Computation of Mass attenuation coefficient, energy absorption and exposure build-up factors for teeth in the energy range 0.015-15 MeV up to 40 mfp

Sandeep Gupta, Gurdeep Singh Sidhu

Abstract— To compute gamma ray energy absorption build-up factor (EABF) and exposure buildup factor (EBF) of teeth (enamel outer surface (EOS), enamel middle (EM), enamel dentin junction towards enamel (EDJE), enamel dentin junction towards dentin (EDJD), dentin middle (DM) and dentin inner surface (DIS)), G-P fitting method has been used for a wide energy range (0.015–15 MeV) up to the penetration depth of 40 mean free paths. Variation in EABF and EBF with incident photon energy and penetration depth has also been studied for which Photon dose multiplication factor K (E,x) plays an important role. The significant variations were also observed in EABF and EBF which may be due to the variation in chemical composition of the chosen teeth. Total mass attenuation coefficient of chosen teeth with incident photon energy in different interaction processes is also studied. Due to predominance of Compton scattering process the maximum percentage difference between EABF and EBF upto 50% has been observed for selected materials at 0.2 MeV energy but for EDJD it is approx. 18%. The computed EABF and EBF are useful to estimate the relative dose distribution in different regions of teeth.

Index Terms—Gamma ray, Buildup factor, Energy absorption, Exposure, Teeth.

I. INTRODUCTION

The tooth consists of connective tissues and mineralized and non-mineralized epithelial, which provide their physical properties. The hardest and the most mineralized tissue in the human body is anatomic crown which is covered by enamel and supported by dentin, an elastic, avascular, hard connective tissue. The root is formed by dentin attached to the cementum, which are mineralized tissues showing inorganic content similar to bone. The dentin is formed from and supported by the dental pulp, a soft connective tissue; sum set the pulp-dentin complex [1]. Considering such properties, Teeth have been selected to visualize the feasibility of using these materials as gamma ray shielding materials. When photons enter the human body they degrade their energy and produce secondary photons. The estimation of these buildups photons is done by buildup factor which explains the interaction of radiation with matter especially with human body [2]. The lambert-beer law which provides the relationship between gamma ray intensities before (I₀) and after (I) passing through the interacting material of linear attenuation coefficient (μ) and finite thickness (x). This law is applicable when the following three conditions are satisfied: (1) radioactive source must be mono-energetic, (2) target/interacting material must be thin , (3) Here must be used narrow beam geometry. In case, any of these conditions were not obey, then the modified lambert-beer law (I= B_{1}e^{-\mu x}) is used for the intensity relation, where B represent the buildup factor its value is 1 for that condition [3].

To calculate buildup factor there are different methods like G.P. fitting method (Harima et al. 1986 [4], invariant embedding method (Shimizu, 2002, Shimizu et al. 2004 [5,6]), iterative method (Suteau and Chiron, 2005 [7] and Monte Carlo method, Sardari et al., 2009 [8]. American National Standards, ANSI/ANS 6.4.3., 1991 [9] calculated buildup factor for 23 elements, one compound and two mixtures viz. water, air and concrete at 25 standard energies in the energy range 0.015-15.0 MeV up to penetration depth of 40 mean free path using G.P. fitting method. The buildup factor data have been computed by various codes such as PALLAS [10], EGS [11] and ASFITT [12], which use a point value for the energy variable and an accurate algorithm for the Compton scattering cross section according to the Klein-nishima formula.


Mass attenuation coefficients of the given materials have been calculated by the DOS based compilation XCOM (Berger and Hubbell, 1987; Hubbell and Seltzer, 1995) [16]-[17] provides total mass attenuation coefficient and total attenuation cross section data for about 100 elements as well as partial cross sections for incoherent and coherent scattering, photoelectric absorption and pair production at energies from 1 keV to 100 GeV.

Recently, different workers had providing data of gamma ray build up factor and attenuation coefficients of different materials such as for flyash concretes (18), teeth (19), enamel and dentin (1), amino acids, fatty acids and carbohydrates (20), polymers (21), low- Z building materials (22), Heavy metal oxide glass (23), human organ and tissues (24), Au alloy (25), soils samples (26) and samples from the earth, moon and mars (27).

In the present work, we study the EABF and EBF by using the G-P fitting method for different types of teeth in the energy
region 0.015-15 MeV up to penetration depth of 40 mfp. The chemical composition of different types of teeth is taken from paper of 'Microhardness and chemical composition of human tooth' by Maria del Pilar et.al. (28) shown in table (1). The generated EABF and EBF data have been studied as a function of incident energy, penetration depth. The comparison of EABF and EBF has been made and significant variation was noted. Also study the variation of mass attenuation coefficients with incident photon energy in different processes like photo electric process, Compton scattering process and pair-production effect.

Table 1. Chemical composition of various types of teeth in (%)

<table>
<thead>
<tr>
<th>Element</th>
<th>Outer Surface</th>
<th>Middle</th>
<th>EDJE</th>
<th>EDI</th>
<th>Middle</th>
<th>Inner</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>38.59</td>
<td>36.28</td>
<td>37.05</td>
<td>59.00</td>
<td>52.27</td>
<td>49.84</td>
</tr>
<tr>
<td>O</td>
<td>32.59</td>
<td>34.21</td>
<td>34.51</td>
<td>30.67</td>
<td>30.57</td>
<td>33.54</td>
</tr>
<tr>
<td>Na</td>
<td>0.24</td>
<td>0.44</td>
<td>0.46</td>
<td>0.47</td>
<td>0.42</td>
<td>0.36</td>
</tr>
<tr>
<td>Mg</td>
<td>0.16</td>
<td>0.23</td>
<td>0.22</td>
<td>0.25</td>
<td>0.34</td>
<td>0.45</td>
</tr>
<tr>
<td>P</td>
<td>10.67</td>
<td>10.86</td>
<td>10.46</td>
<td>4.41</td>
<td>6.23</td>
<td>6.32</td>
</tr>
<tr>
<td>Cl</td>
<td>0.39</td>
<td>0.25</td>
<td>0.09</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Ca</td>
<td>17.36</td>
<td>17.74</td>
<td>16.99</td>
<td>5.19</td>
<td>9.15</td>
<td>9.50</td>
</tr>
</tbody>
</table>

II. THEORY

Buildup factor is one of those important properties of a material used for beam collimation, tissue compensation or radiation shielding and protection. It directly affects the dose quantity.

\[ B = \frac{\text{Total broad beam count}}{\text{Total narrow beam counts}} \]

The value of \( B \) is a function of radiation type and energy, attenuating medium and thickness, geometry, and measured quantity [29]. Buildup factors depend on the incident photon energy of the radiations via total linear attenuation coefficient which is needed for the correction of the attenuation of the uncollided beam. Buildup factor also depends on the detector response function. There are two types of buildup factor:-

Energy absorption buildup factor: - It is defined as that photon buildup factor in which the quantity of interest is the absorbed or deposited energy in the medium, and the detector response function is that of absorption in the material. Exposure buildup factor: - It is defined as that photon buildup factor in which the quantity of interest is the exposure and the detector response function is that of absorption in air.

Mass attenuation coefficient is widely used in calculations of photon penetration and energy deposition in the teeth materials.

A narrow beam of mono-energetic photons having an initial intensity \( I_0 \) is attenuated to an intensity ‘I’ after passing through a layer of material with mass-per-unit-area ‘\( x \)’, according to the exponential law:

\[ I = I_0 e^{-\mu \rho x} \]

Where \( \mu \) is the mass attenuation coefficient. That equation can be rewritten as:

\[ \frac{\mu}{\rho} = \ln \left( \frac{I_0}{I} \right) / x \]

Therefore, mass attenuation coefficient \((\mu_m = \mu / \rho)\) can be calculated by substituting the measured values of \( I_0, I \) and \( x \) in above equation. The mass attenuation coefficient \((\mu_m = \mu / \rho)\) is of more fundamental importance than linear attenuation coefficient \((\mu)\) because all mass attenuation coefficients are independent of the density and physical state (gas, liquid or solid) of the absorber.

The attenuation coefficient is a measure of the average number of interactions between incident photon and matter that occur in a given mass-per-unit area thickness of the material encountered i.e. the attenuation coefficient is interaction process dependent. It is distinguished sharply from the absorption coefficient which is always a smaller quantity and absorption coefficient measures the energy absorbed by the medium. The theoretical values of \((\mu_m = \mu / \rho)\) for present teeth materials were calculated by XCOM program.

III. COMPUTATIONAL WORK

To calculate the buildup factors, the G-P fitting parameters were obtained by the method of interpolations from the equivalent atomic number \((Z_{eq})\). That computation is divided into three steps as follows:

A. Calculation of the equivalent atomic number \((Z_{eq})\)

The equivalent atomic number \(Z_{eq}\) for particular material has been calculated by matching the ratio, \((\mu / \rho)_{\text{Compton}} / (\mu / \rho)_{\text{Total}}\), of that material at a specific energy with the corresponding ratio of an element at the same energy. Thus, firstly the Compton partial mass attenuation coefficient, \((\mu / \rho)_{\text{Compton}}\) and the total mass attenuation coefficients, \((\mu / \rho)_{\text{Total}}\), were obtained for the elements of \(Z=4-40\) and for the teeth materials in the energy region 0.015-15 MeV, using the XCOM [16]-[17] computer program. For the interpolation of \(Z_{eq}\) for which the ratio \((\mu / \rho)_{\text{Compton}} / (\mu / \rho)_{\text{Total}}\) lies between two successive ratios of elements, the following formula has been employed (30):

\[ Z_{eq} = \frac{Z_1 (\log R_2 - \log R) + Z_2 (\log R - \log R_1)}{\log R_2 - \log R_1} \]  

(1)

Where \(Z_1\) and \(Z_2\) is the elemental atomic numbers corresponding to the ratios \((\mu / \rho)_{\text{Compton}} / (\mu / \rho)_{\text{Total}}\) respectively and \(R_1\) and \(R_2\) are the ratio for given teeth materials at a particular energy. The value of \(Z_{eq}\) for the selected teeth materials so obtained.

A. Calculation of geometric progression (G-P fitting parameters)

Calculate the G-P fitting parameter a similar interpolation procedure was adopted as in the case of the equivalent atomic number. The G-P fitting parameter for elements were taken from the ANSI/ANSI-6.4.3 [9] standard reference data base which provides the G-P fitting parameters for elements from beryllium to iron in the energy region 0.015-15 MeV up to 40 mfp. Formula given below is used in interpolation of G-P fitting buildup coefficient of the used materials:

\[ C = \frac{C_1 (\log Z_2 - \log Z_{eq}) + C_2 (\log Z_{eq} - \log Z_1)}{\log Z_2 - \log Z_1} \]  

(2)

Where \(C_1\) and \(C_2\) are the values of coefficients (G-P fitting parameters) corresponding to the atomic numbers of \(Z_1\) and \(Z_2\) respectively, at a given energy and \(Z_{eq}\) is the equivalent atomic number of the given material.
B. Calculation of energy absorption and exposure buildup factors

The G.P fitting parameters were then used to generate energy absorption and exposure buildup factor data for these materials using the following G.P fitting formula given by Harima et al. [4]

\[ B(E, x) = 1 + \frac{(b - 1)(K^x - 1)}{K - 1} \quad \text{for} \quad K \neq 1 \]  

\[ B(E, x) = 1 + (b - 1)x \quad \text{for} \quad K = 1 \]  

Where

\[ K(E, x) = cx^a + d \frac{\tanh(x / X_k) - \tanh(-2)}{1 - \tanh(-2)} \]  

for \( x \leq 40 \text{mfp} \)  

Where \( E \) is the incident photon energy, \( x \) is the penetration depth in mean free path, \( a, b, c, d \) and \( X_k \) are the G-P fitting parameters and \( b \) is the value of buildup factor at 1 mfp. The parameter \( K(E, x) \) is the photon dose multiplication factor and change in the shape of the spectrum.

IV. RESULT AND DISCUSSION

A. The validity of our interpolation method

In order to check the validity of present method, EABF and EBF values for air are compared with EABF and EBF values given by ANSI/ANS-6.4.3 data [9] for some selected energies ranges from 0.015 MeV-15 MeV, and up to penetration depths of 40 mfp. From fig. 1-2 we conclude that the energy absorption and exposure buildup factors generated by our computational procedure are in good agreement with those given by ANSI/ANS (1991) [9] database for air. This shown reliability in our results.

![Fig. 1 and 2. EABF and EBF of air obtained in the present work (G-P method) compared with those of the ANSI/ANS-6.4.3 standard [9] at selected incident photon energies (in MeV)](image)

A. Buildup factor of teeth

The computed energy absorption and exposure GP fitting parameters are used to generate energy absorption and exposure buildup factors values. We discuss below in subsequent sections how the buildup factors vary with penetration depths and incident photon energy

1) Buildup factor of selected materials as a function of chemical compositions and penetration depths

The variation of generated energy absorption and exposure buildup factor have been studied with penetration depth for all the selected teeth materials for some selected incident photon energies 0.015, 0.1, 1 and 15 MeV up to a penetration depths of 40 mfp shown in Figs. 3 to 10 respectively.

However at low and higher incident photon energies, the increase in value of buildup factor is at lesser rate as compared to the intermediate energy region.

It is observed that in figs. 3 and 7, the variation of buildup factors of selected teeth materials is low at low incident photon energy (15 keV), buildup factor lies within the range of 1-1.5, even for the large penetration depth of 40 mfp. It may be due to the reason that in this energy region, photoelectric process is the dominant one and the small variation is due to the coherent scattering process.

It is shown in figs. 4-6 and 8-10. Further with the increase in incident photon energy, the increasing rate of the buildup factor with the penetration depth first becomes more rapid at the certain incident photon energy range of 100 keV, where the Compton scattering process is most dominant and afterward the increasing trend of buildup factor becomes slower and slower for higher energies up to 15 MeV is due to dominance of pair-production process.

The slower increasing trend in the lower and higher incident photon energy region was obvious as the dominant process in these incident photon energy regions were photoelectric effect and pair-production respectively, which results gamma photons are completely absorbed in the interacting medium, whereas in the intermediate energy region the dominant process is the Compton scattering, which results energy of mostly gamma photons is degrade. Among the selected teeth, EM with highest Z_equ shows the minimum value for the EABF and EBF in lower energy region upto 1 MeV whereas maximum values are observed at energy 15 MeV. Whereas in case of EDJ with lowest Z_equ exact reverse trend can be seen.
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Fig. 3 to 10. Variation of generated EABF and EBF for selected teeth for some selected incident photon energies 0.015, 0.1, 1 and 15 MeV up to a penetration depths of 40 mfp

2) Buildup factor of selected materials as a function of incident photon energy

The variation of energy absorption and exposure buildup factors for selected teeth materials with incident photon energy for some randomly selected penetration depths (5, 15, 25, 35, 40 mfp) are shown in figs. 11 to 22.

Starting from 1 to 40 mfp in both the cases, it is observed that, buildup factor values are relatively low for energy less than $E_{pc}$. Where $E_{pc}$ is the energy value at which photoelectric attenuation coefficient matches the Compton attenuation coefficient and $(\mu_m)_{pc}$ is mass attenuation coefficient where take $E_{pc}$. The buildup factor values are low at energies less than or equal to $E_{pc}$ because photoelectric process is the most dominant process in this energy range.

It is observed that in the medium energy range $E_{pc} < E < E_{pp}$ the values of buildup factors are high due to dominance of Compton effect, where $E_{pp}$ is the energy value at which the compton attenuation matches the pair-production coefficient and $(\mu_m)_{pp}$ is mass attenuation coefficient where take $E_{pp}$.

It is also observed that with the incident photon energy range 100 keV to 400 keV, the buildup factor value is very high because of exclusive dominance of compton effect. This results a broad peak occurs in the build-up factor around a particular value of incident photon energy ($E_{peak}$) for all the selected teeth materials.

Furthermore it is observed that for incident photon energies greater than 1.0 MeV, the dominance of the pair-production phenomenon over Compton Effect increases, resulting in lowering of buildup factor values at these incident photon energies of all the selected materials.

The reason for this behavior of changing trend in buildup factor with incident photon energy is because of the
Z-dependence of different photon interaction processes. In comparison to the linear Z-dependence of compton scattering, the Z-dependence of photoelectric and pair-production processes respectively are $Z^{1.5}$ and $Z^2$. As a result of photoelectric and pair-production processes, there is probability of the fast removal of photon in low and high energy regions because of their absorption behavior. Where as in the medium energy region the probability of buildup of photons is large which results in a broad peak having maxima ($E_{\text{peak}}$) around 0.2 MeV. This indicates that maximum multiple scattering occurs around this incident photon energy value which results in the accumulation of photons, because larger number of compton scatterings is required to degrade the energy of the photons. Therefore degraded energy photons exist for a longer time which results in their buildup in the material. That is why there is a high value of buildup factor in the medium energy region. Among the selected teeth, EM (highest $Z_{\text{eq}}$) shows the minimum value for the EABF and EBF, whereas maximum values are observed for EDJ (lowest $Z_{\text{eq}}$). It may be due to the reason that EM, which is a teeth of phosphorous (% weight fraction = 10.86) and calcium (weight fraction = 17.74), has the maximum equivalent atomic number due to the major contribution of phosphorous and calcium. Whereas EDJ consists of phosphorous (weight fraction = 4.41) and calcium (weight fraction = 5.19) and has the minimum equivalent atomic number. From this observation, it can be concluded that EABF and EBF is inversely proportional to the equivalent atomic number of the ceramics at lower penetration depths (below 15 mfp).
in the energy range 5-15 MeV the value of EABF is less than EBF. Max. difference approximately up to 13% for EOS, EM and EDJE but for EDJ, DM and DIS that value is approx. 4%, where the pair production process is the main interaction process, where values of photon buildup factors are lower.

The Z$_\text{eq}$ of air lies between 7 and 8, where Z$_\text{eq}$ values of selected teeth EOS, EM, EDJE, EDJ, DM and DIS are in range 9-13, 9-13, 7-10, 8-12 and 8-12 respectively, so all the teeth materials have higher values of Z$_\text{eq}$ when compared to air. Thus, the absorption inside the medium is much more than the absorption in air, therefore EABF values are greater than EBF in energy region, where photons are more buildup.

The EABF refers to that absorbed of deposited energy in the attenuating material while EBF is based on the energy absorption response of air, there are significant differences between EABF and EBF in the continuous energy region. From Figs. 23-28 the values of EABF are more than EBF in the energy range of 0.04-2.0 MeV and max. % difference upto 50% has been observed for selected materials at 0.2 MeV energy but for EDJ approx. 18% max. difference for at 0.17 MeV energy, where the Compton scattering is the main interaction process. As we go ahead we observed from graph
VI. MASS ATTENUATION COEFFICIENT

The elemental composition of the various types of teeth are tabulated in Table 1. Mass attenuation coefficient for the total photon interaction processes $\mu_{\text{total}}$ includes the contributions of different photon interaction processes (photoelectric absorption, Compton scattering and pair-production). The total mass attenuation coefficients versus incident photon energy of the various teeth are shown in Fig. 29.

It is observed that $\mu_{\text{total}}$ of each selected material is initially high and decreases sharply with increase in incident photon energy up to 100 keV. This is due to the fact that cross-section for photoelectric absorption is proportional to $Z^2$. In the incident photon energy region above 100 keV to 4 MeV the $\mu_{\text{total}}$ of selected materials have almost same values due to the dominance of Compton scattering process in this incident photon energy region. This is attributed to the linear Z-dependence of Compton scattering process. Above 4 MeV, there is again slight variation in $\mu_{\text{total}}$ with incident photon energy because in this energy region pair-production process dominance. It is due to the fact that the cross-section of pair-production process is $Z^2$ dependent.

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Compensation of Mass attenuation coefficient, energy absorption and exposure build-up factors for teeth in the energy range 0.015-15 MeV up to 40 mfp


Sandeep Gupta, M.Sc, NET-JRF and M.Phil Physics, 4 international journal paper, membership of ISC.

Gurdeep Singh Sidhu, M.Sc, Ph. D Physics, 14 international journals paper and 5 national journal paper and Total Number of Supervision: Ph. D.: 5 M.Phil: 10.