Tuning of a PD-controller used with Second Order Processes

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***Abstract*— PID controllers are used for decades in controlling processes in linear feedback control systems. Their use requires accurate and effective tuning to satisfy an acceptable performance for the control system.**

Large number of processes are classified as or approximated by a second order model. They may me underdamped, critically damped or overdamped.

This paper presents the tuning of PD-controllers used with second order processes. The process damping ratio is from 0.05 to 10 and its natural frequency is from 2.5 to 10 rad/s. The tuning technique depends or minimizing the integral of square of error (ISE) between the time response of the system to a unit step input and its steady-state response. It was possible to achieve a PD-controlled system with very small overshoot, settling time and steady-state error. The tuning results are listed in tables for direct use depending on process damping ratio and natural frequency.

The tuning results are compared with controller tuning using standard forms showing the better performance of the proposed tuning in the present work.

***Index Terms*— PD Controller Tuning , Second-order Processes , ISE Error Criterion , Tabulated Tuning Results..**

I. INTRODUCTION

PD-controllers have simple construction using operational and new generation amplifiers [1,2,3].

Sung and Lee (2000) proposed an identification algorithm for the automatic tuning of PID controllers to guarantee better accuracy and to provide more frequency data sets of the process [4]. Vrancic, Strmcnik and Turicic (2001) used the multiple integration of the process time response for calculating the parameters of the PID controller [5]. Shen (2002) proposed a tuning method for PID controller providing the performance assessment formulas. His method is based on a genetic algorithm-based design technique [6]. Tavakoli and Tavakoli (2003) presented a tuning technique for PID controllers controlling first-order plus delay processes based on dimensional analysis and optimization techniques. They used ISE, IAE and ITAE performance criteria [7]. Gaing (2004) presented a design method for determining the optimal PID controller parameters of an AVR system using the particle swarm optimization algorithm providing high-quality solution [8].

Syrcos and Kookes (2005) presented a general mathematical formulation for the development of customized PID control tuning. The presented a number of case studies clarifying their proposed methodology [9]. Chen and Chang (2006) studied the optimal design of a PID controller for an active magnetic bearing using genetic algorithms providing experimental and simulation results [10]. D'Emilia, Marra and Natale (2007) described a theoretical-experimental approach allowing the evaluation of the adequateness of new methods for auto-tuning with respect to traditional ones [11]. Ramasamy and Sundaramoorthy (2008) used the impulse response instead of the step response of the plant to tune the PID controller, requiring no approximation of the plant by any model. They derived formulae for the calculation of PID controller tuning parameters [12]. Malwatker, Sonawane and Waghmane (2009) proposed a model-based design of PID controllers for higher-order oscillating systems. They obtained the controller parameters from a reduced third-order model and presented examples to demonstrate the effectiveness of their proposed method [13].

Pai, Chang and Huang (2010) presented a simple calculation method of a PI/PID controller tuning for integrating processes with dead-time and inverse response based on a model [14]. Matausek and Sekara (2011) proposed a tuning procedure for ideal PID controller in series with a first-order noise filter based on the extension of Ziegler-Nichols frequency domain dynamics of a process [15]. Ayala and Foelh (2012) presented the design and tuning of two PID controllers using the non-dimensional sorting genetic algorithm approach offering simple and robust solutions providing good reference tracking performance [16]. Saglam, Tutomo and Kurtulan (2013) presented a PI controller tuning method for cooling of the hydrogen production unit within the polymer electrolyte membrane fuel cell based micro-cogeneration system. Their proposed analytical tuning rules are based totally on the model parameters using the digital control theory and experiences in industrial control problems [17].

Tiwari, Pakhri and Chile (2013) described the tuning of PD controllers using genetic algorithm for steering control of AUV in pure horizontal plane. They compared the performance of the system using PID and PD controllers [18]. Bazregar, Piltan , Nabaee and Ebrahimi (2014) proposed a fuzzy PD gravity controller for a continuum robot manipulator. The proposed PD controller was a partly model-free type [19]. Perreira, Crisostoma and Coimbra (2014) developed an intelligent computing technique based on support vector regression to overcome the problem of control of an eight-link robot. They used a PD controller tuned initially using the second Ziegler-Nichols method [20].

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II. ANALYSIS

Process:

The process has the transfer function:

$$M_p(s) = \omega_n^2 / (s^2 + 2\zeta\omega_n s + \omega_n^2) \quad (1)$$

Controller:

The controller used in this study is a proportional + derivative (PD)- controller.

The PD-controller of has a transfer function $G_c(s)$ given by:

$$G_c(s) = K_p + K_d s \quad (2)$$

Where: K_p = controller proportional gain
K_d = controller derivative gain

Control System Transfer Function:

Assuming that the control system is a unit feedback one, its transfer function becomes:

$$M(s) = (b_0 s + b_1) / (s^2 + a_1 s + a_2) \quad (3)$$

where:

$b_0 = \omega_n^2 K_d$
$b_1 = \omega_n^2 K_{pc}$
$a_1 = 2\zeta\omega_n + \omega_n^2 K_d$
$a_2 = \omega_n^2(1 + K_{pc})$

System Step Response:

A unit step response is generated by MATLAB using the numerator and deniminator of Eq. 3 providing the system response c(t) as function of time.

III. CONTROLLER TUNING

The sum of absolute error (ISE) is used as an objective function, F required by the optimization process. Thus:

$$F = \int [c(t) - c_{ss}]^2 \, dt \quad (4)$$

where c_{ss} = steady state response of the system ($c_{ss} = b_1/a_2$).

The performance of the control system is controlled using three functional constraints:

(a) The maximum percentage overshoot, OS_{max}:

$$C_1 = OS - OS_{des} \quad (5)$$

Where:

OS is the maximum percentage overshoot of the control system obtained by the *stepinfo* command of MATLAB [21].

OS_{des} is the desired maximum percentage overshoot

(b) The settling time, T_s:

$$C_2 = T_s - T_{sdes} \quad (6)$$

Where:

T_s is the time after which the response enters a band of ± 5 % of c_{ss} and stays within it obtained by the *stepinfo* command of MATLAB [21].

T_{sdes} is the desired settling time of the system time response to a step input.

(c) Steady-state error, e_{ss}:

$$C_3 = e_{ss} - e_{ssdes} \quad (7)$$

Where:

e_{ss} is the steady-state error of the control system defined for a unit step input as:

$$e_{ss} = 1 - c_{ss}$$

e_{ssdes} is the desired steady-state error of the closed-loop control system.

Parameters and functional constraints limits:

- A lower limit of 0.01 is set for the controller parameters: K_p, and K_d.
- An upper limit of 400 is set for the controller parameters.
- An 0.1 % is set for the desired system maximum overshoot.
- A 1 second upper limit is set for the control system settling time.
- An 0.01 is set for the desired steady-state error.

IV. TUNING RESULTS

The MATLAB command "*fmincon*" is used to minimize the optimization objective function given by Eq.4 subjected to the functional inequality constraints given by Eqs. 5 - 7 to provide the controller optimal parameters. The results depends on the process damping ratio and natural frequency. Table 1 presents the results of the optimal tuning procedure used in this work.

Table 1: PD-controller tuning for a second order process.

ζ \ ω_n	2.5		5.0		7.5		10.0	
	K_{pc}	K_d	K_{pc}	K_d	P_{pc}	K_d	K_{pc}	K_d
0.05	592.12	592.12	492.61	492.61	404.38	216.88	455.84	455.84
0.1	577.21	577.21	488.93	488.93	403.66	192.58	400.67	278.79
0.2	436.05	536.05	400.00	400.00	400.15	210.12	400.00	400.00
0.4	416.87	342.15	400.11	180.87	400.00	343.37	324.71	199.83
0.6	400.00	259.55	400.35	136.31	400.00	106.34	375.29	58.35
0.8	328.19	274.27	362.38	176.02	400.00	149.45	400.00	47.37
1	400.00	325.96	315.51	189.36	400.00	213.39	343.19	112.57
2	362.21	400.00	342.62	400.00	400.00	135.91	296.53	195.83
4	260.18	400.00	295.19	111.48	287.78	73.94	214.65	51.63
6	260.14	337.04	288.13	172.87	354.89	120.56	238.37	82.29
8	250.91	400.00	286.04	232.93	371.54	164.64	332.32	120.65
10	277.09	400.00	282.77	291.93	285.78	195.14	397.71	157.52

V. COMPARISON WITH STANDARD FORMS TUNING

To examine the effectiveness of the tuning procedure used in this work, it has been compared with the results of controller tuning using the ITAE-based standard forms [22]. The standard characteristic equation of a second-order closed-loop control system is [22]:

$$s^2 + 3.2\omega_o s + \omega_o^2 = 0 \quad (8)$$

The PD-controller parameters using Table 1 for ω_n = 10 rad/s and ζ = 0.4 are:

$K_{pc} = 324.71$
$K_d = 199.83$

Using Eqs.3 and 8, the derivative gain of the PD-controller using the standard forms for ω_n = 10 rad/s, ζ = 0.4 and K_{pc} = 324.71 is:

$$K_d = 5.6952$$

The un-controlled process has the time response to a unit step input shown in Fig.3.

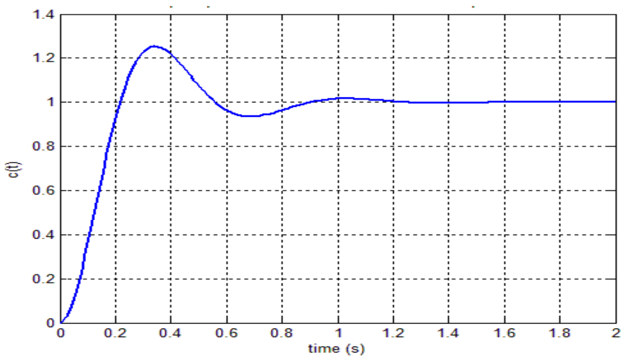

Fig.3 Step response of a the un-controlled second-order process.

It has:

- Maximum percentage overshoot: 25.38 %
- Settling time: 0.76 s

The time response of the control system using both sets of PD-controller parameters is shown in Fig.4:

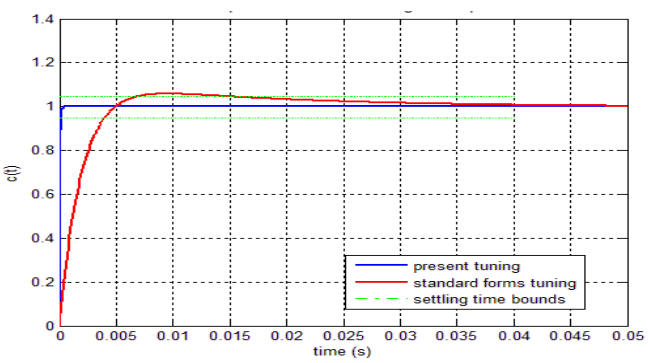

Fig.4 Step response of a PD controlled second-order process.

Characteristics of the control system using the tuned PD-controller:

- Maximum percentage overshoot: 0 % (compared with 5.7 % using the standard forms tuning approach)
- Settling time: 0.00014 s (compared with 0.015 s using the standard forms tuning approach)
- Steady-state error: 0.003 for both sets of tuned parameters.

VI. CONCLUSION

- It is possible to suppress higher oscillations and decreese very large settling time in second order processes through using PD-controllers.

-Through using a PD-controller it was possible to cancel completely the maximum overshoot of the uncontrolled process.

-Through using a PD-controller it was possible reduce the settling time to as low as 2 ms indicating the fast settlement of the controlled process.

-It was possible to control the steady-state error of the closed-loop system to less than 0.01.

-The tuned PD-controller gives superior step response for the closed-loop control system compared with that tuned using standard forms.

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