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Static Deformation of a Uniform Half- Space with Rigid Boundary Due to a Long Dip-Slip Fault of Finite Width

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***Abstract*— The Airy stress function for a long dip-slip fault of arbitrary dip and finite width buried in a homogeneous, isotropic, perfectly elastic half-space with rigid boundary is obtained. This Airy stress function is used to derive closed-form analytical expressions for the stresses and displacements at an arbitrary point of the half-space with rigid boundary caused by a long vertical dip-slip fault of finite width located at an arbitrary distance from the interface of uniform half-space. The variation of the displacement and stress fields with distance from the fault and with depth of the fault for different values of distances of fault from the interface are discussed numerically.**

***Index Terms*— Half- space, Rigid-boundary, Static-deformation, Dip- slip fault.**

I. Introduction

In the field of seismology, Steketee (1958a, b) applied the elasticity theory of dislocations. Steketee dealt with a semi-infinite, non-gravitating, isotropic and homogenous medium. Okada (1985) presented analytical expressions for the surface displacements, strains and tilts due to inclined shear and tensile faults in a half-space for both point and finite rectangular sources. Okada (1992) extended the results to internal deformation. Maruyama (1966) calculated all sets of Green's function for obtaining displacements and stresses around faults in a half-space. Jungles and Frazier (1973) described a finite element variational method applied to plane strain analysis which presented a suitable tool for analysis of permanent displacements and strains caused by seismic waves.

Singh and Garg (1986) obtained the integral expressions for the Airy stress function in an unbounded medium due to various two-dimensional seismic sources. Singh and Rani (1991) obtained closed-form analytical expressions for displacements and stresses at any point of a two phase medium consisting of a homogenous, isotropic, perfectly elastic half-space in welded contact with a homogeneous, orthotropic, perfectly elastic half-space caused by two-dimensional seismic sources located in the isotropic half-space. Singh et. al (1992), following the procedure of Singh and Garg (1986), obtained closed-form analytical expressions for the displacements and stresses at any point of either of two homogenous, isotropic, perfectly elastic half-spaces in welded contact due to two-dimensional sources.

Kumari et al. (1992), obtained the elastic residual field for two half-spaces in welded contact caused by a point dislocation sources. Rani et al. (2006), obtained closed form expressions for displacements and stresses of two half-spaces in welded contact due to a long blind dip slip fault. Again, Rani et al. (2009), obtained closed form analytical expressions for displacements and stresses at any point of a two phase medium consisting of a homogenous, isotropic, perfectly elastic half-space in welded contact with a homogeneous, orthotropic, perfectly elastic half-space due to a long dip-slip fault of finite width located in the isotropic half-space.

The simplest model to consider the effect of a material discontinuity is that of a dip-slip dislocation in an elastic half-space (medium 1 with rigidity μ_1) in contact with another elastic half-space (medium 2 with rigidity μ_2). If $m = (\mu_1/\mu_2)$, then the two particular cases of special interest are for $m=0$ and $m\to\infty$. In case $m=0$, we have a dip slip dislocation in an elastic half -space with free boundary. On the other hand, when $m\to\infty$, we have the case of dip-slip dislocation in an elastic half-space with a rigid boundary, considered in the present study. This implies that our model consists of a dip slip dislocation in an elastic half-space in contact with a rigid half-space. This model is useful when the medium on the other side of the material discontinuity is very hard. Singh et al. (2011) obtained analytical expressions for stresses at an arbitrary point of homogenous, isotropic perfectly elastic half-space with rigid boundary caused by a long tensile fault of finite width.

The aim of the present paper is to study the two-dimensional deformation of a uniform half-space with rigid boundary caused by a long dip-slip fault of finite width. Beginning with the closed-form expression for the Airy stress function for a vertical dip slip line source and dipping fault at $45°$ located in a uniform half-space with rigid boundary, given by Malik et al. (2012,13), we obtained Airy stress function for a long inclined dip-slip line source. Analytic integration over the width of the fault yields the Airy stress function for a dip-slip fault of arbitrary dip and finite width. The expressions for the stresses and displacements at any point of the half-space caused by a long dip-slip fault follow immediately.

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II. THEORY

Let the Cartesian co-ordinates be denoted by (x_1, x_2, x_3) with x_3-axis vertically upward. Consider a homogeneous, perfectly elastic half-space with rigid boundary. The half-space is assumed to be isotropic with stress-strain relation.

$$p_{ij} = 2\mu \left[e_{ij} + \frac{\sigma}{1 - 2\sigma} \delta_{ij} e_{kk} \right], \qquad (1)$$

Where, $(i, j = 1,2,3)$ and p_{ij} are the components of stress tensor, e_{ij} are the components of strain tensor, μ is the shear modulus and σ is Poisson's ratio.

Consider a two dimensional approximation in which displacement component u_1, u_2, u_3 are independent of x_1 so that $\partial/\partial x_1 \equiv$ Under this assumption, the plane strain problem $(u_1 = 0)$ as well as anti-strain problem $(u_2 = 0$ and $u_3 = 0)$ are decoupled and therefore, can be solved separately. The plane strain problem for an isotropic medium can be solved in terms of Airy stress function U such that

$$p_{22} = \partial^2 U / \partial x_3^2,$$

$$p_{33} = \partial^2 U / \partial x_2^2, \qquad (2)$$

$$p_{23} = -\partial^2 U / \partial x_2 \partial x_3$$

and

$$\nabla^2 \nabla^2 U = 0 \qquad (3)$$

Using the expressions of Airy stress function for vertical dip-slip line source and dip-slip on 45^{o} dipping line source given by Malik et al. (2012, 2013), we have Airy stress function due to a dip-slip line source parallel to x_1-axis and passing through the point (y_2, y_3) located in isotropic half-space with rigid boundary as given below:

$$U = \frac{\alpha\mu b ds}{\pi}\Bigg[\cos 2\delta \Bigg\{ \frac{(x_2 - y_2)(x_3 - y_3)}{R_1^2} + \frac{2(\alpha - 1)}{2(2-\alpha)} \tan^{-1}\left(\frac{x_2 - y_2}{x_3 + y_3}\right) + \frac{\alpha}{2-\alpha}\frac{(x_2 - y_2)(x_3 - y_3)}{R_2^2} - \frac{4\alpha}{2-\alpha}\frac{x_3 y_3 (x_2 - y_2)(x_3 + y_3)}{R_2^4} \Bigg\} + \sin 2\delta \Bigg\{ \frac{(x_3 - y_3)^2}{R_1^2} - \frac{2(1-\alpha)}{\alpha(2-\alpha)} \ln R_2 + \frac{\alpha}{2-\alpha}\frac{(y_3^2 - x_3^2 - 2x_3 y_3)}{R_2^2} + \frac{4\alpha}{2-\alpha} x_3 y_3 (x_3 + y_3)^2 \Bigg\}\Bigg] \qquad (4)$$

where, $\alpha = \dfrac{1}{2(1-\sigma)}$,

$b =$ displacement discontinuity,

$ds =$ width of line source,

$\delta =$ dip-angle,

$(x_2, x_3) =$ receiver location,

$$R_1^2 = (x_2 - y_2)^2 + (x_3 - y_3)^2,$$

$$R_2^2 = (x_2 - y_2)^2 + (x_3 + y_3)^2.$$

Using polar co-ordinates (s, δ), fig.1(a)

$$y_2 = s \cos \delta,$$

$$y_3 = d + s \sin \delta,$$

where, s is distance from lower edge of fault measured in dip direction.

Now, integrating U over s between the limits $(0, L)$, we will obtain the following expressions for the Airy stress function for a long dip-slip fault of width L and infinite length passing through the point (y_2, y_3) located in isotropic half-space with rigid boundary with lower edge of fault at distance 'd' from interface as given below:

$$U = \frac{\alpha\mu b}{\pi}\Bigg[\frac{s \sin 2\delta}{\alpha} + \Bigg\{ \frac{2(1-\alpha)}{\alpha(2-\alpha)}(x_2 \sin\delta - X'\cos\delta - s\sin 2\delta) + \frac{\alpha}{2-\alpha}(x_2 \sin\delta - X\cos\delta) \Bigg\} \ln R_2 + (x_2 \sin\delta - X\cos\delta)\ln R_1 - \frac{2(1-\alpha)}{\alpha(2-\alpha)} s\cos 2\delta \tan^{-1}\left(\frac{x_2 - s\cos\delta}{X' + s\sin\delta}\right) - \frac{2\alpha}{(2-\alpha)} x_3 (x_2 \sin\delta + X'\cos\delta)(d + s\sin\delta)\frac{1}{R_2^2} - \frac{2(1-\alpha)}{\alpha(2-\alpha)}(x_2\cos\delta - X'\sin\delta)\tan^{-1}\left(\frac{s - x_2\cos\delta + X'\sin\delta}{x_2\sin\delta + X'\cos\delta}\right)\Bigg]\Bigg|_0^L \qquad (5)$$

where,

$$R_1^2 = (x_2 - s\cos\delta)^2 + (X - s\sin\delta)^2,$$

$$R_2^2 = (x_2 - s\cos\delta)^2 + (X' + s\sin\delta)^2,$$

$$X = x_3 - d,$$

$$X' = x_3 + d,$$

$$f(s)\big|_0^L = f(L) - f(0).$$

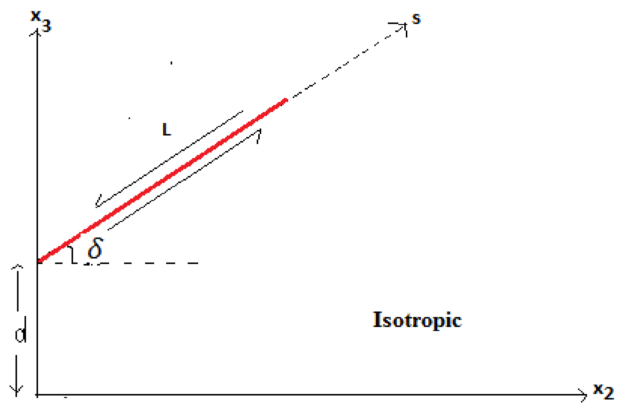

Figure 1(a). Geometry of dip-slip fault of width L at a distance d from boundary of uniform half-space.

III. STRESSES

Using equations (2) and (5), we will obtain the following expressions for stresses of the uniform half-space with rigid boundary due to a long dip-slip fault of width L and infinite length passing through the point (y_2, y_3) located in isotropic half-space with rigid boundary with lower edge of fault at distance 'd' from interface as given below:

$$\begin{aligned}
p_{22} = \frac{\alpha\mu b}{\pi}\Bigg[& (x_2 \sin\delta - 3X\cos\delta + s\sin 2\delta)\frac{1}{R_1^2} - 2(x_2\sin\delta - X\cos\delta)(X - s\sin\delta)^2\frac{1}{R_1^4} \\
& + \Bigg\{\frac{2(-\alpha^2+\alpha-1)}{\alpha(2-\alpha)}(x_2\sin\delta + X'\cos\delta) \\
& + \frac{3\alpha}{2-\alpha}(x_2\sin\delta - d\cos\delta - s\sin 2\delta) - \frac{\alpha}{2-\alpha}x_3\cos\delta\Bigg\}\frac{1}{R_2^2} \\
& - 2\Bigg\{-\frac{\alpha}{2-\alpha}(X' + s\sin\delta)(X - s\sin\delta)(x_2\sin\delta + X'\cos\delta) \\
& + \frac{2\alpha}{2-\alpha}x_3^2\sin\delta(x_2 - s\cos\delta) \\
& - \frac{2\alpha}{2-\alpha}(d + s\sin\delta)(X' + s\sin\delta)[2(x_2\sin\delta + X'\cos\delta) + 3x_3\cos\delta]\Bigg\}\frac{1}{R_2^4} \\
& - \frac{16\alpha}{2-\alpha}x_3(x_2\sin\delta + X'\cos\delta)(d + s\sin\delta)(X' + s\sin\delta)^2\frac{1}{R_2^6}\Bigg]\Bigg|_0^L
\end{aligned} \tag{6}$$

$$\begin{aligned}
p_{23} = \frac{\alpha\mu b}{\pi}\Bigg[& (x_2\cos\delta + X\sin\delta - s)\frac{1}{R_1^2} + 2(X - s\sin\delta)^2(s - x_2\cos\delta - X\sin\delta)\frac{1}{R_1^4} \\
& + \Bigg\{\frac{2(1-\alpha)}{\alpha(2-\alpha)}(x_2\cos\delta - X'\sin\delta - s) + \frac{\alpha}{2-\alpha}(x_2\cos\delta + X\sin\delta - s)\Bigg\}\frac{1}{R_2^2} \\
& + \Bigg\{-\frac{2\alpha}{2-\alpha}(X' + s\sin\delta)[x_3\sin\delta(X' + s\sin\delta) + X'\sin\delta(X' + s\sin\delta) \\
& - d\cos\delta(x_2 - s\cos\delta) + (s\sin\delta - x_3)(s - x_2\cos\delta + X'\sin\delta)] \\
& + \frac{4\alpha}{2-\alpha}(d + s\sin\delta)[(x_3 + X' + s\sin\delta)(s - x_2\cos\delta + X'\sin\delta) + 2x_3\sin\delta(X' + s\sin\delta)]\Bigg\}\frac{1}{R_2^4} \\
& + \frac{16\alpha}{2-\alpha}x_3(d + s\sin\delta)(X' + s\sin\delta)^2(x_2\cos\delta - X'\sin\delta - s)\frac{1}{R_2^6}\Bigg]\Bigg|_0^L
\end{aligned} \tag{7}$$

$$\begin{aligned}
p_{33} = \frac{\alpha\mu b}{\pi}\Bigg[& (x_2\sin\delta + X\cos\delta - s\sin 2\delta)\frac{1}{R_1^2} + 2(x_2\sin\delta - X\cos\delta)(X - s\sin\delta)^2\frac{1}{R_1^4} \\
& + \Bigg\{\frac{2(1-\alpha)}{\alpha(2-\alpha)}(x_2\sin\delta + X'\cos\delta) + \frac{\alpha}{2-\alpha}(x_2\sin\delta + X\cos\delta - s\sin 2\delta)\Bigg\}\frac{1}{R_2^2} \\
& + \Bigg\{\frac{2\alpha}{2-\alpha}(X' + s\sin\delta)[(X' + s\sin\delta)(x_2\sin\delta + d\cos\delta) \\
& - x_3(2x_2\sin\delta - s\sin\delta\cos\delta + X'\cos\delta)] \\
& + \frac{4\alpha}{2-\alpha}x_3[x_3\sin\delta(x_2 - s\cos\delta) - 3\cos\delta(X' + s\sin\delta)(d + s\sin\delta)]\Bigg\}\frac{1}{R_2^4} \\
& + \frac{16\alpha}{2-\alpha}x_3(x_2\sin\delta + X'\cos\delta)(d + s\sin\delta)(X' + s\sin\delta)^2\frac{1}{R_2^6}\Bigg]\Bigg|_0^L
\end{aligned} \tag{8}$$

IV. DISPLACEMENTS

The displacements, for the isotropic half-space, are given by the expressions (Singh and Rani (1991))

$$2\mu u_2 = -\frac{\partial U}{\partial x_2} + \frac{1}{2\alpha}\int (p_{22} + p_{33})\,dx_2,$$
$$2\mu u_3 = -\frac{\partial U}{\partial x_3} + \frac{1}{2\alpha}\int (p_{22} + p_{33})\,dx_3$$

Using equations (9), (5), (6) and (8), we will obtain the following expressions for displacements of the uniform half-space with rigid boundary due to a long dip-slip fault of width L and infinite length with lower edge of fault at distance 'd' from interface as given below:

$$\begin{aligned}
u_2 = \frac{\alpha b}{2\pi}\Bigg[& \left(1 - \frac{1}{\alpha}\right)\sin\delta \ln\frac{R_2}{R_1} - \frac{\cos\delta}{\alpha}\Bigg\{\tan^{-1}\left(\frac{x_2 - s\cos\delta}{X - s\sin\delta}\right) \\
& + \frac{\alpha}{2-\alpha}\tan^{-1}\left(\frac{x_2 - s\cos\delta}{X' + s\sin\delta}\right)\Bigg\} + \frac{2(1-\alpha)}{\alpha(2-\alpha)}\cos\delta\tan^{-1}\left(\frac{s - x_2\cos\delta + X'\sin\delta}{x_2\sin\delta + X'\cos\delta}\right) \\
& + (X - s\sin\delta)(x_2\cos\delta + X\sin\delta - s)\frac{1}{R_1^2} \\
& + \Bigg\{\frac{\alpha}{2-\alpha}x_3(x_2\cos\delta + X\sin\delta - s) - (s - x_2\cos\delta + X'\sin\delta)(d + s\sin\delta)\Bigg\}\frac{1}{R_2^2} \\
& + \frac{4\alpha}{2-\alpha}x_3(s - x_2\cos\delta + X'\sin\delta)(d + s\sin\delta)(X' + s\sin\delta)\frac{1}{R_2^4}\Bigg]\Bigg|_0^L
\end{aligned} \tag{10}$$

$$\begin{aligned}
u_3 = \frac{\alpha b}{2\pi}\Bigg[& \left(1 - \frac{1}{\alpha}\right)\cos\delta \ln\frac{R_1}{R_2} + \frac{\sin\delta}{\alpha}\Bigg\{\tan^{-1}\left(\frac{X - s\sin\delta}{x_2 - s\cos\delta}\right) + \frac{\alpha}{2-\alpha}\tan^{-1}\left(\frac{X' + s\sin\delta}{x_2 - s\cos\delta}\right)\Bigg\} \\
& + \frac{2(1-\alpha)}{\alpha(2-\alpha)}\sin\delta\tan^{-1}\left(\frac{s - x_2\cos\delta + X'\sin\delta}{x_2\sin\delta + X'\cos\delta}\right) \\
& - (x_2\sin\delta - X\cos\delta)(X - s\sin\delta)\frac{1}{R_1^2} \\
& + \Bigg\{\frac{2\alpha}{2-\alpha} + x_3(s\sin 2\delta - x_2\sin\delta + d\cos\delta) \\
& + \frac{\alpha}{2-\alpha}x_3(x_2\sin\delta + X'\cos\delta) - (d + s\sin\delta)(x_2\sin\delta + X'\cos\delta)\Bigg\}\frac{1}{R_2^2} \\
& - \frac{4\alpha}{2-\alpha}x_3(d + s\sin\delta)(x_2\sin\delta + X'\cos\delta)(X' + s\sin\delta)\frac{1}{R_2^4}\Bigg]\Bigg|_0^L
\end{aligned} \tag{11}$$

V. NUMERICAL RESULTS AND DISCUSSION

We will compute the stresses and displacements numerically due to a vertical long dip-slip fault of width L at various points of a uniform half space with rigid boundary.

We define the following quantities

$$Y = \frac{x_2}{L};\; Z = \frac{x_3}{L};\; D = \frac{d}{L} \qquad (12)$$

where, L is the width of dip-slip fault. And also we take $\alpha = \frac{2}{3}$ and $\delta = 90^{o}$ for simpler calculations.

The displacements and stresses are calculated in units of $\frac{b}{2\pi}$ and $\frac{\mu b}{\pi L}$ respectively, where b is the slip.

Let the dimensionless stresses and displacements be denoted by U_i and P_{ij}. Then,

$$U_i = \frac{2\pi}{b}u_i \quad \text{and} \quad P_{ij} = \frac{\pi L}{\mu b}p_{ij} \qquad (13)$$

From equation (5) to (11), we will obtain the following expressions for dimensionless stresses and displacements for a vertical dip-slip fault of finite width.

$$P_{22} = \frac{2}{3}\Bigg[\frac{Y}{A_1^2} - \frac{2Y(Z-D-1)^2}{A_1^4} + \frac{5Y}{4B_1^2} - \frac{Y\{Z^2-3(D+1)^2-4Z(D+1)\}}{B_1^4} - \frac{8YZ(D+1)(Z+D+1)^2}{B_1^6} - \frac{Y}{A_2^2} + \frac{2Y(Z-D)^2}{A_2^4} - \frac{5Y}{4B_2^2} + \frac{Y(Z^2-3D^2-4ZD)}{B_2^4} + \frac{8YZD(Z+D)^2}{B_2^6}\Bigg] \qquad (14)$$

$$P_{23} = \frac{2}{3}\Bigg[\frac{(Z-D-1)}{A_1^2} - \frac{2(Z-D-1)^3}{A_1^4} - \frac{(Z+5D+5)}{4B_1^2} + \frac{2(Z+D+1)(D+1)(4Z+D+1)-(Z+D+1)^3}{B_1^4} - \frac{8Z(D+1)(Z+D+1)^3}{B_1^6} - \frac{(Z-D)}{A_2^2} + \frac{2(Z-D)^3}{A_2^4} + \frac{(Z+5D)}{4B_2^2} - \frac{2D(Z+D)(4Z+D)-(Z+D)^3}{B_2^4} + \frac{8ZD(Z+D)^3}{B_2^6}\Bigg] \qquad (15)$$

$$P_{33} = \frac{2}{3}\Bigg[\frac{Y}{A_1^2} + \frac{2Y(Z-D-1)^2}{A_1^4} + \frac{5Y}{4B_1^2} - \frac{Y\{Z^2-4Z(D+1)-3(D+1)^2\}}{B_1^4} + \frac{8YZ(D+1)(Z+D+1)^2}{B_1^6} - \frac{Y}{A_2^2} - \frac{2Y(Z-D)^2}{A_2^4} - \frac{5Y}{4B_2^2} + \frac{Y(Z^2-4ZD-3D^2)}{B_2^4} - \frac{8YZD(Z+D)^2}{B_2^6}\Bigg] \qquad (16)$$

$$U_2 = \frac{2}{3}\Bigg[-\frac{1}{2}\ln\frac{B_1}{A_1} + \frac{(Z-D-1)^2}{A_1^2} + \frac{1}{2}\frac{\{Z^2-3Z(D+1)-2(D+1)^2\}}{B_1^2} + \frac{2Z(D+1)(Z+D+1)^2}{B_1^4} - \frac{(Z-D)^2}{A_2^2} + \frac{1}{2}\ln\frac{B_2}{A_2} - \frac{1}{2}\frac{(Z^2-3ZD-2D^2)}{B_2^2} - \frac{2ZD(Z+D)^2}{B_2^4}\Bigg] \qquad (17)$$

$$U_3 = \frac{2}{3}\Bigg[\frac{3}{2}\tan^{-1}\left(\frac{Z-D-1}{Y}\right) + \frac{3}{2}\tan^{-1}\left(\frac{Z+D+1}{Y}\right) - \frac{Y(Z-D-1)}{A_1^2} + \frac{Y\left(\frac{1}{2}Z-D-2\right)}{B_1^2} - \frac{2YZ(D+1)(Z+D+1)}{B_1^4} - \frac{3}{2}\tan^{-1}\left(\frac{Z-D}{Y}\right) - \frac{3}{2}\tan^{-1}\left(\frac{Z+D}{Y}\right) + \frac{Y(Z-D)}{A_2^2} - \frac{Y\left(\frac{1}{2}Z-D-1\right)}{B_2^2} + \frac{2YZD(Z+D)}{B_2^4}\Bigg] \qquad (18)$$

Where, $A_1^2 = Y^2 + (Z-D-1)^2$, $\quad A_2^2 = Y^2 + (Z-D)^2$

$B_1^2 = Y^2 + (Z+D+1)^2$, $\quad B_2^2 = Y^2 + (Z+D)^2$

Figure 1.1 - 1.2 diplays the variation of dimensionless horizontal displacement U_2 with dimensionless distance (Y) from the fault at dimensionless depth Z=2 and Z=3 respectively for distances D=0, 1, 2 of fault from interface. The displacement is symmetric about Y=0. In fig. 1.1, at Z =2 and D=0, U_2 varies along a smooth curve but at D=1, 2 U_2 varies strongly in the range -2 <Y< 2 and tends to zero as Y approaches to infinity. In fig 1.2, at Z =3 and D = 2. U_2 attains minima at origin and varies in the range - 2< Y <2 . At D=0,1 variation is smooth and U_2 tends to zero as Y approaches to infinity. In this case, displacement field is affected when depth from the fault is changed at D=1. Fig. 1.3 shows the variation of U_2 with depth from the fault Z at Y=1 and D=0, 1, 2. In this case, displacement is not symmetric. At D=0, the curve is smooth, at D=1, U_2 varies for -3< Z < -1 otherwise smooth and at D=2, U_2 varies strongly in the range -4 < Z < -2, but both have the same pattern. Moreover, U_2 tends to zero as Z approaches to infinity.

Fig. 1.4 – 1.5 shows the variation of dimensionless vertical displacement U_3 with Y at Z=2 and Z=3 respectively. In fig. 1.4 at D=0, U_3 varies in range -1< Y <1 ; at D=1, 2 curve is discontinuous at origin. In fig. 1.5 at D=0,1, U_3 varies in range -1< Y <1 ; at D=2, curve is discontinuous at origin. U_3 tends to zero as Y approaches to infinity in fig. 1.4-1.5 . Fig. 1.6 shows the variation of U_3 with depth from the fault Z at Y=1 and D=0, 1, 2. All curves have the same pattern but varies differently.

In fig 1.7 at Z = 2.5; D = 0, P_{22} exhibit very smooth curve , at D=1 varies in the range -3< Y <3 and at D = 1/10, varies in in the range -2< Y <2. P_{22} tends to zero as Y approaches to infinity. In fig. 1.8 at Y = 4; D =1/10, 1, 2 pattern is same but varies strongly in different range. Fig. 1.10 at Y = 1.5, P_{33} follows the pattern of fig. 1.8. In fig. 1.9 at Z = 4; D = 3.5, P_{33} shows smooth curve; at D = 2 curve varies in range -2< Y <2 . The curve at D = 1/10 is more smoother than at D = 2. In fig 1.11 at Z = 4.5; at D = 1.5, 1/10, P_{23} varies smoothly and at D = 3 varies in range -2< Y <2. In fig. 1.12 at Y=2; at D=1/10 P_{23} shows smooth curve, at D=3 varies for negative values of Z and for D=1.5 varies in the range -3< Z <-1, otherwise smooth. Moreover , all stresses and displacements tends to zero as distances or depths approaches to infinity.

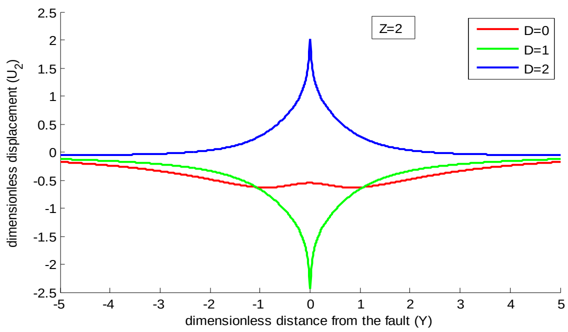

Figure 1.1 - Variation of horizontal displacement (U_2) with distance from the fault.

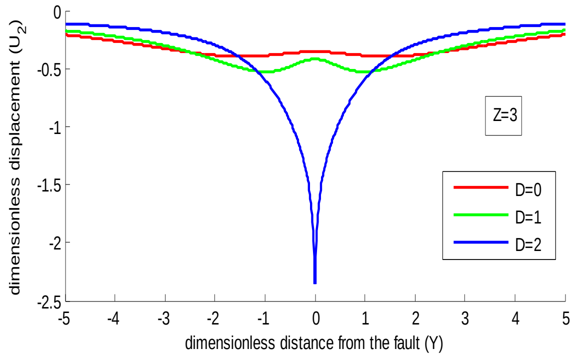

Figure 1.2 - Variation of horizontal displacement (U_2) with distance from the fault.

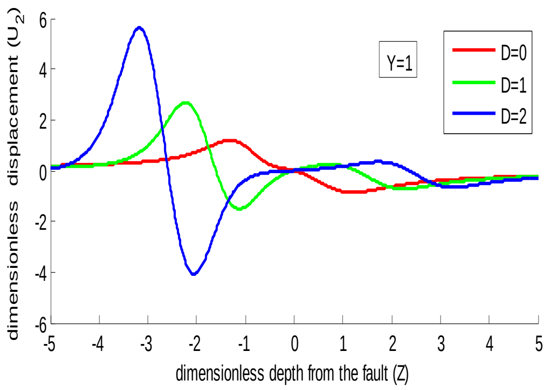

Figure 1.3 - Variation of horizontal displacement (U_2) with distance from the fault.

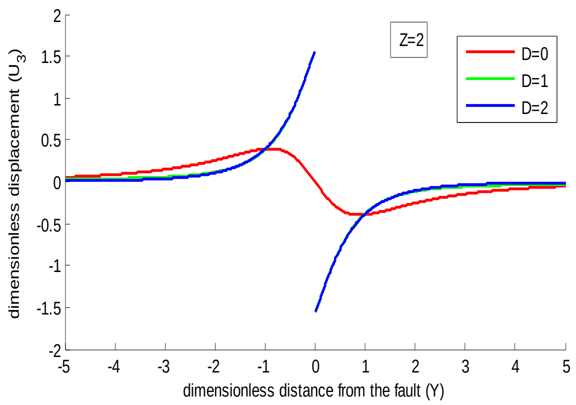

Figure 1.4 - Variation of vertical displacement (U_3) with distance from the fault.

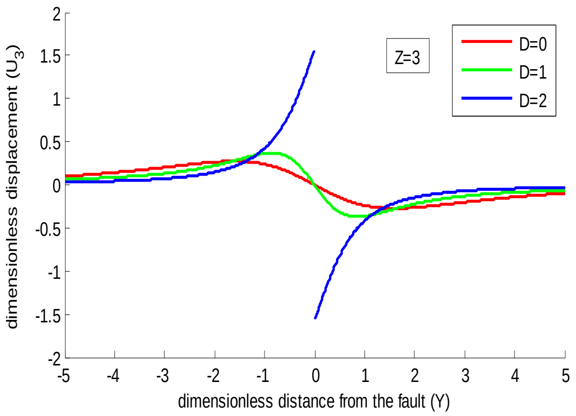

Figure 1.5 - Variation of vertical displacement (U_3) with distance from the fault .

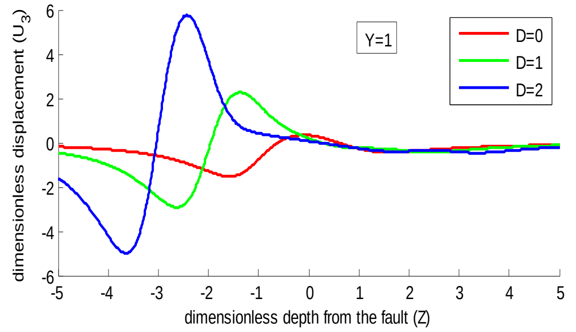

Figure 1.6 - Variation of vertical displacement (U_3) with distance from the fault.

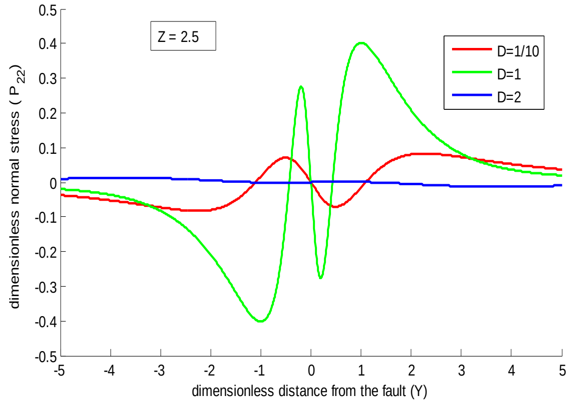

Figure 1.7 - Variation of Dimensionless normal stress (P_{22}) with distance from the fault .

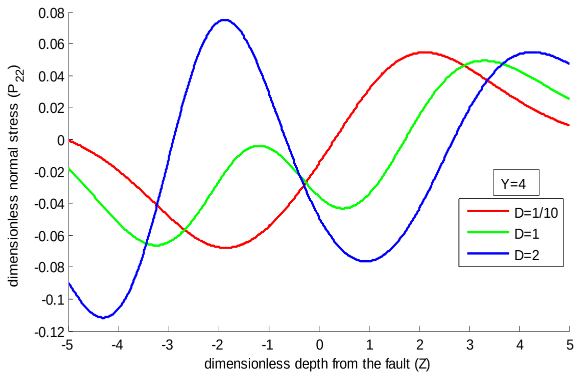

Figure 1.8 - Variation of Dimensionless normal stress (P_{22}) with depth from the fault.

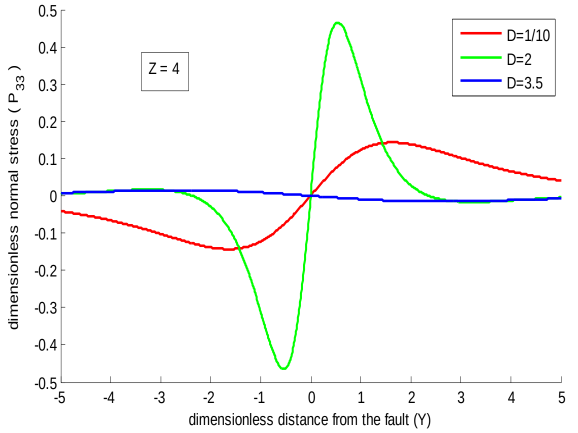

Figure 1.9 - Variation of Dimensionless normal stress (P_{33}) with distance from the fault.

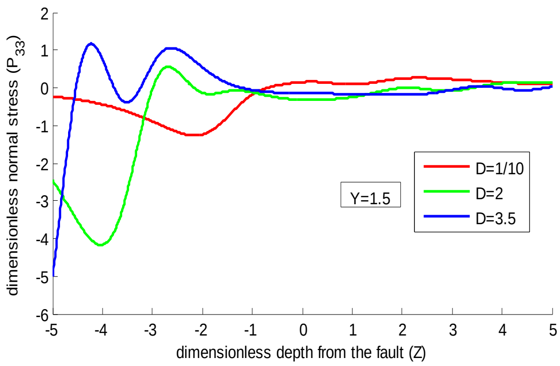

Figure 1.10 - Variation of Dimensionless normal stress (P_{33}) with depth from the fault.

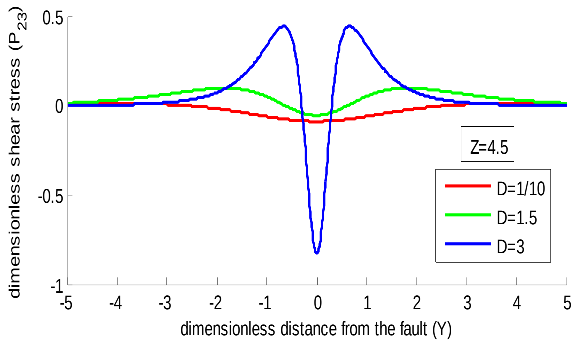

Figure 1.11 - Variation of Dimensionless normal stress (P_{23}) with distance from the fault.

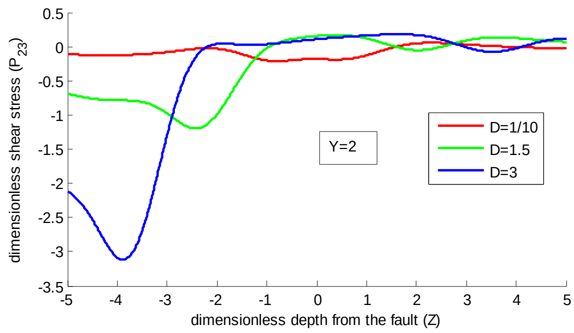

Figure 1.12 - Variation of Dimensionless normal stress (P_{23}) with distance from the fault.

VI. CONCLUSION

Analytical expressions for elastic residual field caused by a long dip-slip fault of finite width L located in a uniform half-space with rigid boundary have been obtained. If we take the particular case of Rani et al. (2009) such as when orthotropic half-space is removed, i.e., if we take, $\mu_2 = 0$, then m$\rightarrow \infty$ and we will get a uniform half-space with rigid boundary. Our results have been verified with displacement and stress fields obtained by Rani et al (2009), for a particular case. Numerical results presented the variation of horizontal displacement, vertical displacement, normal and shear stresses with distance and depth fom the fault for different distances of fault from the interface. This model is useful when the medium on the other side of the material discontinuity is very hard. High-rigidity layers are generally present at depth below a volcanic edifice, covered by much softer volcanic-sedimentary layers composed of a mixture of ash, mud and lava (Bonafede & Revalta, 1999). Results obtained may find applications in geophysical phenomenon for above mentioned areas.

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