

# A Look-ahead Control Algorithm with Arc Transition for High-speed Machining of Continuous Micro-segments

Jun Zhang, Liqiang Zhang

**Abstract**—To diminish the vibration and shock produced in NC machining and realize the high-speed smooth transition, a corner transition model based on quadratic NURBS arc fitting for continuous micro-segments is established. The new inserted arc is everywhere curvature-continuous, and can realize the corner path smooth transition. Based on the arc model, a look-ahead control algorithm is proposed. The algorithm aimed to obtain optimal transition speed on the conditions of the chord error and machine dynamics, control acceleration/deceleration, avoid the motor starting and stopping frequently, and achieve the high-speed transfer smoothly. The simulation results demonstrate that the proposed algorithm can attain higher and more stable transition speed, improve the machining efficiency, and satisfy the requirements of high-speed machining.

**Index Terms**—High-speed machining, smooth transition, arc transition, look-ahead control algorithm

## I. INTRODUCTION

High-speed and high-precision machining is mainly used in machining a series of continuous micro-line segments which are discrete formed by complex curve surface [1]. When processing micro-segments, the traditional numerical control (NC) system often tends to slow down to zero at the transfer points. NC system frequently starts and stops, accelerates and decelerates, so it is not conducive to high-speed machining and is easy to have a great impact and vibration to the machine tool, then affects the machining precision and machining efficiency [2]. Therefore, smooth transition speed at the corner of adjacent segments has become a key technology for high-speed and high-precision NC machining.

In order to overcome this problem, many scholars make an intensive study in recent years. Wang established a switching feedrate model for micro-segments, which based on the linear acceleration and deceleration (Acc/Dec), and implement the optimal transition feed by setting the pre-disposal section [3]. Zhang [4] proposed an assumptive arc transition method to deal with the feedrate on the corner. Due to the interpolation of parametric curve has the advantages of smooth speed, less

data storage, easy to express, the parametric curve, such as Bezier curve, B-spline and NURBS curve, is applied to

continuous line processing by scholars. Zhang et al. [5] established a quintic corner transition curve model, and proposed an interpolation algorithm based on the transition of the curve. Ning [6] proposed a quintic polynomial curve to obtain continuous change of acceleration and jerk. Huang [7] constructed an adjustable form of the cubic spline between the adjacent segments to realize the smooth speed transition. Bi [8] inserted a curvature-continuous of cubic Bezier curve between the adjacent segments to realize the smooth corner transition. Zhao constructed a curvature-continuous B-spline corner transition model, and proposed a real-time look-ahead interpolation algorithm [9].

Now most of the NC systems only support line interpolation and arc interpolation. The above parametric curves methods can provide a smooth transition speed, but cannot be widely applied in the NC systems. Therefore, a corner transition model based on the quadratic NURBS arc fitting for continuous micro-segments is established in this paper, which support both arc interpolation and NURBS interpolation so that it can be widely used. The inserted arc is everywhere curvature-continuous, so it can effectively avoid the machine vibration and shock caused by the velocity and acceleration mutation. Then, a look-ahead control algorithm is proposed to control Acc/Dec, and realize high-speed transition smoothly for continuous micro-segments. Finally, the algorithm is analyzed and verified by the simulation example.

## II. CORNER ARC TRANSITION MODEL

### A. Quadric NURBS curve

The arc can be represented by NURBS curve of Quadric and higher [10]. Higher order NURBS curves are often used to fit the special combination of curves and surfaces, which need more control vertices and weights and lead to more computational complexity. In order to facilitate calculation, an arc corner model based on the quadratic NURBS fitting is used to realize the smooth transition at the adjacent corner.

A  $p$ th-degree NURBS is defined by

$$C(u) = \frac{\sum_{k=0}^n N_{k,p}(u)\omega_k P_k}{\sum_{k=0}^n N_{k,p}(u)\omega_k}, \quad a \leq u \leq b \quad (1)$$

Where  $P_k$  are the control vertices ( $k = 0, 1, \dots, m-1$ ),  $\omega_k$  is the weights. The knot vector is designed as

Manuscript received Aug 17, 2014.

Jun Zhang, College of Mechanical Engineering, Shanghai University of Engineering Science, Shanghai China, +86-21-67791180.

Liqiang Zhang, College of Mechanical Engineering, Shanghai University of Engineering Science, Shanghai China, +86-21-67791180.

$$U = \{\underbrace{a, \dots, a}_{p+1}, u_{p+1}, \dots, u_{m-p-1}, \underbrace{b, \dots, b}_{p+1}\},$$

where  $a$  is set to zero and  $b$  is equal to one and  $\omega_k$  is greater than zero for all  $k$ . The  $N_{k,p}(u)$  is the  $k$ th B-spline basis function of  $p$ th degree defined on the normalized knot vector  $U$ . The basis functions  $N_{k,p}(u)$  is given by the Cox-DeBoor recursion formulas. Specifically,

$$\left\{ \begin{aligned} N_{k,p}(u) &= \begin{cases} 1, & \text{if } u_k \leq u \leq u_{k+1} \\ 0, & \text{otherwise} \end{cases} \\ N_{k,p}(u) &= \frac{u - u_k}{u_{k+p} - u_k} N_{k,p-1}(u) \\ &\quad + \frac{u_{k+p+1} - u}{u_{k+p+1} - u_{k+1}} N_{k+1,p-1}(u) \\ \text{set } \frac{0}{0} &= 0 \end{aligned} \right.$$

(2)

B. Construction of the transition arc

1) Angle between adjacent segments

In figure 1, the  $\theta$  is the angle between adjacent segment where  $\theta \in (0, \pi)$ ,

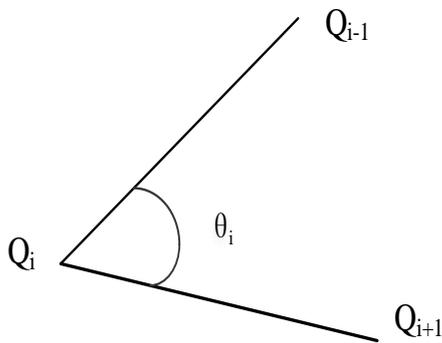


Fig.1 Angle of adjacent segment

According to the formula of vectorial angle,  $\theta_i$  is defined as:

$$\theta_i = \arccos\left(\frac{\overrightarrow{Q_i Q_{i-1}} \cdot \overrightarrow{Q_i Q_{i+1}}}{\|Q_i Q_{i-1}\| \cdot \|Q_i Q_{i+1}\|}\right), i = 1, \dots, N$$

(3)

The NURBS arc must satisfy five requirements in practical applications, that is the least number of control points, the good parameter, the compact convex hull, and each central angle should not exceed  $90^\circ$ . For  $0 < \theta \leq \frac{\pi}{2}$ , the arc should be divided into two stages and fitted by the NURBS curves. For  $\frac{\pi}{2} < \theta < \pi$ , arc can be fitted by only a single NURBS curve.

2) Quadric NURBS arc for  $\frac{\pi}{2} < \theta < \pi$

As shown in figure 2, the arc  $C(u)$  is fitted by a quadric NURBS curve, and used as the corner transition curve between two adjacent processing paths:  $Q_0Q_1$  and  $Q_1Q_2$ . The control points are  $P_0, P_1$  and  $P_2$ . The line  $P_0P_1$  and  $P_1P_2$  are respectively tangent to the arc, and each point of tangency is  $P_0$  and  $P_2$ . The positive weights are  $\omega_0 = \omega_2 = 1$  and  $\omega_1 = \cos(\theta/2)$ . The knot vector is  $U = \{0, 0, 0, 1, 1, 1\}$ . In this way, arc is actually fitted by the rational quadratic Bezier curve which is the special case of the quadratic NURBS curve.  $Q_1O_1$  is the angle bisector of  $\angle Q_0Q_1Q_2$ .  $B_1$  is the midpoint of  $C(u)$ .

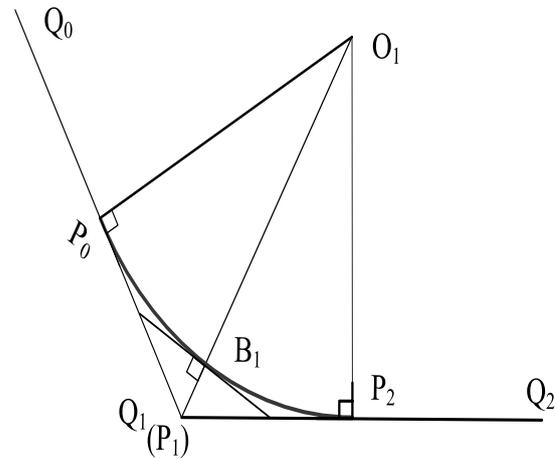


Fig.2 Quadric NURBS arc for  $\frac{\pi}{2} < \theta < \pi$

3) Quadric NURBS arc for  $0 < \theta \leq \frac{\pi}{2}$

As shown in Fig. 3, By using the results of the rational quadratic Bezier curve, a new knot  $u (u = 0.5)$  is inserted into knot vector  $U$ . Thus the new knot vector is  $U = \{0, 0, 0, 0.5, 1, 1, 1\}$ . As a result, the arc  $C(u)$  can be fitted by two quadric NURBS curves. The control points is  $P_0, P_1, P_2$  and  $P_3$ . The midpoint of arc is  $B_1$ , and the line  $P_1P_2$  is tangent to the midpoint of arc. The positive weights are  $\omega_0 = \omega_3 = 1, \omega_1 = \omega_2 = \cos^2(\theta/4)$ . The line  $P_0P_1$  and  $P_2P_3$  respectively belong to the adjacent machining path  $Q_0Q_1$  and  $Q_1Q_2$ , and are respectively tangent to the arc with each tangent point:  $P_0$  and  $P_2$ . The length of line  $P_0P_1$  is equal to line  $P_2P_3$ , and both of them is half of  $P_1P_2$ , that is:  $|P_1P_2| = 2|P_0P_1| = 2|P_2P_3|$ .

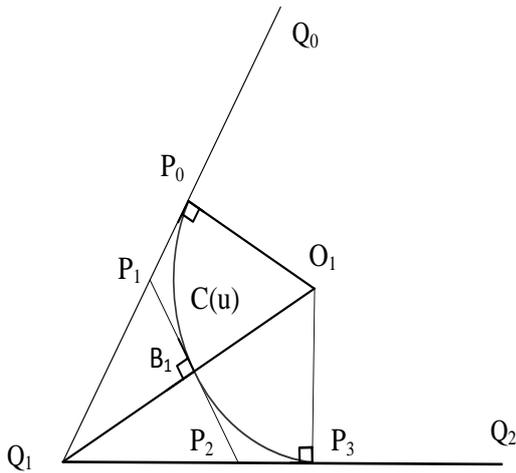


Fig.3 Quadratic NURBS arc for  $0 < \theta \leq \frac{\pi}{2}$

#### 4) Curvature continuous

Curvature continuous, that is  $G^2$  continuity, two continuous curves at the endpoint have the same coordinate, the same tangent vector, and the same center of curvature. According to the former two sections, the arc based on the quadratic NURBS fitting at the joint has the same tangent vector, and the curvature center is the circle center. Thus, the arc transition curve is everywhere curvature-continuous, so that it is bale to effectively avoid the machine vibration and impact which is caused by the velocity and acceleration mutation, and realize a smooth transition.

### III. A LOOK-AHEAD CONTROL ALGORITHM FOR ARC TRANSITION

When the machine processes the continuous micro-segments, feedrate will mutate at the corner of adjacent micro-segments. If not to control the speed for advance planning, it will lead to the overcut due to beyond the machine limits of acceleration/ deceleration, and then it has a bad effect on the processing quality. Therefore, in order to adjust the processing speed and make the acceleration/deceleration of processing speed satisfy the changes of transition path, so a look-ahead control function need to be added. In this paper, the look-ahead control processing mainly achieves three functions: to establish a corner arc transition model based on the quadratic NURBS fitting, to obtain optimal transition speed under the conditions of the chord error and machine dynamics, to realize the acceleration/deceleration in the processing of the line and arc. The flow chart of look-ahead control function is shown in Figure 4.

#### A. Machining error

As shown in Fig. 5, after inserting transition arc at the adjacent segment, due to the inconsistency of machining path it leads to machining error. So it requires the machining error does not exceed the systematic error. The machining error  $\varepsilon_i$  mainly comes from two aspects. One is the contour

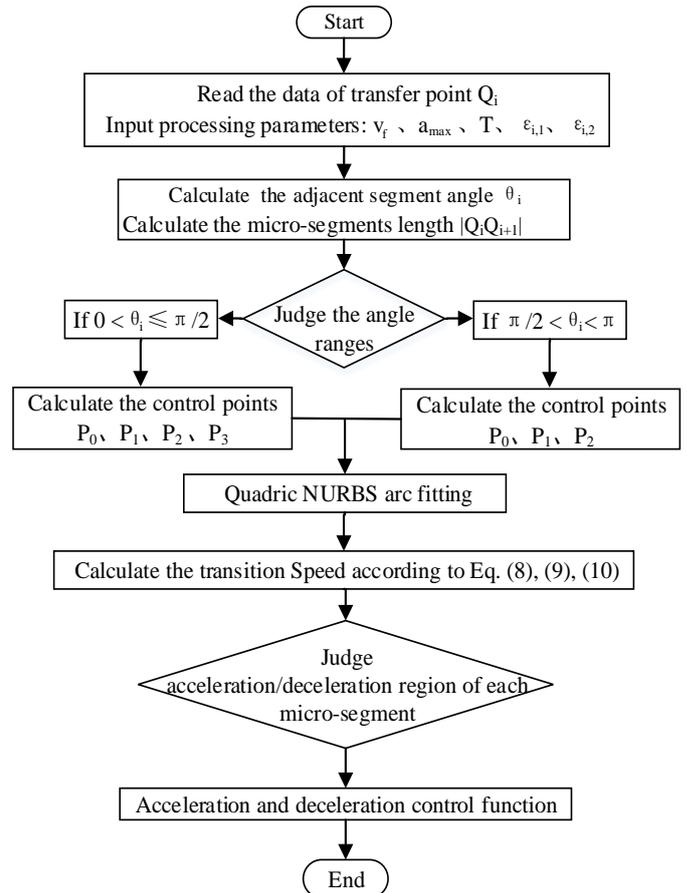


Fig.4 Flow chart of the look-ahead control function

error  $\varepsilon_{i,1}$  between the inserted arc and adjacent lines, another is the chord error  $\varepsilon_{i,2}$  that generated by arc interpolation.

The contour error  $\varepsilon_{i,1}$  is defined as:

$$\varepsilon_{i,1} = \frac{L_{1,i}[1 - \sin(\theta_i / 2)]}{\cos(\theta_i / 2)} \quad (4)$$

Where  $L_{1,i}$  is the distance of arc transition, that is, the distance from corner point to transition point.

The distance of arc transition can be calculated from the Eq. (4) as:

$$L_{1,i} = \frac{\varepsilon_{i,1} \cos(\theta_i / 2)}{1 - \sin(\theta_i / 2)} \quad (5)$$

The chord error  $\varepsilon_{i,2}$  is given as follows:

$$\varepsilon_{i,2} = r_i - \sqrt{r_i^2 - (v_{i,2} T / 2)^2} \quad (6)$$

Where  $v_{i,2}$  is the machining velocity under constraint of the chord error. T is the interpolation period.  $r_i$  is the radius of quadratic NURBS arc, and it is derived from the Eq. (6) as:

$$r_i = \frac{\varepsilon_{i,1} \sin(\theta_i / 2)}{1 - \sin(\theta_i / 2)} \quad (7)$$

(For the convenience of calculation, in this paper the contour error and chord error is set to the maximum error of the machine tool.)

B. Turning speed transition speed

When the transition arc is processed, the transition speed  $v_i$  is calculated under the limitation of chord error and machine dynamics.

Under the limitation of contour error, the transition speed is derived from the Eq. (6) as:

$$v_{i,2} = \frac{2}{T} \sqrt{r_i^2 - (r_i - \varepsilon_{i,2})^2} \tag{8}$$

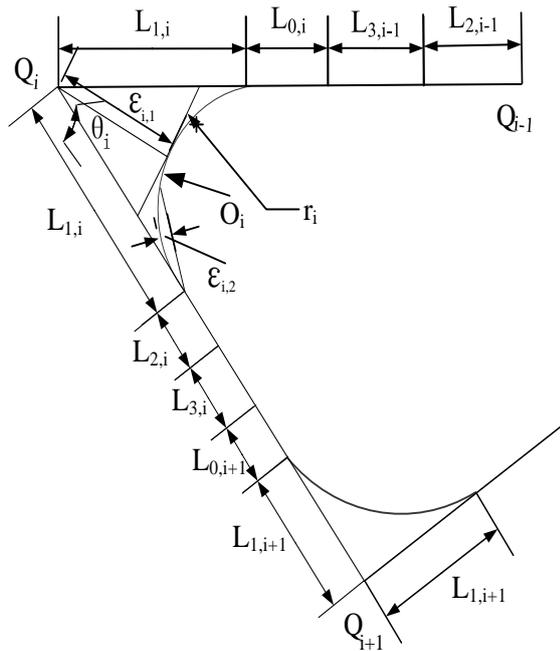
Under the constraint of machine dynamic, a constant speed of the arc transition method is proposed, so only the normal acceleration has an effect on the transition speed. Thus, the transition speed is defined as:

$$v_{i,1} = \sqrt{a_{\max} r_i} \tag{9}$$

Where  $a_{\max}$  is the maximum acceleration of machine tool.

The transition speed  $v_i$  can not only satisfy the prescribed tolerance of chord error and machine dynamics, but also can't exceed the machine program speed  $v_f$ . Therefore, the transition speed must be the minimum among the three velocities, which is defined as:

$$v_i = \min(v_{i,1}, v_{i,2}, v_f) \tag{10}$$



Transition arc  $O_i$ , Transition distance  $L_{1,i}$ , Deceleration distance  $L_{0,i}$ , Acceleration distance  $L_{2,i}$ , Uniform distance  $L_{3,i}$ , Arc radius  $r_i$ , Contour error  $\varepsilon_{i,1}$ , Chord error  $\varepsilon_{i,2}$

Fig.5 The arc transition path

C. Arc transition speed smoothing

Due to the inconsistency of programming speed and arc transition speed during processing segment, it is necessary to adjust the machining velocity for Acc/Dec.

According to the arc transition speed  $v_i, v_{i+1}$ , programming speed  $v_f$ , and the length of each micro-segment  $|Q_i Q_{i+1}|$ , there are seven acceleration/deceleration modes to consider, that is: only the acceleration period, only deceleration period, both acceleration period and deceleration period, all the acceleration period and uniform period and deceleration period, both the acceleration period and uniform period, both the uniform period and deceleration period, only the uniform period. As shown in Fig. 5,

① Only the acceleration period, if  $v_i < v_{i+1} \leq v_f$ , the machining path is given as follows:

$$\begin{cases} L_{1,i} + L_{2,i} + L_{1,i+1} = |Q_i Q_{i+1}| \\ L_{3,i} = L_{0,i+1} = 0 \end{cases}$$

② Only deceleration period, if  $v_{i+1} < v_i \leq v_f$ , the machining path is given as follows :

$$\begin{cases} L_{1,i} + L_{0,i+1} + L_{1,i+1} = |Q_i Q_{i+1}| \\ L_{2,i} = L_{3,i} = 0 \end{cases}$$

③ Both acceleration period and deceleration period, if  $\max(v_{i+1}, v_i) < v_f$ , the machining path is given as follows :

$$\begin{cases} L_{1,i} + L_{2,i} + L_{0,i+1} + L_{1,i+1} = |Q_i Q_{i+1}| \\ L_{3,i} = 0 \end{cases}$$

④ All the acceleration period and uniform period and deceleration period, if  $\max(v_{i+1}, v_i) < v_f$ , the machining path is given as follows :

$$\begin{cases} L_{1,i} + L_{2,i} + L_{0,i+1} + L_{1,i+1} < |Q_i Q_{i+1}| \\ L_{3,i} = 0 \end{cases}$$

⑤ Both the acceleration period and uniform period, if  $v_i < v_{i+1} = v_f$ , the machining path is given as follows :

$$\begin{cases} L_{1,i} + L_{2,i} + L_{1,i+1} < |Q_i Q_{i+1}| \\ L_{0,i+1} = 0 \end{cases}$$

⑥ Both the uniform period and deceleration period, if  $v_{i+1} < v_i = v_f$ , the machining path is given as follows :

$$\begin{cases} L_{1,i} + L_{3,i} + L_{0,i+1} + L_{1,i+1} < |Q_i Q_{i+1}| \\ L_{2,i} = 0 \end{cases}$$

⑦ Only the uniform period, if  $v_i = v_{i+1} = v_f$ , the machining path is given as follows :

$$\begin{cases} L_{1,i} + L_{1,i+1} < |Q_i Q_{i+1}| \\ L_{2,i} = L_{0,i+1} = 0 \end{cases}$$

Thus, the machining acceleration/deceleration period for every micro-segment is judged through the transition speed and the length of each micro-segment. Then, the acceleration/deceleration methods provided by references [3, 4] can be used to realize the smooth speed.

#### IV. SIMULATION AND VALIDATION

In order to verify the correctness and feasibility of the proposed algorithm, a path segment is established for machining comparison, as shown in Fig.6. The NC machining parameters are: The maximal machining speed is  $v_f = 100$  mm/s, the maximal acceleration is  $a_{max} = 3000$  mm/s<sup>2</sup>, and the interpolation period is  $T=2$ ms, the maximal contour error is  $\varepsilon_{i,1}=0.5$ mm, the maximal chord error is  $\varepsilon_{i,2}=0.002$ mm,

In this paper, the simulation is performed by using MATLAB software. Based on the linear Acc/Dec method, the comparison of machining error chart is shown in Fig. 7 and the speed curve contrast figure Fig.8 can be obtained on account of the quadric NURBS arc transition algorithm and directly transition method respectively.

As shown in Fig.6, the inserted arc makes the transition corner path smoother, avoids processing corner directly, can greatly lessen the machine tool vibration and impact from machining corner, and improves product quality of the parts and tool life.

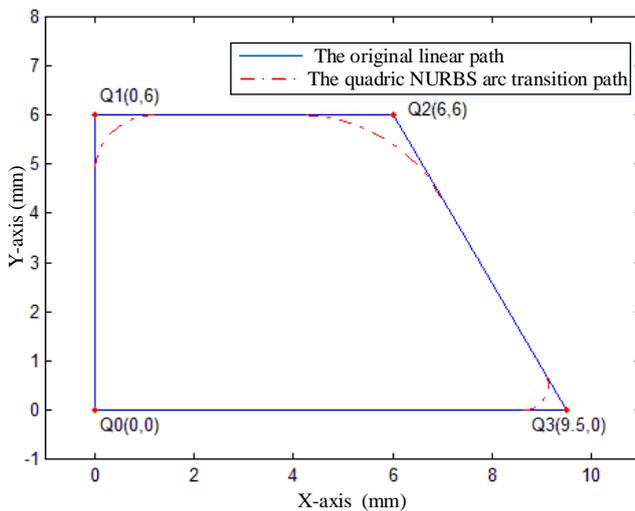


Fig.6 The transition path comparison

It must produce the contour error between the inserted arc and the original linear toolpath. However, according to Fig. 7, the results analysis show that: The machining error of the proposed algorithm can be controlled under the limit range of the maximal programming error, so the algorithm does well in validity and can meet the precision requirements of processing workpieces.

From Fig. 8, it is seen that the time used for machining the transition arc paths are 0.34s, while those for the original linear paths are 0.39s. Thus, the look-ahead control algorithm proposed in this paper can improve the machining efficiency significantly. Furthermore, the arc transition speed and the average velocity for using the proposed scheme are faster and smoother. Therefore, the proposed look-ahead control scheme can meet the requirements of high speed and high accuracy. Especially, when  $90^\circ < \theta_i < 180^\circ$ , the larger the angle is, the faster transition speed is. For example, the transition velocity is the maximal machining speed with 100mm/s at  $Q_2$  (Fig.6), thus it can avoid the frequent

acceleration/deceleration effectively, and achieve more stable transition and higher machining productivity.

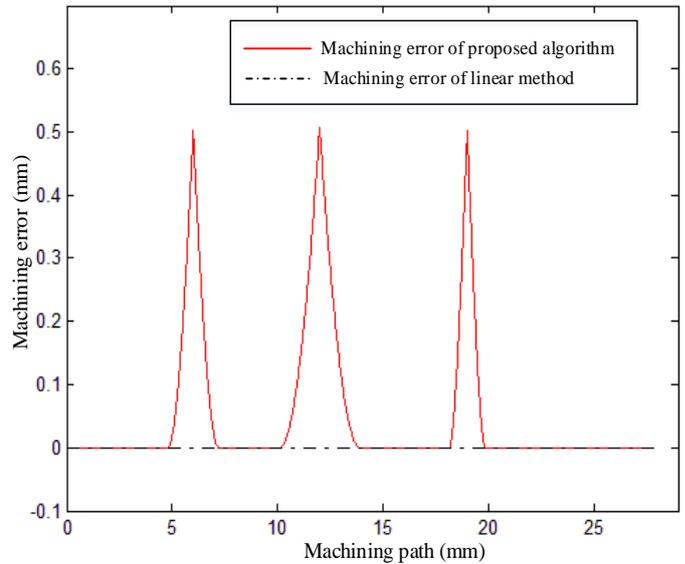


Fig.7 Machining error comparison

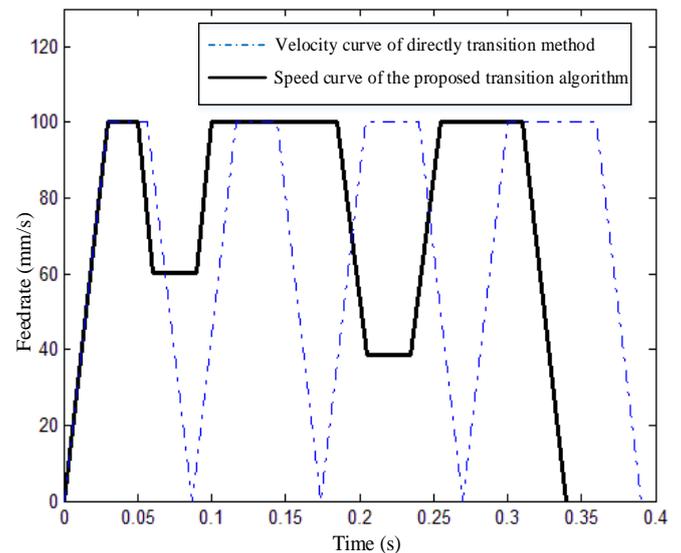


Fig.8 Machining speed comparison

#### V. CONCLUSION

In this paper, a corner transition model based on the quadratic NURBS arc fitting for continuous micro-segments is established.

The model is everywhere curvature-continuous, can decrease the speed fluctuation and the machine tool significantly, and realize the smooth transition. Meanwhile, it can be interpolated by the arc interpolation, which can be widely used in NC manufacturing. Then, according to the arc corner transition model, a look-ahead control algorithm is proposed to achieve acceleration/deceleration control. Under the restriction of the machining error and machine dynamics, the algorithm can obtain optimal transition speed, and attain the acceleration/deceleration control. Furthermore, it can avoid starting/stopping motor frequently, realize high-speed smooth machining of continuous micro-segments, and improve the quality and efficiency of machining significantly. The simulation results demonstrate that, compared with

traditional algorithms, the proposed algorithm is simple and practical, can achieve the transition speed higher and more stable at adjacent segment, can improve the machining speed greatly, and satisfy the demands of high speed machining.

### ACKNOWLEDGMENT

The authors wish to thank the helpful comments and suggestions from my teachers and colleagues in College of Mechanical Engineering. The research is sponsored by the project supported by high level cultivation project of Shanghai University of Engineering Science (No. 2012gp01).

### REFERENCES

- [1] Q.L.Zhong, J.LI, S.H.Huang, "Interpolation of the micro-line at high continuum speed with high accuracy in RP system", *J Huazhong Univ of Sci & Tech*, 2000, 28(3): 39-41.
- [2] J.Hu, L.J.Xiao, Y.H.Wang, et al, "An optimal feedrate model and solution algorithm for high speed machine of small line blocks with look-ahead", *International Journal of Advanced Manufacturing Technology*, 2006, 28(9): 930-935.
- [3] Y.H.Wang, L.J.Xiao, Sh.Sh.Zeng, Z.Y.Wu, Sh.B.Zhong, et, "An optional feedrate model and solution for high-speed machining of small line blocks with look-ahead", *Journal of Shanghai Jiaotong University*, 2004, 38(6): 901-904.
- [4] D.L.Zhang, L.S.Zhou, "Adaptive algorithm for feedrate smoothing of high speed machining", *Acta Aeronautica et Astronautica Sinica*, 2006, 27(1): 125-130.
- [5] X.H.Zhang, D.Yu, D.Sh.Yang, Y.Hu, W.Y.Han, "Corner curve transition interpolation algorithm for high speed machining of micro-line segment", *Journal of mechanical engineering*, 2010, 46(19): 183 – 191.
- [6] P.Z.Ning, Q.Z.Bi, Y.H.Wang, J.Shi, "A high-flexible acc/dec look-ahead algorithm based on quintic polynomial federate curve", *Modular machine tool & Automatic manufacturing technique*, 2014, 4(4): 15-18.
- [7] J.Huang, A.P.Song, J.M.Tao, W.ZH.Yin, "Smoothing transfer for the corner of adjacent NC processing segment", *Journal of Shanghai Jiaotong University*, 2013, 47(5): 734-739.
- [8] Q.Z.Bi, Y.Q.Jin, Y.H.Wang, "An analytical curvature-continuous Bézier transition algorithm for a linear tool path", *International Journal of Machine Tools & Manufacture*, 57(2012): 55-65.
- [9] H.Zhao, L.M.Zhu, H.Ding, "A real-time look-ahead interpolation methodology with curvature-continuous B-spline transition scheme for CNC machining of shortline segments", *International Journal of Machine Tools & Manufacture*, 65(2013): 88-98.
- [10] F.Zh.Shi, "Computer-Aided Geometric Design and NURBS", Beijing: Higher Education Press, 2013(in Chinese).

**Jun Zhang** received his BS degree from Changchun Institute of Technology in 2012, China. He is currently a master student of mechanical engineering in Shanghai University of Engineering Science. His research interests include CAD/CAM/CNC, manufacturing automation and control.

**Liqliang Zhang** received his PhD from Shanghai Jiaotong University in 2008, MS degree from Qingdao Technological University and BS degree from Shandong University, China. He is currently an associate professor of mechanical engineering in Shanghai University of Engineering Science. His research interests include CAD/CAM/CNC, manufacturing automation and control.