International Journal of Engineering and Technical Research (IJETR)
ISSN: 2321-0869, Volume-2, Issue-5, May 2014

Theoretical and Software analysis of Crack detection in Structure

Mr. Patil Amit V., Prof. Meghna Pathak, Dr. P.K. Sharma

***Abstract*— the aim of this paper is on the problem of crack detection for a cracked beam using theoretical and software analysis. Crack is the discontinuation in a body. Location of crack through the method of vibration is great advantages over traditional methods. Presence of crack in a structural member induces local flexibility which affects vibration response of the structure. This property may be used to detect location and depth of the crack in structural member. The presence of crack leads to changes in some of the lower natural frequencies and mode shapes. Method based on measurement of natural frequencies is presented for detection of the location and size of a crack in a cantilever beam. Numerical calculations has been done by solving the Euler equation for un-crack beam and cracked beam to obtain first three natural frequencies of different modes of vibration considering various crack positions for the beam. CATIA software is used for designing cantilever beam. ANSYS software is used for analysis of crack and un-crack cantilever beam. Total 7 models has been tested according to theoretical and software method.**

***Index terms*— analysis, Finite element analysis, structure.**

I. I. INTRODUCTION

A. *Vibration:*

Motion which repeats itself after a certain interval of time is called vibration. A vibration can caused due to external unbalanced force also. A vibratory system, in general, includes elastic member for storing potential

B. *Lateral vibration:*

The vibration in which particles of the system vibrates in the direction perpendicular to the axis of system is known as lateral vibration. Since the cantilever beam has infinite number of masses, we need infinite number of coordinates to specify the deflected configuration. Thus, cantilever beam is infinite degree of freedom system and it is necessary to study lateral vibration of cantilever beam.

C. *Need of Crack Detection:*

Crack detection is one of the important aspects in structural engineering for safety reasons and because of economic benefits that can result.

Manuscript received May 14, 2014.

Mr. Patil Amit V., MTECH (Scholar) ,Mechanical Department, N.R.I. Institute of Information & Technology Bhopal, Madhya Pradesh, India.

Prof. Meghna Pathak, Guide, Professor, Mechanical department, N.R.I. Institute of Information & Technology Bhopal, Madhya Pradesh, India.

Dr. P.K.Sharma, H.O.D. , Mechanical department ,N.R.I. Institute of Information & Technology Bhopal, Madhya Pradesh, India.

D. *Specifications:*

Cantilever beam height- 10mm
Crack depth in cantilever beam - 1mm, 2mm, and 3mm
Crack location- in cantilever beam 150mm, 300mm
E = modulus of elasticity (N/m^2)
m = mass of the beam per unit length (kg/m)
I= moment of inertia (m^4).
ωi = natural frequency of the i^{th} mode (rad/s)
Cantilever beam length- 600mm
Cantilever beam width- 10mm
L1=Distance between the left node and crack,
I_0=Moment of inertia of the beam cross section,
I_c= Moment of inertia of the beam with crack.
q=displacement vector of the beam
$\left[K_c^e\right]$ is the Stiffness matrix of the cracked element
$\left[K^e\right]$ is the Element stiffness matrix,
L= length of the beam,
$\left[K_c\right]$ is the Reduction in stiffness matrix due to the crack.

II. LITERATURE REVIEW

Vibration analysis of a cracked beam was performed by S Orhan et al. in order to identify the crack in a cantilever beam. Single- and two-edge cracks were evaluated. The study results suggest that free vibration analysis provides suitable information for the detection of single and two cracks, whereas forced vibration can detect only the single crack condition. The Euler– Bernoulli beam model was assumed. The crack is assumed to be an open crack and the damping has not been considered in this study.

E Cam, S Orhan and M Lüy proposed method of analysis of cracked beam structure using impact echo method. They analyzed vibrations as a result of impact shocks. ANSYS software was used for experimental results and simulations. R. Tiwari and M. Karthikey developed identification procedure for the detection, localization, and size of a crack in a beam based on forced response measurements. They used circular beam supported by rolling bearings at both ends for this experiment.

C. P. Filipich, M B Rosales and F S Buezas et al. determined sensibility analysis of the inverse problem of the crack parameters (location and depth). M. Jabalpur, N. Khaji, M. Shafiei developed analytical approach for crack identification procedure in uniform beams with an open edge crack, based on bending vibration measurements. S. Loutridis, E.Douk developed new method for crack detection in beams based on instantaneous frequency and empirical mode decomposition. It follows that the harmonic distortion increases with crack depth following definite trends and can be also used as an effective indicator for crack size. The research work by W Zhang, Z Wang and H Ma et al. illustrates the crack identification method combining wavelet analysis with transform matrix.

III. THEROTICAL FORMULATIONS

Dynamic response of the structure affected by the following aspects of the crack

- Position of crack
- Depth of crack
- Number of cracks

A. *Equation of free vibration of beam:*

The beam with a transverse edge crack is clamped at left end, free at right end and has different cross section The crack in this case is assumed to be an open crack and the damping is not being considered in this theory.

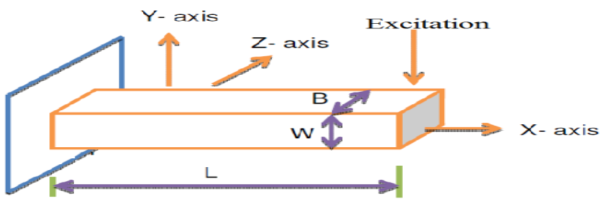


Figure A: Cantilever Beam without Crack.

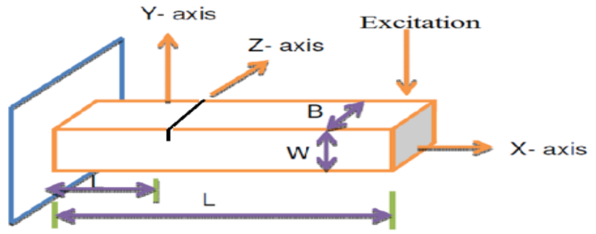


Figure B: Cantilever Beam with Crack.

The free bending vibration of an Euler-Bernoulli beam of a constant rectangular cross section is given by the following equation:

$$EI\frac{d^4y}{dx^4} - m\omega_i^2 y = 0$$

…………………. (1)

By defining $\lambda^4 = \frac{m\omega_i^2}{EI}$ equation is rearranged as a fourth-order differential equation as follows:

$$\frac{d^4y}{dx^4} - \lambda^4 y = 0$$

......……...……… (2)

The general solution to the equation is:

$$y = A\cos\lambda_i x + B\sin\lambda_i x + C\cosh\lambda_i x + D\sinh\lambda_i x$$

..... (3)

Where A, B, C, D are constants and 'λ_i' is a frequency parameter.

The element stiffness matrix of the un-cracked beam is given as:

$$[K^e] = \int [B(x)]^T EI[B(x)]dx$$

$$[B(x)] = \{H_1(x)H_2(x)H_3(x)H_4(x)\}$$

…… (4)

Where the Hermitian shape functions are defined as:

$$H_1(x) = 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3}$$

$$H_2(x) = x - \frac{2x^2}{l} + \frac{x^3}{l^2}$$

$$H_3(x) = \frac{3x^2}{l^2} - \frac{2x^3}{l^3}$$

$$H_4(x) = -\frac{x^2}{l} + \frac{x^3}{l^2}$$

Assuming the beam rigidity EI is constant and is given by EI_0 within the element, and then the element stiffness is:

$$[K^e] = \frac{EI_0}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

$$[K_c^e] = [K^e] - [K_c]$$

……......…….. (5)

Here,

The matrix [Kc] is:

$$[K_c] = \begin{bmatrix} K_{11} & K_{12} & -K_{11} & K_{14} \\ K_{12} & K_{22} & -K_{12} & K_{24} \\ -K_{11} & -K_{12} & K_{11} & -K_{14} \\ K_{14} & K_{24} & -K_{14} & K_{44} \end{bmatrix}$$

.....…… (6)

Where:

$$K_{11} = \frac{12E(I_0 - I_c)}{L^4}\left[\frac{2l_c^3}{L^2} + 3l_c\left(\frac{2L_1}{L^2} - 1\right)^2\right]$$

$$K_{12} = \frac{12E(I_0 - I_c)}{L^3}\left[\frac{l_c^3}{L^2} + l_c\left(2 - \frac{7L_1}{L} + \frac{6L_1^2}{L^2}\right)^2\right]$$

$$K_{14} = \frac{12E(I_0 - I_c)}{L^3}\left[\frac{l_c^3}{L^2} + l_c\left(2 - \frac{5L_1}{L} + \frac{6L_1^2}{L^2}\right)^2\right]$$

$$K_{22} = \frac{12E(I_0 - I_c)}{L^3}\left[\frac{3l_c^3}{L^2} + 2l_c\left(\frac{3L_1}{L} - 2\right)^2\right]$$

$$K_{24} = \frac{12E(I_0 - I_c)}{L^2}\left[\frac{3l_c^3}{L^2} + 2l_c\left(2 - \frac{9L_1}{L} + \frac{9L_1^2}{L^2}\right)^2\right]$$

$$K_{44} = \frac{12E(I_0 - I_c)}{L^2}\left[\frac{3l_c^3}{L^2} + 2l_c\left(\frac{3L_1}{L} - 1\right)\right]$$

Here, lc=1.5W, It is supposed that the crack does not affect the mass distribution of the beam. Therefore, the consistent mass matrix of the beam element can be formulated directly as:

$$[M^e] = \int_0^1 \rho A[H(x)]^T [H(x)]dx \qquad \text{................ (7)}$$

$$[M^e] = \frac{\rho Al}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \qquad \text{.......... (8)}$$

The natural frequency then can be calculated from the relation:

$$[-\omega^2[M] + [K]]\{q\} = 0 \qquad \text{.................. (9)}$$

From Euler's Beam theory theoretical formula for natural frequency given as follows:

$$\omega_n = C^* \sqrt{\frac{EI}{mL^4}}$$

Where C is Constant,
C = 0.56 for First mode, 3.51 for Second, 9.82 for Third mode for Cantilever Beam.

IV. FINITE ELEMENT ANALYSIS

Finite element analysis has been carried out by ANSYS software.
In general, a finite-element solution may be broken into the following three stages.

A. Pre-processing:

The major steps in pre-processing are
(i) Define key points/lines/areas/volumes,
(ii) Define element type and material properties and (iii) mesh lines/areas/ volumes as required.

B. Solution:

Assigning loads, constraints and solving. Here, it is necessary to specify the loads, constraints and finally solve the resulting set of equation.

C. Post processing:

In this stage we can see (i) lists of nodal displacements, (ii) element forces and moments, (iii) deflection plots and (iv) frequencies and temperature maps.

Following steps show the guidelines for carrying out Modal analysis.

Define Materials
1. Set preferences. (Structural)
2. Define constant material properties.

Model the Geometry
3. Follow bottom up modeling and create/import the geometry

Generate Mesh
4. Define element type.
5. Mesh the area.

Apply Boundary Conditions
6. Apply constraints to the model.

Obtain Solution
7. Specify analysis types and options.
8. Solve.

Total 7 models has been tested .The result of ANSYS analysis for the beams have first natural frequencies is shown below:

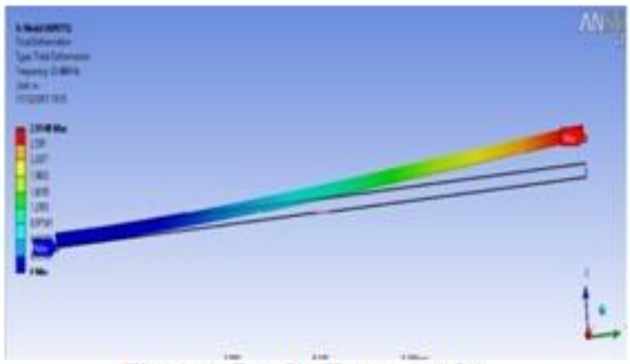


Figure A: Result of beam model 1

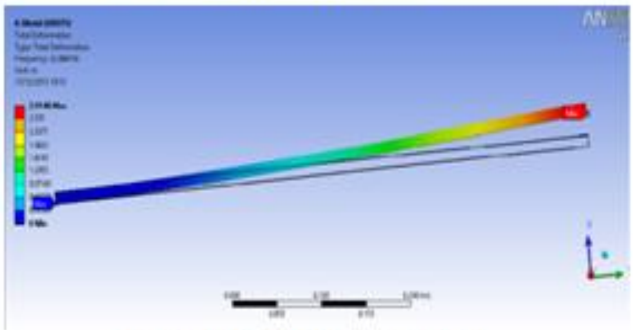


Figure B: Result of beam model 2

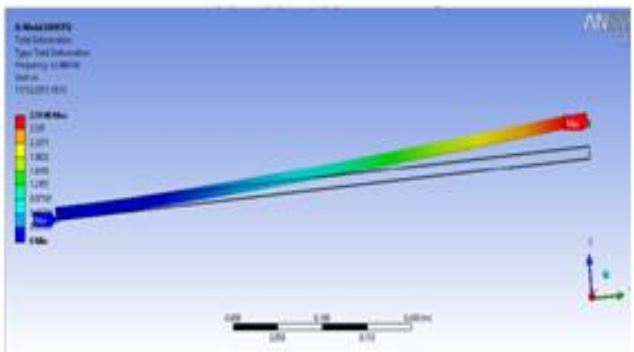


Figure C: Result of beam model 3

V. RESULTS

The table 1 shows theoretical Natural Frequency, Crack Depth and Location
Where FNF = first natural frequency.
SNF = second natural frequency.
TNF= third natural frequency.

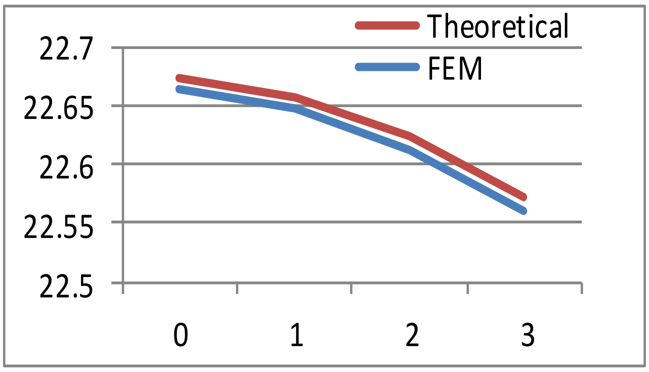
Table 1

Sr. No.	Location of crack (mm)	Depth of crack (mm)	FNF (Hz)	SNF (Hz)	TNF (Hz)
1	0	0	22.661	142.441	397.376
2	150	1	22.652	142.433	397.375
3	150	2	22.566	141.839	395.712
4	150	3	22.350	40.506	391.981
5	300	1	22.651	142.433	397.374
6	300	2	22.566	141.837	395.715
7	300	3	22.353	40.506	391.981

The table 2 shows results of beams tested by finite element modeling.

Table 2

Sr. No.	Location of crack (mm)	Depth of crack (mm)	FNF (Hz)	SNF (Hz)	TNF (Hz)
1	0	0	22.661	141.84	396.32
2	150	1	22.648	141.84	395.82
3	150	2	22.613	141.82	394.41
4	150	3	22.561	141.77	395.16
5	300	1	22.641	141.75	396.34
6	300	2	22.623	140.78	396.33
7	300	3	22.572	141.17	396.33



Graph A: First Natural Frequency Vs Crack Depth

We plot the Graph A.
Where X axis = Crack Depth (mm)
Y axis= First Natural Frequency (Hz)

VI. CONCLUSION

From Table 1 , Table 2 and Graph A:

- Natural frequency of the beam decreases as the crack depth increases and the crack location is constant.
- Changes in natural frequency observed at the area of crack location
- The results show that the values of natural frequencies by theory and ANSYS are close to the agreement.
- Natural frequencies vary as the crack location from the cantilever end varied

ACKNOWLEDGMENT

The author would like to thank Prof. Meghna Pathak (Guide) and Dr. P. K. Sharma (HOD), Mechanical Department, N.R.I. Institute of Information and Technology, Bhopal, Madhya Pradesh. India. I am also thankful to the blessing of Family, Teacher's and my friends are the main cause behind the successful completion of this paper. I wish to acknowledge great moral support given by management of N.R.I. Institute of Information and Technology, Bhopal, Madhya Pradesh. India.

REFERENCES

[1] Meysam Siyah Mansoory, 'Automatic crack detection in eggshell based on SUSAN Edge Detector using Fuzzy Thresholding', Journal of Modern Applied Science, Vol.5 No.6, Dec 2011, pp. 1-9.
[2] N. V. NarasimhaRao, 'Fault diagnosis of dynamic cracked structure using fuzzy interface system', International Journal of Emerging Trends and Engineering Development, Vol.5, Jul 2012, pp. 1-10.
[3] Dayal R. Parhi And Sasanka Choudhary,' Smart crack detection of a cracked cantilever beam using fuzzy logic technology with hybrid membership functions', Journal of Engineering And Technology Research, Vol. 3(8), Aug. 2011, pp. 270-300.
[4] Amiya Kumar Dash, 'Analysis of adaptive fuzzy technique for multiple crack diagnosis of faulty beam using vibration signatures', Journal of Advances in Fuzzy Systems, Vol. 1, 2013.
[5] Mohammad R. Jahanshahi, 'An innovative methodology for detection and quantification of cracks through incorporation of depth perception', Journal of Machine Vision and Applications, Feb 2011, pp. 3-11.

Author profile: Passed B.E. in Mechanical Engineering from Pune University. M.TECH (Scholar) ,Mechanical Department, N.R.I. Institute of Information & Technology Bhopal, Madhya Pradesh, India.