_{i,p}(v_i) q_i$, here q_i are unknown control points.

First, it can get v_i from $v_i = v_{i-1} + \frac{|Q_i - Q_{i-1}|}{d}$, where $d = \sum_{i=1}^{m} |Q_i - Q_{i-1}|$, i=（2,3，...，m-1）, then according to: $\begin{cases} u_0 = \ldots = u_p = 0, u_{t-p} = \ldots = u_t = 1 \\ u_{j+p} = \frac{1}{p}\sum_{i=j}^{j+p-1} v_i, j = 1,2,\ldots,m-p \end{cases}$, the node of vector U={ u_0 , u_1 ,......, u_t }will be obtained. According to

$$\begin{cases} B_{i,p}(v_i) = \begin{cases} 1, u_i \le v_i \le u_{i+1} \\ 0, otherwise \end{cases} \\ B_{i,p}(v_i) = \frac{v_i - u_i}{u_{i+p} - u_i} \cdot B_{i,p-1}(v_i) + \frac{u_{i-p+1} - v_i}{u_{i+p+1} - u_{i+1}} \cdot B_{i+1,p-1}(v_i) \end{cases}$$

the matrix equations can be expressed as : $B_{i,p}(v_i) \cdot q_i = Q_i$. Then, control points q_i can be obtained and B-spline curves of quaternion be expressed as: $Q_i = \sum_{i=1}^{m} B_{i,p}(v_i) q_i$. As shown in Fig.5, the quaternion points Q_i are fitted by B-spline curve:

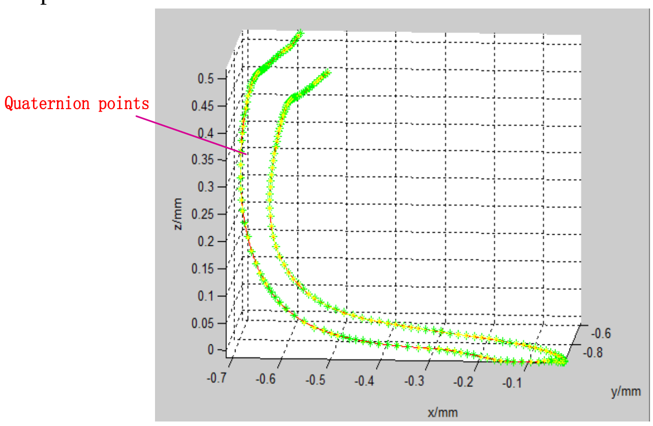


Fig.3 B-spline fitting quaternion points

V. FITTING TOOL POSTURE VECTOR

When get the B spline curves of quaternion, it can interpolate and optimized the discrete tool posture vector in quaternion space, so as to get the tool motion curve smoother. Now it starts to optimize tool orientation as following:

When the quaternion point as $Q_i = \sum_{i=1}^{m} B_{i,p}(v_i) q_i$ was obtained, and according to the expression $p_i = Q_i p_1 Q_i^{-1}$, the other expression of tool posture can be derived as follows:

$$p_i = \sum_{i=1}^{m}\sum_{j=1}^{m} B_{i,p}(v_i) B_{j,p}(v_i) q_i q_j q_1 =$$

$$\sum_{i+j=1}^{2m} \frac{C_i^p C_j^p}{C_{i+j}^{2p}} B_{i+j,2p}(v_i) q_i q_j q_1 ,$$

Let $W=\sum_{i+j}\frac{C_i^p C_j^p}{C_{i+j}^{2p}} q_i q_j q_1=\sum_{k}\frac{C_i^p C_j^p}{C_k^{2p}} q_i q_j q_1$, and tool orientation curve expression could be got as $p_i=\sum_{k=1}^{2m} B_{k,2p}(v_i)\cdot W$.

Based on the algorithm stated above, the tool orientation curve will be got and described in the unit sphere as illustrated in the following Fig.4, we can see that tool orientation curve fitting is performed based on quaternion points.

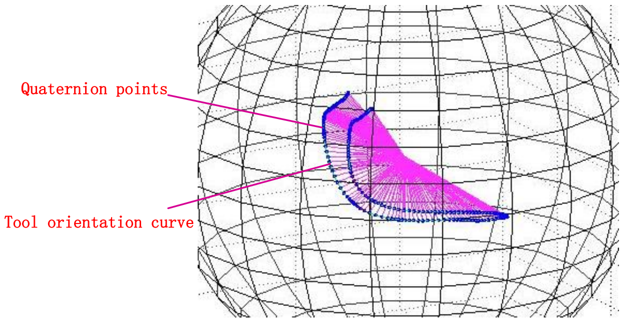
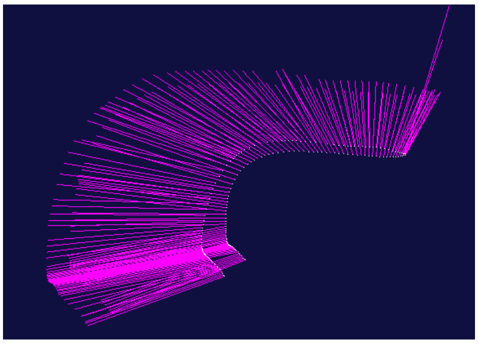


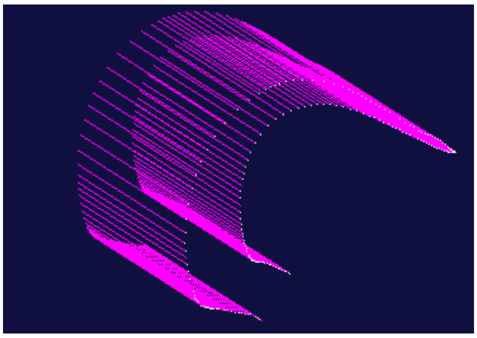
Fig.4 The tool orientation curve in the unit sphere

VI. ANALYSIS OF THE ALGORITHM

In order to test the validity of this algorithm, the following work is on a track of the actual simulation. Fig.5 compared the normal vector of tool posture points between before and after optimized as following:



(a). the normal vector before optimization



(b). the normal vector after optimization

Fig.5 The normal vector of the tool posture point

Fig.5 (a) shows the normal vector before optimization, and Fig.5 (b) is the normal vector after optimization. It is evident from these figures that the cutter axis vector becomes smoother after optimized. This confirms that the tool axis vector optimization method in this article is effective.

VII. CONCLUSION

For five-axis machining, if the angles of cutter shaft between two adjacent cutter locations are too large, or the cutter axis vectors are uneven, all these would make the cutter axis influence the surface quality. In this article, through the use of quaternion on the rigid body rotation and convenient description, the tool posture can be converted into quaternion and transform the discrete tool posture interpolation problem into the curve design interpolation problem. In this way, the tool vector can be optimized and the quality of machining can be improved.

ACKNOWLEDGMENT

The authors wish to thank the helpful comments and suggestions from my teachers and colleagues in College of Mechanical Engineering. The research is sponsored by the project supported by high level cultivation project of shanghai university of engineering science (No. 2012gp01).

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