International Journal of Engineering and Technical Research (IJETR)
ISSN: 2321-0869, Volume-2, Issue-6, June 2014

System Identification and State Feedback Controller Design of Magnetic Levitation System

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***Abstract*— The aim of the paper is about system identification, modeling with validation and controller design of a magnetic levitation system. Magnetic levitation system is an electromagnetic device that suspends a ferromagnetic ball using electromagnetism. The Magnetic levitation system is a nonlinear, unstable system with fast dynamics. This report contains an overview in modeling of maglev system using least square estimation taking change in current as input and change in ball position as an output and state feedback controller design as well as implementation in real time Maglev system. The maglev system include an in build controller that levitated and stabilized a steel ball about an operating region. The main objectives are to obtain good model for the maglev system and to implement a different controller to stabilize the ball.**

***Index Terms*— Magnetic Levitation system, Least Square Estimation, Linear Quadratic Regulator, Electromagnetic Levitation force.**

I. INTRODUCTION

Magnetic Levitation Systems (MLS) have gained considerable interest due to their great Practical importance in many engineering fields.

The object of this project is to keep a metal ball suspended in mid-air by adjusting the field strength of an electromagnet. The electromagnet current may be increased until the Magnetic force produced is equal to or greater than, the gravitational force acting on the ball. Variations in the electromagnet current cause the ball to either fall (when current is decreasing) or be attached to the electromagnet (when current is increasing). The Feedback path control introduced aims to stabilize the ball when current disturbance occurs.

From the control engineering point of view, an MLS (or Maglev as it is sometimes called) is a quite complex system since it presents non-linearity and it is naturally unstable. The easiest (but not the best) way to handle this system is to consider only small variations around a given operating point. This allows linear control principles to be applied.

The Magnetic Levitation System allows the study of various control strategies. For Example :

- Classical analogue lead controllers.
- Classical discrete PID controllers.
- State feedback control.

Manuscript received June 20, 2014.

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Additionally, the MLS is a good illustrative process to perform system identification from closed-loop experimental data.

II. PROPOSED WORK

The identification, modeling with validation and controller design of a magnetic levitation system has been described below.

A. Maglev mechanical unit description

The Maglev Mechanical unit consists of a base, with a connection interface. On that base the mechanical unit is placed with the coil being mounted on top of the construction. An IR sensor is placed on the two side of the construction.

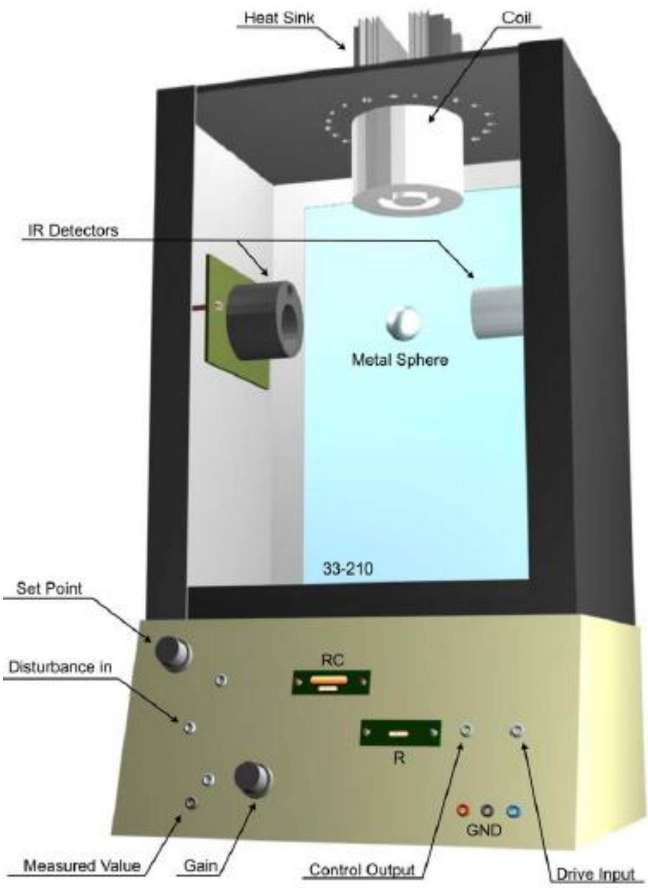


Fig.1. Maglev Mechanical Unit

B. Maglev control system

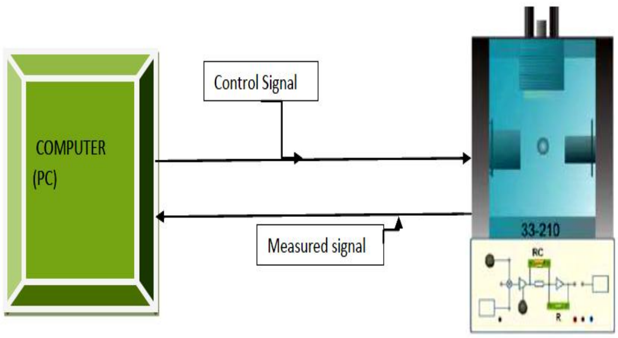
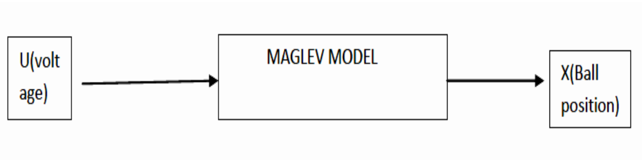


Fig.2

The computer with MATLAB and SIMULINK environment serve as the main control unit. The control signal which is voltage between [-5V to 5V] is transferred to the maglev unit, which causes current flow through the coil and thus magnetic field produce. The position of the ball is measured using IR sensor. The position information transferred to the PC via interface unit, where the entire control algorithm is placed in Simulink environment.



C. *System description:*

The control system consists of three inputs and one output. The inputs are:
1. Set point – adjusts the vertical position of the ball.
2. Reference input signal
3. Disturbances – such as power supply fluctuations, coil temperature variations and external forces applied to the ball.
The final output of the system is the actual ball position.

The applied control is voltage, which is converted into the urrent via a driver embedded within the unit. The current passes hrough an electromagnet, which creates the corresponding magnetic field in its vicinity. The sphere is placed along the vertical axis of the electromagnet. The measured position is determined from an array of infrared transmitters and detectors, positioned in such a way that the infrared beam is intersected by the sphere.

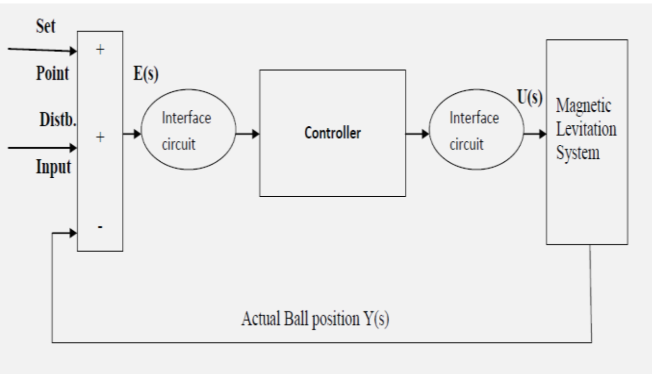


Fig.3. Control Architecture

D. *Coil characteristics:*

Description	Value
Core	Iron
Core Diameter	25mm
Coil Diameter	80mm
Number of Turns	2850
Resistance	22Ω
Inductance(L)	277 mH at 1Khz 442 mH at 120Khz

E. *Linear Quadratic Regulator*

For the derivation of the linear quadratic regulator, we assume the plant to be written in state-space form $\dot{x} = Ax + Bu$, and all the states are available for the controller. The feedback gain is a matrix K is implemented as $u = -K\,(x - x_{desired})$. The system dynamics are then written as
$\dot{x} = (A - BK)\,x + BK\,x_{desired}$
$x_{desired}$ represents the vector of desired states and serves as the external input to the closed loop system. The 'A' matrix of the closed loop system is '(A-BK)' and the 'B' matrix of the closed loop system becomes BK. The column dimension of 'B' equals the number of channels available in u, and must match the row dimension of K.

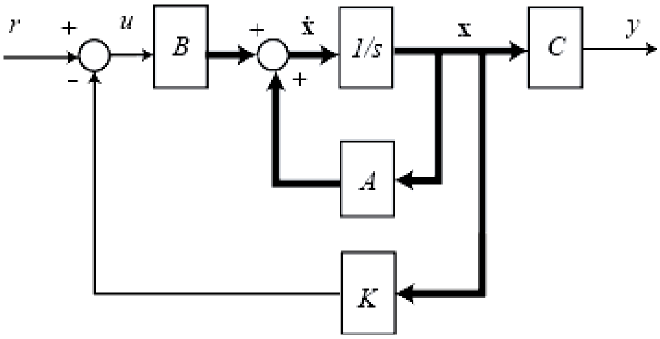


Fig.4

A system can be expressed in state variable form as:
$\dot{x} = Ax + Bu$ and $Y = Cx + Du$

With $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$. The initial condition is x (0). We assume here that all the states are measurable and seek to find a state-variable feedback (SVFB) control $u = -Kx$ that gives desirable closed-loop properties.

The closed-loop system using this control becomes:
$\dot{x} = (A-BK)\,x$; $(A-BK) = A_c$ = the closed-loop plant matrix.
The design procedure for finding the LQR feedback K is:
• Select design parameter matrices Q and R.
• Solve the algebraic Riccati equation for P.
• Find the SVFB using $K = R^{-1}B^{T}P$.
The Riccati equation= $A^{T}P + PA - PBR^{-1}B^{T}P + Q = 0$

✓ The system is full controllable, as [B AB] = rank (2)
So LQR method stands .

a) *Steps behind designing the controller:*

Considering the system, $\frac{\Delta x}{\Delta i} = \frac{-0.5145}{s^2+0.274}$

The steady state form of the Transfer Function is:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -0.274 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 0 & -0.5145 \end{bmatrix}, D = 0$$

We have chosen the value of LQR parameters like R as unity and Q as identity matrix to calculate the state feedback gain.

$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $R = 1$

State feedback gain as: $K = [-1.5893 \quad -0.7629]$

The two states of the 2^{nd} order system are displacement of the ball (first row element of gain matrix) and velocity of the ball (second row element of gain matrix).

The closed loop Transfer functions as:

$$\frac{\Delta x}{\Delta i} = \frac{-0.5145}{s^2 + 1.5893s + 1.0369}$$

III. Mathematical Modeling & Validation

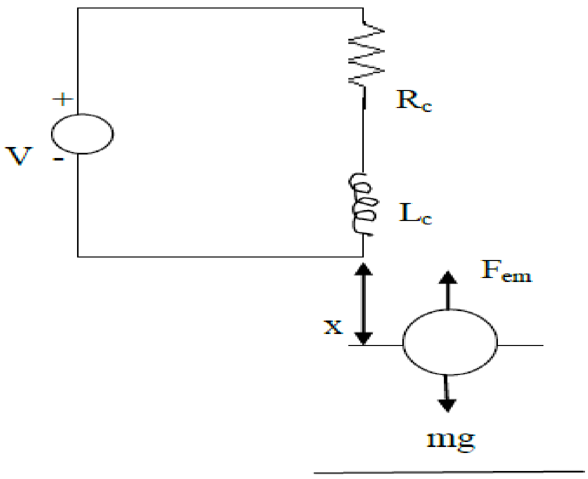


Fig.5

From the diagram and using basic equations we can write that:

$m\frac{d^2x}{dt^2} = mg - F_{em}$ …..(1)

And, $V = R_c i + L_c \frac{di}{dt}$ ….(2)

Where, i= electric current[A]

x= ball position[m]

m= Mass of ball[kg]

g= gravity[m/s^2]

R_c= coil resistance[Ω]

L_c= coil inductance [H]

[∴ we know Magnetic energy of a coil is $\omega_m = \frac{1}{2} Li^2$.

The electromagnetic levitation force can be determined using the theorems of the generalized forces:

$F_{em} = -[\frac{\partial \omega_m}{\partial x}]$;

The relation between coil inductivity and position of the ferromagnetic ball is:

$L(x) = L_1 + \frac{2k}{x}$;

where, L=inductivity[H],

x= position of the ball[m],

L_1= inductivity when the ball is present(x=0)

k- Coil constant [$N\text{-}m^2/A^2$]

$\therefore F_{em} = k\frac{i^2}{x^2}$]

At equilibrium point($x=x_0$ and $i=i_0$), constant current will flow to the maglev system. So we can write that $\frac{di}{dt} = 0$.

$\therefore V = R_c i + L_c \frac{di}{dt}$ in this equation we put the value of $\frac{di}{dt} = 0$ and we get : $V=Ri_0$.

So, $i_0 = \frac{V}{R}$ ….(3)

At equilibrium point, we assume acceleration is zero, so $\frac{d^2x}{dt^2}$ =0.

Now, the equation no (2) becomes: $mg = k\frac{i_0^2}{x_0^2}$

$\therefore x_0^2 = k\frac{i_0^2}{mg}$ $\therefore x_0 = i_0\sqrt{\frac{k}{mg}}$ ……(4)

Now, linearize the system about the point (x_0 and i_0)

Using Taylor's series expansion to the equation no (1) we get:

$\frac{d^2x}{dt^2} = -(\frac{\partial f(i,x)}{\partial i})|(i_0,x_0)\ \Delta i - (\frac{\partial f(i,x)}{\partial x})|(i_0,x_0)\ \Delta x$; (higher order terms are neglected)

Now, applying Laplace Transformation we get:

$S^2\, \Delta x = -(k_i\, \Delta i + k_x\, \Delta x)$

⇨ $\Delta x\,(S^2+k_x) = -k_i\, \Delta i$

⇨ $\frac{\Delta x}{\Delta i} = \frac{-k_i}{s^2+k_x}$;

Where, $k_i = \frac{\partial f(i,x)}{\partial i} = \frac{k.2i}{i_0} = \frac{2mg}{i_0}$ (from equation (4))

And, $k_x = \frac{\partial f(i,x)}{\partial x} = \frac{-k.2i^2}{mx^3} = -\frac{2mg}{x_0}$ (from equation (4))

∴ So the Transfer function of the system becomes $\frac{\Delta x}{\Delta i} = \frac{-k_i}{s^2+k_x}$

A. *Calculation:*

R_c= 21.5 Ω [From practical measurement]

$k = 1.477\times 10^{-4}\ N\text{-}m^2/A^2$ [Given]

m = mass of the ball = 0.021 kg [Given]

$L = \frac{\mu N^2 A}{l}$; $\mu = 4\pi \times 10^{-7}$, N= number of turns on coil =2850,

$A = \pi\,(0.038)^2 = 4.53\times 10^{-3}$,

l= 0.065m.

∴ L = 0.711 H

$i_0 = \frac{1}{R} V = 0.23$ [A]

$x_0 = i_0\sqrt{\frac{k}{mg}} = 0.15\times 10^{-2}$ [m]

$\therefore k_1 = \frac{2mg}{i_0} = 0.5145$

$\therefore k_x = \frac{2mg}{x_0} = 0.274$

∴ So the incremental Transfer function of the system becomes $\frac{\Delta x}{\Delta i} = \frac{-0.5145}{s^2+0.274}$

B. *Validation of Maglev model using least square estimation technique:*

$$\frac{Y(s)}{U(s)} = \frac{as+b}{s^3+cs^2+ds+e}$$

$\dddot{y}(t) + c\ddot{y}(t) + d\dot{y}(t) + ey(t) = a\dot{u}(t) + bu(t)$ …….(1)

Let, t=kT

So now we can write:

$u(t) = u(kT)$

$\dot{u}(t) = \frac{u(k+1)T - u(kT)}{T}$

$\frac{dy(t)}{dt} = \dot{y}(t) = \frac{y(k+1)T - y(kT)}{T}$

$\frac{d^2y(t)}{dt^2} = \ddot{y}(t) = \frac{y(k+2)T - 2y(k+1)T + y(kT)}{T^2}$

$\frac{d^3y(t)}{dt^3} = \dddot{y}(t) = \frac{y(k+3)T - 3y(k+2)T + 3y(k+1)T - y(kT)}{T^3}$

Now after putting the values in equation no (1) we get:

$\frac{y(k+3)T - 3y(k+2)T + 3y(k+1)T - y(kT)}{T^3} + c\,\frac{y(k+2)T - 2y(k+1)T + y(kT)}{T^2} + d\,\frac{y(k+1)T - y(kT)}{T} + e\,y(kT) = a\,\frac{u(k+1)T - u(kT)}{T} + b\,u(kT)$

$\cong y[(k+3)T] = (3-cT)\,y[(k+2)T] + (2cT-dT^2-3)\,y[(k+1)T] + (1-cT+dT^2-eT^3)\,y(kT) + aT^2\,u[(k+1)T] + (bT^3 - aT^2)\,u(kT)$

$\equiv y[(k+3)T] =$

$$[y[(k+2)T]\ \ y[(k+1)T]\ \ y(kT)\ \ u[(k+1)T]\ \ u(kT)] \quad X \quad \begin{bmatrix} (3-cT) \\ (2cT-dT2-3) \\ (1-cT+dT2-eT3) \\ aT2 \\ (bT3-aT2) \end{bmatrix}$$

a) Least square estimation:

Consider the case where fβ is a linear function of β, that is,

$f\beta(x) = x_1\beta_1 + \ldots\ldots + x_p\,\beta_p$.

Here $(x_1\ldots\ldots x_p)$ stand for the observed variables.

To write down the least squares estimator for the linear regression model, it will be convenient to use Matrix notation. Let $y = (y_1,\ldots, y_n)^T$ and let x be the n × p data matrix of the n observations on the p variables

$$x = \begin{bmatrix} x_{1,1} & \cdots & x_{1,p} \\ \vdots & \ddots & \vdots \\ x_{n,1} & \cdots & x_{n,p} \end{bmatrix}$$

where, x_j is the column vector containing the *n* observations on variable j , j = 1, . . . , n.

Which is the squared distance between the vector y and the linear combination b of the columns of the Matrix x. The distance is minimized by taking the projection of y on the space spanned by the columns of x .Suppose now that x has full column rank, that is, no column in x can be written as a linear Combination of the other columns. Then, the least squares estimator β is given by:

$\beta = (x^Tx)^{-1}\,x^Ty$

IV. WAVEFORMS, RESULTS & DISCUSSIONS

Characteristics Curve of Output Ball Position as sensed by Infra red Sensor and Sinusoidal Disturbance Signal applied at the input side-

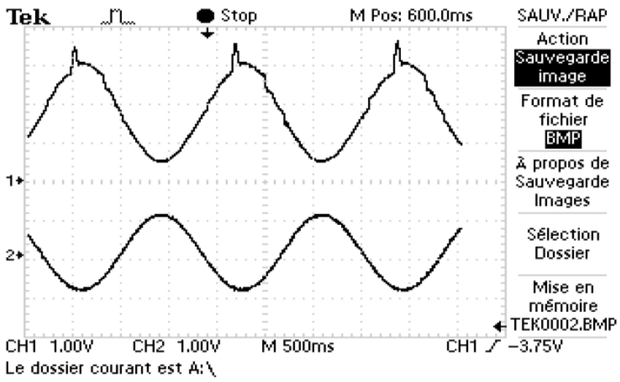


Fig.6 Characteristics Curve

The first waveform is the Output Ball Position with X-axis as Voltage and Y-axis as time. Second one is the Sinusoidal Disturbance Signal with same axis as first one.

Set point voltage applied: -1 V

Sampling time chosen : 100 millisecond.

A. Data points for closed loop ball position and disturbance Signal

Time (milli second):	Measured Ball Position (V):	Disturbance signal (V):	Input Signal Applied (V):
0100	1.2	0.4	-0.6
0200	1.6	0.0	-1.0
0300	2.1	-0.2	-1.2
0400	2.8	-0.6	-1.6
0500	3.6	-0.8	-1.8
0600	3.0	-0.8	-1.8
0700	3.0	-0.6	-1.6
0800	2.8	-0.6	-1.6
0900	2.2	-0.2	-1.2
1000	2.8	-0.1	-1.1
1100	1.2	0.2	-0.8
1200	0.8	0.6	-0.4
1300	0.6	0.0	-1.0
1400	0.6	1.2	0.2
1500	0.6	1.0	0.0
1600	0.8	1.0	0.0
1700	1.0	0.8	-0.2
1800	1.4	0.2	-0.8
1900	2.0	0.0	-1.0
2000	2.5	-0.3	-1.3
2100	2.4	-0.8	-1.8
2200	3.6	-0.6	-1.6
2300	3.0	-0.8	-1.8
2400	3.0	-0.6	-1.6
2500	2.8	-0.4	-1.4
2600	2.0	0.0	-1.0
2700	1.6	0.2	-1.2
2800	1.2	0.6	-1.6
2900	0.8	1.0	0.0
3000	0.6	1.1	0.1
3100	0.6	1.2	0.2

3200	0.6	1.2	0.2
3300	0.9	1.0	0.0
3400	1.2	0.6	-0.4
3500	1.6	0.3	-0.7
3600	1.2	0.0	-1.0
3700	0.6	1.1	0.1
3800	2.6	-0.6	-1.6
3900	3.0	-0.6	-1.6
4000	3.0	-0.6	-1.6
4100	2.8	-0.6	-1.6
4200	2.4	-0.4	-1.4
4300	2.0	-0.2	-1.2
4400	2.4	0.6	-1.6
4500	1.0	0.6	-1.6

Parameters of the closed loop transfer function using least square estimation:

a=142.75
b=-120050
c=148.22
d=7293.75
e=64812.75

Closed loop transfer function for estimated plant with controller:

Controller transfer function is: $\frac{Vo(s)}{Vi(s)} = \frac{1.1\times10^{-7}s+1}{5\times10^{-10}s+1}$

Closed loop transfer function becomes:

Transfer Function = $\frac{\Delta x}{\Delta v} = \frac{142.75s-120050}{s^3+148.22s^2+7293.75s+64812.75}$

Where; Δx = Change in ball position (in terms of voltage).

ΔV = Change in applied voltage signal.

DC Gain of the estimated system =1.852

Z= Zero of the system = 840.9807

P = Poles of the system are:

-68.4742 +32.5793i
-68.4742 -32.5793i
-11.2715

Open loop Estimated Transfer function :

$$\frac{\Delta x}{\Delta v} = \frac{0.648(s-841)}{(s+339.4)(s^2-191.2s+5148e^4)}$$

a) Discussions:

1. The incremental input is actually the increase in applied Sinusoidal disturbance Signal with respect to the DC voltage applied as a reference signal (-1 V).
2. The operating point chosen for output ball position as 2 V. The incremental output ball position is taken as the change in ball position with respect to operating point.
3. Poles of estimated closed loop transfer function are all on the left half of the transfer functions, indicates the system is stable which can only be due to the lead compensator in forward path which is a good sign for validation.

B. Frequency response of the Estimated Open Loop Plant and Given Modeled Plant in same Reference:

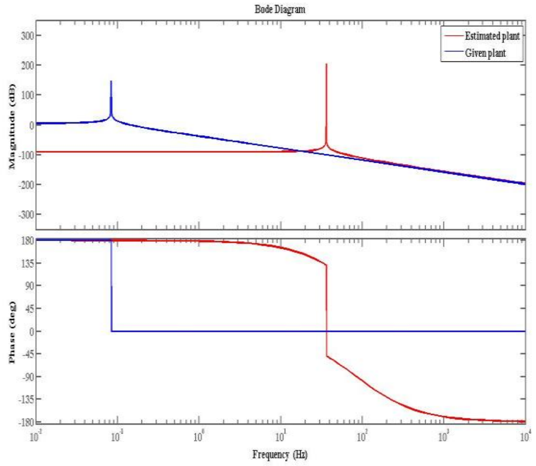


Fig.7. Frequency Response

a) Discussion :

1. For both plots the magnitude are exactly matched in low frequency region i.e. up to 0.1 Hz and after that although they are not exactly matched i.e. at high frequency region, but their negative decaying pole nature remains almost same, parallel with each other.
2. For low frequency range the Phase Plots is almost zero for original CL whereas for estimated CL it is phase shifted by 180 degrees, but at high frequency range, plots do not match at all.
3. Overall, we can say our model has validated in terms of :
   - DC gain with 10% Error.
   - Both model magnitude plots matches in low freq region and decays same in high freq zone ,i.e. same magnitude difference.
   - Phase plot of both model matches in low frequency zone with 180^o phase shift.

4. A notch signal has been found in both the magnitude and phase plot at about 0.8 Hz of original modeled closed loop which can be estimated just by a noise signal.

The Closed loop simulation result:

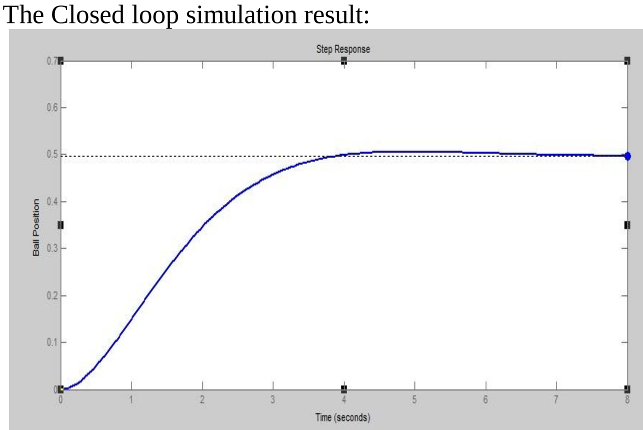


Fig.8

From closed loop Response:

1. The ball position finally reaches steady state of amplitude 0.5 with settling time of 7 seconds without any overshoot.

2. Although the settling time is 7 seconds but we can use this feedback gain in real time simulation and can check the stability of the ball position.

b) Real time simulation results:

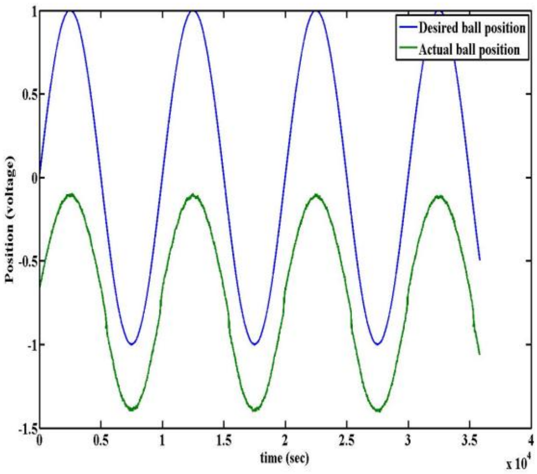


Fig.9

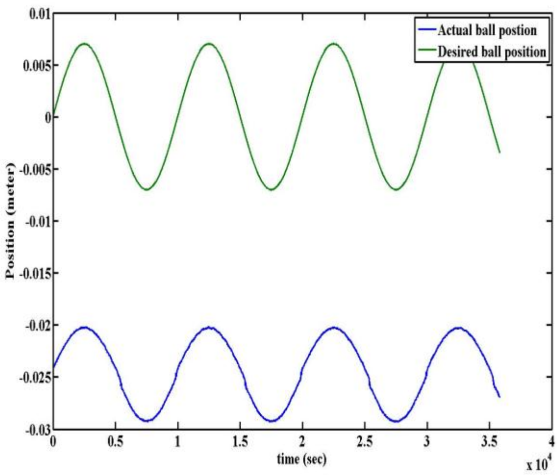


Fig. 10

V. Conclusion

System identification, mathematical modeling with validation and controller design has been done in this paper successfully. Designed controller has some big limitation in terms of frequency; it will only work on low frequency zone. It is a better controller with comparison to the existing PID controller in terms of stability performance, more robust. As it is a linearised model at a certain equilibrium point, we are not considering many aspects otherwise it may work for all frequency zones.

Robustness can be improved by incorporating h-infinity control which we have not considered here. The plant can be linearized at other equilibrium points, so that stability performance in terms of frequency can more aptly studied. The choice of equilibrium points is an important consideration; otherwise ir can give erroneous results. The plant is assumed to be fully state controllable so that the linear state feedback controller can be designed. A notch signal has been found out which can be estimated by a noise signal. There is a scope of improvement in the design of the controller in terms of frequency response so that the controller can work for all frequency range.

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