

# Optimized Iterative Clipping and Filtering for PAPR Reduction of OFDM Signals

Priyanka Jain, Ramesh Bharti

**Abstract**— Orthogonal Frequency Division Multiplexing (OFDM) has become the most widely adopted technology in wireless communication systems. OFDM is limited mainly by its high Peak-to-Average Power Ratio (PAPR). The optimized iterative clipping and filtering technique (optimized ICF) efficiently reduces PAPR after only 1 or 2 iterations. Also, the clipped OFDM symbols obtained by the optimized ICF technique have low out-of-band radiation. However, clipping the peak envelope of the input signal to a predetermined value causes in-band distortion. Thus, the optimized ICF method reduces PAPR at the cost of increase in bit error rate (BER). This work proposes the distortion compensated optimized ICF technique, in which the distortion due to clipping is encoded as clipping information and is transmitted in the next block. In the receiver, the in-band distortion is compensated by the clipping information. The proposed scheme can achieve nearly same PAPR reduction effect, same out-of-band radiation and better BER performance as that of the optimized ICF technique, at the cost of slight loss of spectral efficiency.

**Index Terms**— Orthogonal frequency division multiplexing, peak-to-average power ratio, optimized iterative clipping and filtering, distortion compensation, bit error rate.

## I. INTRODUCTION

Recently, multimedia applications for wireless communications are widespread. The requirements of high data rate transmission for wireless communications become necessary. Orthogonal frequency division multiplexing (OFDM) is a promising technique owing to the high spectral efficiency, good immunity against multipath effects and narrow band interferences, and easy implementation [1].

However, due to the inherent multicarrier nature of OFDM, the summation of subcarriers with the same phase will usually lead to a high peak-to-average power ratio (PAPR) of the time-domain signal [2]. This high PAPR signal, when transmitted through a nonlinear power amplifier, creates spectral broadening and also will increase the dynamic range of the digital to analog converter (DAC). The result will be an increase in the cost of the system and efficiency degradation.

Accordingly, reducing the PAPR of an OFDM signal is imperative. The PAPR problem of OFDM has received much interest from the research community and a number of techniques have been developed to reduce it [2]–[3]. The iterative clipping and filtering (ICF) procedure maybe the simplest to approach a specified PAPR threshold in the

processed OFDM symbols among all these existing techniques [4], [5]. However, clipping the input signal to a specified value in the time-domain will cause in-band distortion and out-of-band radiation. Filtering can reduce out-of-band radiation, but it cannot reduce in-band distortion and this result in bit error rate (BER) performance degradation. Also, the use of a fixed rectangular-window for filtering in the frequency-domain requires many iterations to approach specified PAPR threshold in the complementary cumulative distribution function (CCDF).

The optimized ICF scheme proposed in [6] modifies the classic ICF by replacing the rectangular-window filter with the one designed by convex optimization techniques. The optimized ICF procedure requires only 1 or 2 iterations to reach a given PAPR level while simultaneously achieving a sharp drop of CCDF and low out-of-band radiation. But the clipped OFDM symbols in the optimized ICF scheme undergo a significant nonlinear distortion degrading the BER performance of the OFDM system.

This paper proposes a distortion compensated optimized ICF scheme, which can achieve nearly same PAPR reduction effect as that of the optimized ICF technique, while maintaining the BER performance of the OFDM system. Simulation results demonstrate the effectiveness of the proposed approach.

## II. SYSTEM MODEL AND PAPR DEFINITION

### A. OFDM Symbol

Let  $c \in \mathbb{C}^N$  be the frequency-domain OFDM symbol,  $\{c(i), i = 1, \dots, N\}$  be the symbol value carried by the  $i$ -th sub-carrier,  $l$  be the over-sampling factor and  $k$  be the time index.

Then, the time-domain OFDM symbol,  $x \in \mathbb{C}^{lN}$ , corresponding to  $c$  is expressed as

$$x(k) = \frac{1}{\sqrt{lN}} \sum_{i=1}^N c(i) e^{j \frac{2\pi}{lN} ki}, k = 1, \dots, lN. \quad (1)$$

The frequency-domain OFDM symbol  $c$  is computed using

$$c(i) = \frac{1}{\sqrt{lN}} \sum_{k=1}^{lN} x(k) e^{-j \frac{2\pi}{lN} ki}, i = 1, \dots, N. \quad (2)$$

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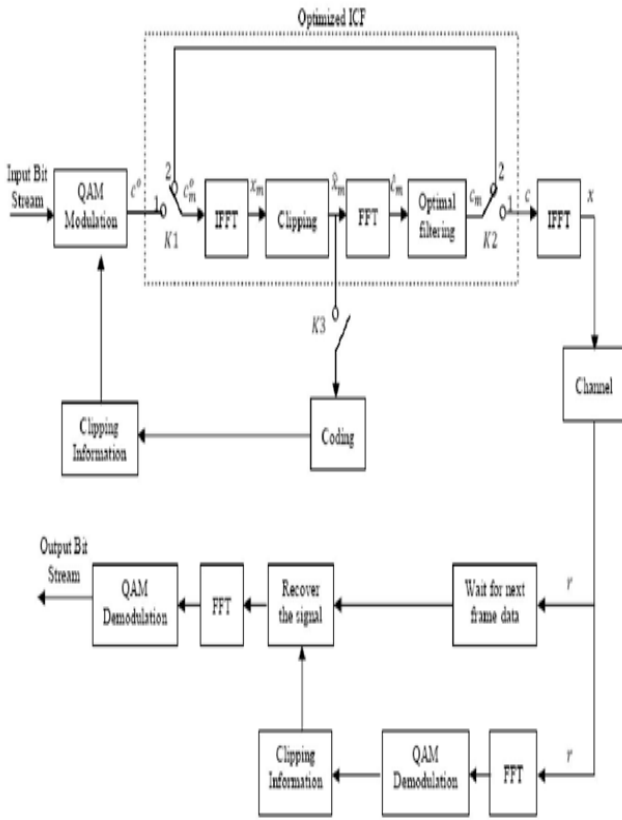


Fig. 1. Block diagram of a baseband OFDM system utilizing the proposed distortion compensated optimized ICF scheme, where  $c^0 \in \mathbb{C}^N$ ,  $c_m^i \in \mathbb{C}^N$ ,  $x_m \in \mathbb{C}^N$ ,  $\hat{x}_m \in \mathbb{C}^N$ ,  $t_m \in \mathbb{C}^N$ ,  $c_m \in \mathbb{C}^N$ ,  $c \in \mathbb{C}^N$ ,  $x \in \mathbb{C}^N$  and  $r \in \mathbb{C}^N$ ,  $m = 1, \dots, Max$  denotes the iteration number and  $Max$  is a preset maximum number of iterations to perform.

B. PAPR

PAPR is defined as the ratio of the maximum instantaneous power to the average power and is related to OFDM constellation reshaping. The PAPR of the time-domain OFDM symbol,  $x$ , can be defined as

$$PAPR = \frac{\max_{k=1 \dots IN} |x(k)|^2}{\frac{1}{\sqrt{IN}} \sum_{k=1}^{IN} |x(k)|^2} = \frac{\|x\|_\infty^2}{\frac{1}{\sqrt{IN}} \|x\|_2^2}, \quad (3)$$

where  $\|\cdot\|_2$  and  $\|\cdot\|_\infty$  stands for the 2-norm and the  $\infty$ -norm respectively.

III. PROPOSED TECHNIQUE

A. An Optimized ICF Method

An optimized ICF method as described in [6] modifies each OFDM symbol one at a time (See optimized ICF block in figure 1). For convenience, we use  $c^0$  and  $x^0$  to represent the original (undistorted) frequency-domain and time-domain OFDM symbols, respectively, and  $c$  and  $x$  for the processed frequency-domain and time-domain OFDM symbols.

In the first iteration of optimized ICF method ( $m = 1$ ), switch K1 is set to 1 and the new OFDM symbol enters the optimized ICF block. Then, both K1 and K2 are set to 2 and clipping and optimal filtering is iteratively performed. In the Max-th (final) iteration, the switches are returned to position

1 and thus the output  $c$  is produced. More specifically, the clipping procedure is performed by

$$\hat{x}_m(k) = \begin{cases} T_m e^{j\theta_m(k)}, & |x_m(k)| > T_m \\ x_m(k), & |x_m(k)| \leq T_m \end{cases}, \quad (4)$$

where  $1 \leq k \leq IN$ ,  $\theta_m(k)$  represents the phase of  $x_m(k)$ , and  $T_m$  the clipping level in the  $m$ -th iteration. The clipping level is recalculated in each iteration according to a constant value called the clipping ratio (CR), which is defined as the ratio of the clipping level to the root mean square value of the unclipped signal and it is related to the desired PAPR (denoted PAPRmax) as follows:

$$CR = \sqrt{PAPR_{max}} = \frac{T_m}{rms_m}, m = 1, 2, \dots, Max, \quad (5)$$

where  $rms_m = \frac{1}{\sqrt{IN}} \|x_m\|_2$  is the root mean square value of the unclipped signal in  $m$ -th iteration.

In the optimized ICF method, the filtering is done using the optimal filter designed by convex optimization technique. Let  $\hat{c}_m$  denote the frequency-domain OFDM symbol at the  $m$ -th iteration (See figure 1) then the filter design problem (6) is given by

$$\min_{H_m \in \mathbb{C}^N, t \in \mathbb{R}} t \quad (6a)$$

$$\text{subject to } \|c^0 - \hat{c}_m \cdot H_m\|_2 \leq \|c^0\|_2 t \quad (6b)$$

$$\|A(\hat{c}_m \cdot H_m)\|_\infty \leq T_{m+1} \quad (6c)$$

Here,  $H_m$  is the frequency response of the optimal filter at iteration  $m$ ;  $c_m^i$  and  $\hat{c}_m^i$  are the in-band components, i.e.,  $c_m^i = [c_m(1) \dots c_m(N)]^T$ ,  $\hat{c}_m^i = [\hat{c}_m(1) \dots \hat{c}_m(N)]^T$ ;  $t = \frac{\|c^0 - \hat{c}_m\|_2}{\|c^0\|_2}$  is an auxiliary variable; the operator ‘ $\cdot$ ’ denotes element-by-element product;  $A$  is a matrix, which consists of the first  $N$  columns of  $IN$ -IFFT twiddle factor matrix and  $T_{m+1} = \frac{1}{\sqrt{IN}} \|\hat{x}_m\|_2 CR$  is an approximated clipping level.

Due to use of optimal filter, the optimized ICF procedure requires only 1 or 2 iterations to reach a given PAPR level. Moreover, the clipped OFDM symbols obtained by the optimized ICF method have low out-of-band radiation.

But these symbols undergo a significant nonlinear distortion, degrading the BER performance of the OFDM system.

B. Proposed distortion compensation scheme

By making several modifications in the distortion compensation scheme given in [7], we propose a distortion compensation scheme for optimized ICF technique. The block diagram of a baseband OFDM system under the AWGN channel with the proposed distortion compensated optimized ICF scheme is depicted in figure 1. In the first iteration ( $m = 1$ ), switch K1 is set to 1 and the new OFDM symbol enters the optimized ICF block. Then, both K1 and K2 are set to 2 and clipping and optimal filtering is iteratively performed. When both K1 and K2 are set to 2, the switch K3 is kept open. In the Max-th (final) iteration, the switches K1 and K2 are returned to position 1 which produces the output  $c$  and the switch K3 is closed to generate the clipping information as explained below

IV. SIMULATION RESULTS

We compare unprocessed time-domain symbol,  $x_1$  and clipped time-domain symbol in the  $Max$ -th iteration,  $\hat{x}_{Max}$  and accordingly record the  $h$  largest amplitude points that are clipped as  $x_p = [x_{p_1}, x_{p_2}, \dots, x_{p_h}]$  and their position as  $P = [p_1, p_2, \dots, p_h]$ . We calculate clipping noise,  $D = [d_{p_1}, d_{p_2}, \dots, d_{p_h}]$  where:

$$d_{p_n} = 20 \log \left( \frac{|x_{p_n}|}{rms_{Max}} \right), n = 1, 2, \dots, h. \quad (7)$$

Here,  $rms_{Max}$ , is the root mean square value of the unclipped time domain symbol in the  $Max$ -th equation. Also calculate  $\Delta d_{p_n} = [d_{p_n} - 20 \log(CR)]$ ,  $n = 1, 2, \dots, h$ , where  $[x]$  is the nearest integer to  $x$ .

While calculating  $\Delta d_{p_n}$  we approximate  $d_{p_n} - 20 \log(CR)$  to its nearest integer value, so  $\Delta d_{p_n}$  can be zero for some

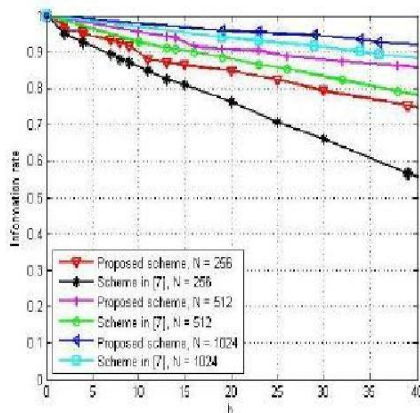


Fig. 2. Information rate comparison for different values of  $N$  for the proposed distortion compensation scheme and the distortion compensation scheme given in [7].

values of  $n$ . Transmitting these zero valued  $\Delta d_{p_n}$  to the receiver do not contribute for distortion compensation but result in lower information rate. Thus, if for any value of  $n$ ,  $\Delta d_{p_n} = 0$ , discard  $\Delta d_{p_n}$  and the corresponding position,  $p_n$  so that  $P' = [p_1, p_2, \dots, p_z]$  and  $\Delta d'_{p_n} = [d_{p_n} - 20 \log(CR)]$ ,  $n = 1, 2, \dots, z$ , where  $z$  is the number of amplitude points for which  $\Delta d_{p_n} \neq 0$ .

Let  $L$  be the length of the clipping information. Encode the clipping information  $L$ ,  $rms_{Max} \cdot \Delta d'_{p_n}$  and  $P'$  by binary code and then this information will be included in the transmit data of the next block.

At the receiver, denote the received signal as  $r = [r_1, r_2, \dots, r_N]$ . Use  $L$  to separate clipping information from the processed OFDM symbol. Recover the received signal  $r$  by the clipping information as:

$$r_{p_n} = \left| 10^{\frac{(\Delta d'_{p_n} + rms_{Max})}{20}} \right| rms_{Max} e^{j\theta(r_{p_n})}, n = 1, 2, \dots, z. \quad (8)$$

where  $\theta(r_{p_n})$  is the phase of  $r_{p_n}$ . Thus the distortion due to clipping at the transmitter is compensated at the receiver by this way.

The information rate for a block of  $N$  symbols, for the proposed distortion compensation scheme, is given as,

$$I = 1 - \frac{L}{N}. \quad (9)$$

In figure 2, the information rate as a function of  $h$  for different values of  $N$  for the OFDM system with 64-QAM modulation and over-sampling factor  $l = 4$  is plotted for the proposed distortion compensation scheme and the distortion compensation scheme given in scheme in [7]. It can be seen from the figure that the information rate of both the proposed distortion compensation scheme and the distortion compensation scheme given in [7] is relatively high for higher

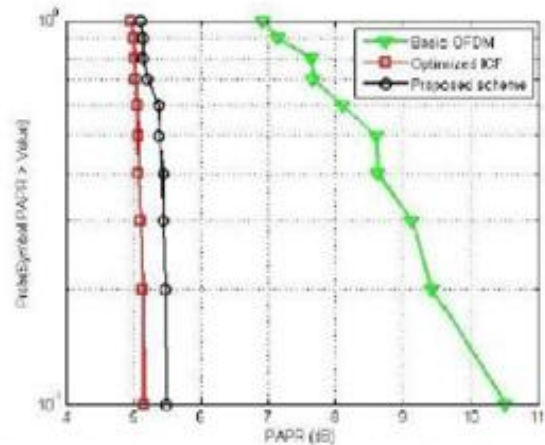


Fig.3. PAPR statistics for an OFDM system (256 sub-carriers, 64-QAM modulation, and over-sampling factor  $l = 4$ ).

values of  $N$ . It is also observed that the information rate of our distortion compensation scheme is better than that of the distortion compensation scheme given in [7] for the given value of  $N$ . This is because in the proposed scheme we transmit clipping information for  $z$  amplitude points instead of  $h$  amplitude points.

We now present simulation results for a 256 sub-carrier OFDM system with 64-QAM modulation and over-sampling factor  $l = 4$ . The convex optimization problem presented in (6) is solved using the public software CVX [8]. Figure 3 plots the CCDFs of the PAPR for basic OFDM symbols, modified symbols using the optimized ICF method, and modified symbols using our proposed method. For the optimized ICF method and our proposed method, the desired PAPR is set to 5dB (or CR=1.7783). From the figure, it can be seen that the proposed scheme can achieve nearly same PAPR reduction effect as that of the optimized ICF technique while simultaneously achieving a comparable sharp drop in the CCDF.

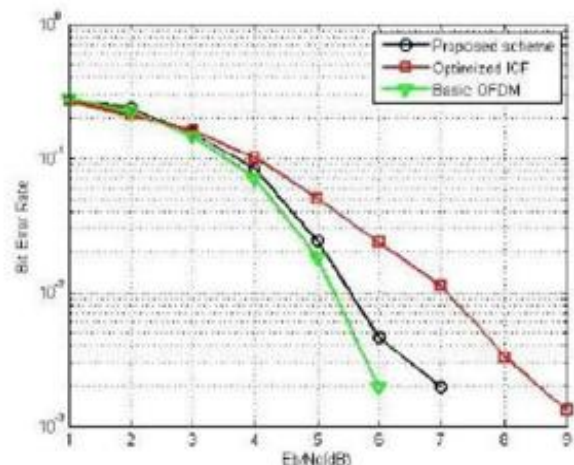


Fig.4. Comparison of BER performance.

Figure 4 plots the bit error rate curves of the basic OFDM signal, the modified signal using the optimized ICF method, and modified signal using our proposed method through an AWGN channel. From figure it is clear that the proposed scheme has 1.5 dB improvement comparing to the optimized ICF method at  $BER = 10^{-2}$ .

Finally, we consider passing the clipped signals through a solid-state power amplifier (SSPA), which is modelled by

$$s_o(t) = \frac{|s_i(t)|}{[1+(\frac{|s_i(t)|}{C})^{2p}]^{1/p}} e^{j\theta(t)}, \quad (10)$$

where  $s_i(t) = Gxe^{j\theta(t)}$  is the input signal,  $s_o(t)$  is the output of SSPA, and  $G$  is gain of SSPA. The out-of-band radiation comparison is shown in figure 5. In the simulation,  $G = 27\text{dB}$  and SSPA parameters  $p$  and  $C$  are set 3 and 1dB, respectively. We can see that the out-of-band radiation for both the optimized ICF method and our proposed method are overlapping and both the methods lead to about 10dB lower out-of-band radiation than without using any PAPR reduction technique at normalized frequency 0.8.

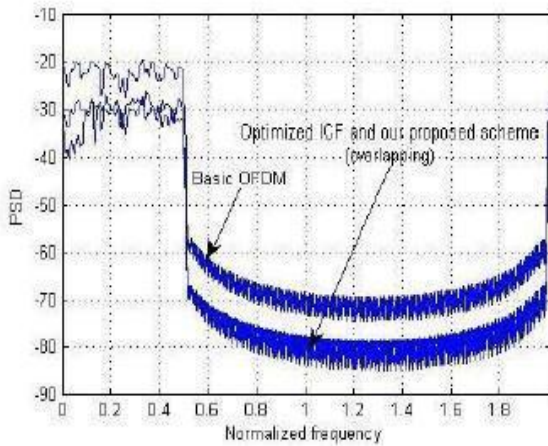


Fig. 5. Out-of-band radiation comparison of OFDM signals through SSPA: optimized ICF, proposed scheme, and none-processed.

## V. CONCLUSIONS

The optimized ICF method can alleviate the high PAPR problem in OFDM systems, but causes BER performance degradation. In this paper, the distortion compensated optimized ICF scheme is proposed to mitigate the clipping effect. Our investigations, performed by considering 256 subcarrier OFDM system with 64-QAM modulation and oversampling factor  $l = 4$ , have shown that this technique can achieve nearly same PAPR reduction effect, same out-of-band radiation and better BER performance than that of the optimized ICF technique. This is achieved at the cost of slight loss of spectral efficiency. It is also shown by simulations that the information rate of our distortion compensation scheme is better than that of the distortion compensation scheme given in [7] for the given value of  $N$ . The information rate of the proposed scheme is relatively high for higher values of  $N$ .

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