The influence of two temperature generalized thermoelastic diffusion inside a spherical shell

D. Bhattacharya, M. Kanoria

Abstract- **In this investigation, the problem of elasto-thermo-diffusion interaction insider a spherical shell is concerned in the context of the mechanism of two temperature generalized thermoelasticity theory. The inner and outer boundaries of the spherical shell are traction free and subjected to heating. The chemical potential is also assumed to be a function of time on the boundary of the shell. The problem is based on the theory of two temperature generalized thermoelastic diffusion with one relaxation time (i.e. two temperature Lord-Shulman (2TLS) model). To obtain the general solution the integral transform technique is used. The solution obtained in the Laplace transform domain by using a direct approach. The inversion of the transformed solution is carried out by applying the method of Bellman. The numerical estimates for the thermophysical medium are obtained for copper material and have been shown graphically. The influence of diffusion on thermoelastic stresses, conductive temperature, thermodynamic temperature, displacement, concentration, chemical potential inside the shell are observed for Lord-Shulman model. The results in the absence of diffusion are also found as a particular case.**

Index terms- **elasto-thermo-diffusion interactions, generalized thermoelasticity, thermoelastic diffusion, two temperature**

I. INTRODUCTION

Diffusion can be defined as the migration of the particles from region of high concentration to region of lower concentration. The recent interest in the study of this phenomenon is due to its extensive applications in geophysics and many industrial applications. In most of these applications, the concentration is calculated using what is known as Fick's law. This is a simple law that does not take into consideration the mutual interaction between the introduced substance and the medium into which it is introduced or the effect of the temperature on this interaction.

Biot [1] develop the coupled theory of thermo-elasticity to deal with defeat of the uncoupled theory that mechanical cause has no effect on the temperature field. In this theory, the heat equation has a parabolic form which predicts an infinite speed for the propagation of mechanical wave.

The theory of generalized thermoelasticity with one relaxation time was introduced by Lord and Shulman [2]. This theory was extended by Dhaliwal and Sherief [3]. In the theory, the Maxwell-Cattaneo law of heat conduction replaces the conventional Fourier's law. For this theory, Ignaczak [4] studied the uniqueness of solution; Sherief [5] proved uniqueness and stability. The fundamental solution was developed by Sherief for spherical regions and by Sherief and Anwar [6] for cylindrical region.

Nowcaki [7] - [10] developed the theory of thermoelastic diffusion. In this theory, classical coupled thermoelastic model is used. Later on, Gawinecki et al [11] proved a theory on uniqueness and regularity of the solution for a nonlinear parabolic thermoelastic diffusion problem. For the same problem a theorem about the global existence of the solution was established by Gawinecki and Szymaniec [12]. Sherief et al. [22] and, later on, Kumar and Kansal [23] introduced the generalized theories of thermoelastic diffusion in the frame of LS and GL theories by introducing thermal relaxation time parameters and diffusion relaxation time parameters into the governing equations, which allow the finite speeds of propagation of waves inside the medium. Sherief and Salah investigated the problem of a thermoelastic half space in the context of the theory of generalized thermoelastic diffusion with one relaxation time. Singh [26], [27] studied the reflection phenomena of waves from the free surface of an elastic solid under theory of generalized thermodiffusion. Aouadi [28]-[33] also gave some attention on thermoelastic diffusion and generalized thermoelastic diffusion. Othman et al. [33] reported some studied on the effects of diffusion on a two-dimensional problem of generalized thermoelasticity in the context of the mechanism of Green-Naghdi theory. The theory introduced by Sherief et al. [22], Kothari and Mukhopadhyay [38], presented the Galerkin-type representation of solutions for thermoelastic diffusion. In the context of the same theory, variational and reciprocity theorems have been established by Kumar et al. [38].

The linearized version of the two-temperature theory (2TT) has been studied by many authors. Chan and Gurtin [13] and Chen et al. [14], [15] have formulated a theory of heat conduction in deformable bodies, which depends on two distinct temperatures (a) the conductive temperature ϕ and (b) the thermodynamic temperature θ. Lesan [17] has established uniqueness and reciprocity theorems for 2TT. The existence, structural stability and spatial behavior of the solution in 2TT have been discussed by Quintanilla [18].

Manuscript received May 14, 2014.
Debarghya Bhattacharya, M.sc., University Of Calcutta, +918961328903
Mridula Kanoria, Ph.D, University of Calcutta, Kolkata, West Bengal, INDIA, Department of Applied Mathematics, University of Calcutta, 92 A. P. C Road, Kolkata-700009, West Bengal, India, University Of Calcutta, +919433775538

The key element that sets the two-temperature thermoelasticity (2TT) apart from the classical theory of thermoelasticity (CTE) is the material parameter χ (≥0), called the temperature discrepancy [15]. Specifically, if χ=0, then ϕ=θ and the field equations of the 2TT reduce to those of CTE. It should be pointed out that 2TT suffer from so-called paradox of heat conduction, i.e., the prediction that a thermal disturbance at some point in a body is felt instantly, but unequally, through-out the body.

The present paper is concerned with the investigation of disturbances in a homogenous, isotropic temperature dependent elastic medium with two temperature generalized thermodiffusion. The analytical expressions for the displacement components, thermoelastic stresses, conductive temperature, thermoelastic temperature, concentration and chemical potential are obtained in the physical domain whose boundaries are traction free and subjected to a time dependent temperature and chemical potential in the context of 2TLS model. The Laplace transform technique is used to obtain the general solution. To get the solution in the physical domain, the inversion of the transformed solution is carried out by applying the method of Bellman. The influences inside the shell is analyzed for a copper like material and depicted graphically. The effect of two temperature parameter in the presence and absence of diffusion are analyzed theoretically and numerically. The most significant points arising out from our analysis are highlighted.

II. MATHEMATICAL FORMULATION OF THE PROBLEM:

We consider an isotropic homogenous thermoelastic spherical shell with inner radius a and outer radius b in a uniform temperature T_0 . Let the body be referred to spherical polar coordinate (r,θ,φ) with the origin at the center O of the cavity. Since we consider thermoelastic interactions which are spherically symmetric, so all the functions considered will be function of the radial distance r and the time t only. It follows that the displacement vector $\vec{u}$ thermodynamic temperature θ and Conductive temperature ϕ have the following forms:

$$\vec{u}=\big(u(r,t),0,0\big),\quad \theta=\theta(r,t),\quad \varphi=\varphi(r,t). \tag{1.1}$$

In the context of two temperature generalized thermoelastic diffusion based on LS theory, the equation of motion, the equation of heat conduction and the equation of mass diffusion in absence of body forces for a linearly isotropic generalized thermoelastic solid are, respectively,

$$\rho\frac{\partial^2 u}{\partial t^2}=(\lambda+2\mu)\frac{\partial e}{\partial r}-\beta_1\frac{\partial\theta}{\partial r}-\beta_2\frac{\partial C}{\partial r} \tag{1.2}$$

$$K\nabla^2\phi=\rho c_E(\dot\theta+\tau_0\ddot\theta)+T_0\beta_1(\dot e+\tau_0\ddot e)+cT_0(\dot C+\tau_0\ddot C) \tag{1.3}$$

$$D\beta_2\nabla^2 e+Dc\nabla^2\theta+\dot C+\tau^0\ddot C=Dd\nabla^2 C, \tag{1.4}$$

where θ is the thermodynamic temperature, ϕ is the conductive temperature, λ and μ are Lame's constants, ρ is the density, C is the mass concentration, β_1, β_2 are the material constants given by

$\beta_2=(3\lambda+2\mu)\alpha_c$, $\beta_1=(3\lambda+2\mu)\alpha_t$,

in which α_t and α_c are, respectively, the coefficient of linear thermal expansion and linear diffusion expansion, K is the thermal conductivity; D is the diffusion coefficient and c_E is the specific heat at constant strain, τ_0 is the thermal relaxation time and τ^0 is the diffusion relaxation time, c and d are the measures of the thermo-diffusion effect and diffusive effect, respectively. ∇^2 is the Laplacian, given by

$$\nabla^2=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right).$$

The strain components are given by

$$e_{rr}=\frac{\partial u}{\partial r},\quad e_{\theta\theta}=e_{\phi\phi}=\frac{u}{r}, \tag{1.5}$$

and thus the cubical dilation will be

$$e=\frac{\partial u}{\partial r}+2\frac{u}{r}=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2u). \tag{1.6}$$

The constitutive equations are given by

$$\sigma_{rr}=2\mu\frac{\partial e}{\partial r}+\lambda e-\beta_1\theta-\beta_2 C \tag{1.7}$$

$$\sigma_{\theta\theta}=\sigma_{\phi\phi}=2\mu\frac{u}{r}+\lambda e-\beta_1\theta-\beta_2 C \tag{1.8}$$

$$P=-\beta_2 e+dC-c\theta, \tag{1.9}$$

Where P is the chemical potential per unit mass of the diffusive material in the elastic body, σ_{ij} is the stress tensor.

The relation between the conductive temperature ϕ and the thermodynamic temperature θ is given by,

$$\phi-\theta=\chi\nabla^2\phi. \tag{1.10}$$

Where χ (>0) is the two temperature parameter.

For convenience, the following dimensionless quantities are used:

$$u'=c_1\eta u,\qquad r'=c_1\eta r,\qquad \theta'=\frac{\beta_1\theta}{(\lambda+2\mu)},$$

$$C'=\frac{\beta_2 C}{(\lambda+2\mu)},\quad \phi'=\frac{\beta_1\phi}{(\lambda+2\mu)},\quad P'=\frac{P}{\beta_2},\quad t'=c_1^2\eta t,$$

$$\left[\tau'^0,\tau_0'\right]=c_1^2\eta\left[\tau^0,\tau_0\right].$$

Where $c_1^2=\dfrac{(\lambda+2\mu)}{\rho}$ and $\eta=\dfrac{\rho c_E}{K}$.

Then, the governing equations are given by equations (1.2) - (1.4) and equations (1.7)-(1.10) can be expressed in the following forms (dropping the primes for convenience):

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial e}{\partial r}-\frac{\partial\theta}{\partial r}-\frac{\partial C}{\partial r}, \tag{1.11}$$

$$\nabla^2\phi=(\dot\theta+\tau_0\ddot\theta)+\varepsilon(\dot e+\tau_0\ddot e)+\varepsilon\alpha_1(\dot C+\tau_0\ddot C), \tag{1.12}$$

$$\phi-\theta=\omega\nabla^2\phi \tag{1.13}$$

$$\nabla^2 e+\alpha_1\nabla^2\theta+\alpha_2(\dot C+\tau^0\ddot C)=\alpha_3\nabla^2 C, \tag{1.14}$$

$$\sigma_{rr} = e - \frac{4}{\beta^2}\frac{u}{r} - \theta - C, \qquad (1.15)$$

$$\sigma_{\theta\theta} = (1 - \frac{2}{\beta^2})e + \frac{2}{\beta^2}\frac{u}{r} - \theta - C, \qquad (1.16)$$

$$P = -e + \alpha_3 C - \alpha_1 \theta. \qquad (1.17)$$

Where $\alpha_1 = \frac{c\rho c_1^2}{\beta_1\beta_2}$, $\alpha_2 = \frac{\rho c_1^2}{\eta D \beta_2^2}$, $\alpha_3 = \frac{d\rho c_1^2}{\beta_2^2}$,

$\varepsilon = \frac{T_0 \beta_1^2}{\rho^2 c_1^2 c_E}$, $\beta^2 = \frac{\lambda + 2\mu}{\mu}$, $\omega = \chi c_1^2 \eta^2$.

III. BOUNDARY CONDITIONS:

From the physical phenomena for the problem, we will take the boundary conditions as:

(1) The boundaries of the shell are assumed to be traction free, i.e.:

$$\sigma_{rr}(r,t) = 0 \quad \text{On } r = a, b \text{ for } t \ge 0. \qquad (1.18)$$

(2) Boundaries of the shell are subjected to a thermal shock in the form

$$\phi = \phi_1 H(t) \quad \text{on } r = a,\ t > 0,$$
$$= \phi_2 H(t) \quad \text{on } r = b,\ t > 0. \qquad (1.19)$$

(3) The chemical potential is also assumed to be a known function of time at the boundaries of the shell, that is:

$$P = P_1 H(t) \quad \text{on } r = a,\ t > 0,$$
$$= P_2 H(t) \quad \text{on } r = b,\ t > 0. \qquad (1.20)$$

IV. SOLUTIONS IN LAPLACE TRANSFORM DOMAIN:

Applying the Laplace transform defined by the relation, $\bar{f}(r,s) = \int_0^\infty f(r,t)e^{-st}dt \quad \text{Re}(s) > 0,$

to equation (1.11) and using homogeneous initial conditions, we get

$$s^2\bar{u} = \frac{\partial \bar{e}}{\partial r} - \frac{\partial \bar{\theta}}{\partial r} - \frac{\partial \bar{C}}{\partial r}. \qquad (1.21)$$

Now applying the Laplace transform on equation (1.12) and using (1.13), we get

$$\bar{\theta} = \frac{1}{1+\omega a_3}\bar{\phi} - \frac{\varepsilon\alpha_1\omega a_3}{1+\omega a_3}\bar{C} - \frac{\varepsilon\omega a_3}{1+\omega a_3}\bar{e}, \qquad (1.22)$$

where $a_3 = s(1+\tau_0 s)$.

Applying divergence operator on (1.21), we get

$$(\nabla^2 - s^2)\bar{e} = \nabla^2\bar{C} + \nabla^2\bar{\theta}. \qquad (1.23)$$

Using the equation (1.22), equations (1.12)-(1.14) and (1.23) becomes

$$\nabla^2\bar{\phi} = L_1\bar{\phi} + L_2\bar{C} + L_3\bar{e}, \qquad (1.24)$$

$$[(1+L_1\varepsilon\omega)\nabla^2\bar{e}] - s^2\bar{e} = \frac{L_1}{a_3}\nabla^2\bar{\phi} + (1-L_2\omega)\nabla^2\bar{C}, \qquad (1.25)$$

$$(1-L_2\omega)\nabla^2\bar{e} + \frac{\alpha_1 L_1}{a_3}\nabla^2\bar{\phi} = [(\alpha_3 + \omega\alpha_1 L_2)\nabla^2 -$$
$$\alpha_2 s(1+\tau^0 s)]\bar{C}, \qquad (1.26)$$

where,

$L_1 = \frac{a_3}{1+a_3\omega}$, $L_2 = \frac{a_3\alpha_1\varepsilon}{1+a_3\omega}$, $L_3 = \frac{a_3\varepsilon}{1+a_3\omega}$,

$$\bar{\sigma}_{rr} = \bar{e} - \frac{4}{\beta^2}\frac{\bar{u}}{r} - \bar{\theta} - \bar{C}, \qquad (1.27)$$

$$\bar{\sigma}_{\theta\theta} = (1 - \frac{2}{\beta^2})\bar{e} + \frac{2}{\beta^2}\frac{\bar{u}}{r} - \bar{\theta} - \bar{C}, \qquad (1.28)$$

$$\bar{P} = -\bar{e} + \alpha_3\bar{C} - \alpha_1\bar{\theta}. \qquad (1.29)$$

Now, using Equations (1.24)-(1.26), we obtain

$$\left(\nabla^6 - b_1\nabla^4 + b_2\nabla^2 - b_3\right)(\bar{e}, \bar{\phi}, \bar{C}) = 0. \qquad (1.30)$$

Where we used the notations

$$b_1 = \frac{1}{a_3^2(1-L_2\omega)^2 - a_3^2(\alpha_3 + L_2\alpha_1\omega)(1+L_1\varepsilon\omega)}[\alpha_1 L_1 a_3 L_3(1-L_2\omega) +$$
$$\alpha_1 L_1 a_3 L_2(1+L_1\varepsilon\omega) + a_3^2\alpha_2 s(1+\tau^0 s)(1+L_1\varepsilon\omega) + L_1 a_3^2(1+L_1 s\omega)$$
$$(\alpha_3 + L_2\alpha_1\omega) + L_1 a_3^2(1+L_1\varepsilon\omega)(\alpha_3 + L_2\alpha_1\omega) + a_3^2 s^2(\alpha_3 + L_2\alpha_1\omega) +$$
$$L_1 L_3 a_3(\alpha_3 + L_2\alpha_1\omega) + L_1 L_3 a_3(1-L_2\omega) - L_1 a_3^2(1-L_2\omega)^2] \qquad (1.31)$$

$$b_2 = \frac{1}{a_3^2(1-L_2\omega)^2 - a_3^2(\alpha_3 + L_2\alpha_1\omega)(1+L_1\varepsilon\omega)}[\alpha_3 L_1$$
$$L_3 a_3 s^2 + L_1 a_3^2 s^2(\alpha_3 + L_2\alpha_1\omega) + a_3^2 s\alpha_2(1+\tau^0 s)L_1 \qquad (1.32)$$
$$(1+L_1\varepsilon\omega) + a_3^2 s^3\alpha_2(1+\tau^0 s) + \alpha_2 L_1 L_3 a_3 s(1+\tau^0 s)]$$

$$b_3 = \frac{1}{a_3^2(1-L_2\omega)^2 - a_3^2(\alpha_3 + L_2\alpha_1\omega)(1+L_1\varepsilon\omega)}[a_3^2\alpha_2 s^3(1+\tau^0 s)L_1] \qquad (1.33)$$

Equation (1.29) can be factorized as

$$(\nabla^2 - k_1^2)(\nabla^2 - k_2^2)(\nabla^2 - k_3^2)(\bar{e}, \bar{\phi}, \bar{C}) = 0, \qquad (1.34)$$

where k_1, k_2, k_3 are the roots with positive real part of the characteristic equation

$$k^6 - b_1 k^4 + b_2 k^2 - b_3 = 0,$$

and are given by [24]

$$k_1=\sqrt{\frac{1}{3}[2p\sin q+b_1},\qquad (1.35)$$

$$k_2=\sqrt{\frac{1}{3}[b_1-p(\sqrt{3}\cos q+\sin q)]}\quad,\qquad (1.36)$$

$$k_3=\sqrt{\frac{1}{3}[b_1+p(\sqrt{3}\cos q-\sin q)]},\qquad (1.37)$$

where

$$p=\sqrt{b_1^2-3b_2},\ q=\frac{\sin^{-1}v}{3},\ v=-\frac{2b_1^3-9b_1b_2+27b_3}{2p^3}.\qquad (1.38)$$

Therefore, the solution of equation (1.30), which is bounded at infinity, is given by

$$\bar{\phi}(r,s)=\frac{1}{\sqrt{r}}\sum_{i=1}^{3}[A_i(s)I_{1/2}(k_ir)+B_i(s)K_{1/2}(k_ir)],\qquad (1.39)$$

$$\bar{e}(r,s)=\frac{1}{\sqrt{r}}\sum_{i=1}^{3}[A_i'(s)I_{1/2}(k_ir)+B_i'(s)K_{1/2}(k_ir)],\qquad (1.40)$$

$$\bar{C}(r,s)=\frac{1}{\sqrt{r}}\sum_{i=1}^{3}[A_i''(s)I_{1/2}(k_ir)+B_i''(s)K_{1/2}(k_ir)],\quad (1.41)$$

where A_i, A_i', A_i'', B_i, B_i', B_i'' (for i =1, 2, 3) are parameters depending only on s. $I_{1/2}$ is the modified Bessel function of the first kind of order $1/2$ and $K_{1/2}$ is the modified Bessel function of second kind of order $1/2$. Now, from Equations (1.24)-(1.26) and (1.39)-(1.40), we obtain the following relations: $A_i'=\frac{p_i}{d_i}A_i$, $A_i''=\frac{f_i}{d_i}A_i$, $B_i'=\frac{p_i}{d_i}B_i$,

$$B_i''=\frac{f_i}{d_i}B_i,\qquad (1.42)$$

where,

$$p_i=a_3(1-L_2\varepsilon\omega)k_i^4+[L_1L_2-a_3L_1(1-L_2\omega)]k_i^2,$$
$$d_i=k_i^2[a_3L_3(1-L_2\omega)+a_3L_2(1+\varepsilon L_1\omega)]-s^2a_3L_2,$$
$$f_i=a_3(1+\varepsilon L_1\omega)k_i^4-[a_3L_1(1+\varepsilon L_1\omega)+s^2a_3$$
$$+L_1L_3]k_i^2+s^2a_3L_1.$$

Thus we have

$$\bar{e}(r,s)=\frac{1}{\sqrt{r}}\sum_{i=1}^{3}\frac{\varsigma_{1i}}{d_i}[A_i(s)I_{1/2}(k_ir)+B_i(s)K_{1/2}(k_ir)],\qquad (1.43)$$

$$\bar{C}(r,s)=\frac{1}{\sqrt{r}}\sum_{i=1}^{3}\frac{\varsigma_{2i}}{d_i}[A_i(s)I_{1/2}(k_ir)+B_i(s)K_{1/2}(k_ir)].\qquad (1.44)$$

Where

$$\varsigma_{1i}=a_3(1-L_2\varepsilon\omega)k_i^4+[L_1L_2-a_3L_1(1-L_2\omega)]k_i^2,$$
$$\varsigma_{2i}=a_3(1+\varepsilon L_1\omega)k_i^4-[a_3L_1(1+\varepsilon L_1\omega)+s^2a_3+L_1L_3]k_i^2+s^2a_3L_1.$$

Using the relation between u and e from (1.5) and (1.43), we get the solution for the dimensionless form of displacement as follows:

$$\bar{u}(r,s)=\frac{1}{\sqrt{r}}\sum_{i=1}^{3}\frac{\varsigma_{1i}}{k_id_i}[A_i(s)I_{3/2}(k_ir)-B_i(s)K_{3/2}(k_ir)].\qquad (1.45)$$

Therefore, from Equations (1.27)-(1.29), (1.39) and (1.43)-(1.45), we get

$$\bar{\sigma}_{rr}(r,s)=\frac{1}{\sqrt{r}}\sum_{i=1}^{3}A_i(s)[\varsigma_{3i}I_{1/2}(k_ir)-\frac{4p_i}{\beta^2rk_id_i}I_{3/2}(k_ir)]+\sum_{i=1}^{3}B_i(s)[\varsigma_{3i}K_{1/2}(k_ir)+\frac{4p_i}{\beta^2rk_id_i}K_{3/2}(k_ir)],\qquad (1.46)$$

$$\bar{\sigma}_{\phi\phi}(r,s)=\frac{1}{\sqrt{r}}\sum_{i=1}^{3}A_i(s)[\varsigma_{4i}I_{1/2}(k_ir)+\frac{2p_i}{\beta^2rk_id_i}I_{3/2}(k_ir)]+\sum_{i=1}^{3}B_i(s)[\varsigma_{4i}K_{1/2}(k_ir)-\frac{2p_i}{\beta^2rk_id_i}K_{3/2}(k_ir)],\qquad (1.47)$$

$$\bar{P}(r,s)=\frac{1}{\sqrt{r}}\sum_{i=1}^{3}\varsigma_{5i}[A_i(s)I_{1/2}(k_ir)+B_i(s)K_{1/2}(k_ir)].\qquad (1.48)$$

Where

$$\varsigma_{3i}=\frac{p_i}{d_i}-1-\frac{f_i}{d_i},\qquad \varsigma_{4i}=(1-\frac{2}{\beta^2})\frac{p_i}{d_i}-1-\frac{f_i}{d_i},$$
$$\varsigma_{5i}=-\frac{p_i}{d_i}-\alpha_1-\frac{\alpha_3f_i}{d_i}.$$

To evaluate the unknown parameters, we shall use the Laplace transformation of the boundary condition (1.18)-(1.20), together with equations (1.39), (1.46) and (1.48); we have the following set of six linear equations in six
unknowns:

$$\sum_{i=1}^{3}A_i(s)[\varsigma_{3i}I_{1/2}(k_ia)-\frac{4p_i}{\beta^2ak_id_i}I_{3/2}(k_ia)]+\sum_{i=1}^{3}B_i(s)[\varsigma_{3i}K_{1/2}(k_ia)+\frac{4p_i}{\beta^2ak_id_i}K_{3/2}(k_ia)]=0,\qquad (1.49)$$

$$\sum_{i=1}^{3}A_i(s)[\varsigma_{3i}I_{1/2}(k_ib)-\frac{4p_i}{\beta^2bk_id_i}I_{3/2}(k_ib)]+\sum_{i=1}^{3}B_i(s)[\varsigma_{3i}K_{1/2}(k_ib)+\frac{4p_i}{\beta^2bk_id_i}K_{3/2}(k_ib)]=0,\qquad (1.50)$$

$$\sum_{i=1}^{3}[A_i(s)I_{1/2}(k_ia)+B_i(s)K_{1/2}(k_ia)]=\frac{\phi_1\sqrt{a}}{s},\qquad (1.51)$$

$$\sum_{i=1}^{3}[A_i(s)I_{1/2}(k_ib)+B_i(s)K_{1/2}(k_ib)]=\frac{\phi_2\sqrt{b}}{s},\qquad (1.52)$$

$$\sum_{i=1}^{3}\varsigma_{5i}[A_i(s)I_{1/2}(k_ia)+B_i(s)K_{1/2}(k_ia)]=\frac{P_1\sqrt{a}}{s},\qquad (1.53)$$

$$\sum_{i=1}^{3}\varsigma_{5i}[A_i(s)I_{1/2}(k_ib)+B_i(s)K_{1/2}(k_ib)]=\frac{P_2\sqrt{b}}{s}.\qquad (1.54)$$

We can obtain $A_1(s)$, $A_2(s)$, $A_3(s)$, $B_1(s)$, $B_2(s)$, $B_3(s)$ by solving the above linear system of equations (1.49)-(1.54). This completes the solution of the present problem in the Laplace transform domain.

IV. SPECIAL CASE: (WITHOUT DIFFUSION)

By putting $C=0$ and $\beta=0$ into equation (1.2), (1.3), (1.7) and (1.8) and neglect the diffusion effect by eliminating Equations (1.4) and (1.9), we get the equations for conductive temperature, displacement and the stresses without the effect of diffusion.In this case, after some simple computations, equation (1.30) reduces to,

$$(\nabla^4 - a_1\nabla^2 + a_2)(\bar{e},\bar{\phi}) = 0. \qquad (1.55)$$

Where $a_1 = \dfrac{a_3L_1(1+L_1\varepsilon\omega)+s^2a_3+L_1L_3}{a_3(1+L_1\varepsilon\omega)}$, $a_2 = \dfrac{L_1s^2a_3}{a_3(1+L_1\varepsilon\omega)}$

Equation (1.55) can be factorized as $(\nabla^2 - k_1^2)(\nabla^2 - k_2^2)(\bar{e},\bar{\phi}) = 0,$ (1.56)

where k_1, k_2 are the roots with positive real part of the characteristic equation $k^4 - a_1k^2 + a_2 = 0$.

Using the solution of equation (1.55), given by

$$\bar{\phi}(r,s) = \frac{1}{\sqrt{r}}\sum_{i=1}^{2}[A_i(s)I_{1/2}(k_ir) + B_i(s)K_{1/2}(k_ir)], \qquad (1.57)$$

$$\bar{e}(r,s) = \frac{1}{\sqrt{r}}\sum_{i=1}^{2}[A_i'(s)I_{1/2}(k_ir) + B_i'(s)K_{1/2}(k_ir)], \qquad (1.58)$$

where A_i, A_i', B_i, B_i' (for i =1, 2) are parameters depending only on s, we get

$$\bar{u}(r,s) = \frac{1}{\sqrt{r}}\sum_{i=1}^{2}\frac{\varsigma_{1i}}{k_id_i}[A_i(s)I_{3/2}(k_ir) - B_i(s)K_{3/2}(k_ir)], \qquad (1.59)$$

$$\bar{\sigma}_{rr}(r,s) = \frac{1}{\sqrt{r}}\sum_{i=1}^{2}A_i(s)[\varsigma_{3i}I_{1/2}(k_ir) - \frac{4p_i}{\beta^2rk_id_i}I_{3/2}(k_ir)] + \sum_{i=1}^{2}B_i(s)[\varsigma_{3i}K_{1/2}(k_ir) + \frac{4p_i}{\beta^2rk_id_i}K_{3/2}(k_ir)], \qquad (1.60)$$

$$\bar{\sigma}_{\phi\phi}(r,s) = \frac{1}{\sqrt{r}}\sum_{i=1}^{2}A_i(s)[\varsigma_{4i}I_{1/2}(k_ir) + \frac{2p_i}{\beta^2rk_id_i}I_{3/2}(k_ir)] + \sum_{i=1}^{2}B_i(s)[\varsigma_{4i}K_{1/2}(k_ir) - \frac{2p_i}{\beta^2rk_id_i}K_{3/2}(k_ir)]. \qquad (1.61)$$

Where

$$p_i = L_1k_i^2,\ d_i = a_3[(1+L_1\varepsilon\omega)k_i^2 - s^2],\ \varsigma_{3i} = \frac{p_i}{d_i} - 1,$$

$$\varsigma_{4i} = (1 - \frac{2}{\beta^2})\frac{p_i}{d_i} - 1.$$

In view of the boundary conditions (1.18) and (1.19) $A_i(s)$ and $B_i(s)$ involved in Equations (1.57), (1.59)-(1.60) can be determined by the solution of following equations:

$$\sum_{i=1}^{2}A_i(s)[\varsigma_{3i}I_{1/2}(k_ia) - \frac{4p_i}{\beta^2ak_id_i}I_{3/2}(k_ia)] + \sum_{i=1}^{2}B_i(s)[\varsigma_{3i}K_{1/2}(k_ia) + \frac{4p_i}{\beta^2ak_id_i}K_{3/2}(k_ia)] = 0, \qquad (1.62)$$

$$\sum_{i=1}^{2}A_i(s)[\varsigma_{3i}I_{1/2}(k_ib) - \frac{4p_i}{\beta^2bk_id_i}I_{3/2}(k_ib)] + \sum_{i=1}^{2}B_i(s)[\varsigma_{3i}K_{1/2}(k_ib) + \frac{4p_i}{\beta^2bk_id_i}K_{3/2}(k_ib)] = 0, \qquad (1.63)$$

$$\sum_{i=1}^{3}[A_i(s)I_{1/2}(k_ia) + B_i(s)K_{1/2}(k_ia)] = \frac{\phi_1\sqrt{a}}{s}, \qquad (1.64)$$

$$\sum_{i=1}^{2}[A_i(s)I_{1/2}(k_ib) + B_i(s)K_{1/2}(k_ib)] = \frac{\phi_2\sqrt{b}}{s}. \qquad (1.65)$$

This completes the solution of the present problem in the Laplace transform domain.

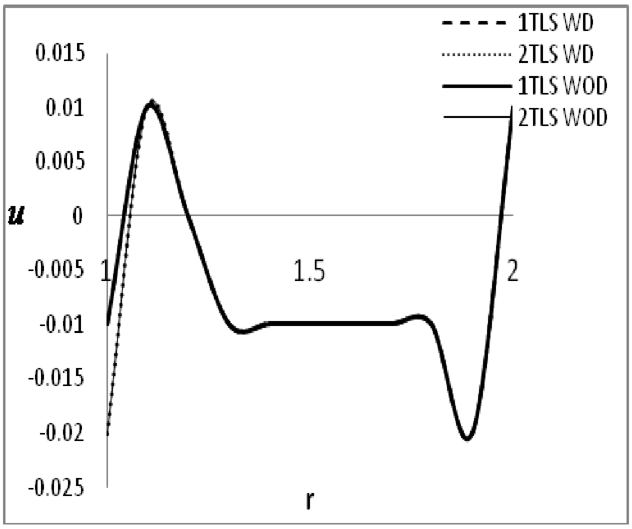
V. NUMERICAL RESULTS AND DISCUSSION:

In order to illustrate theoretical results in the preceding sections, we now present some numerical results. To get the solutions for the displacement, radial stress, shear stress, conductive temperature, thermodynamic temperature, chemical potential and mass concentration in the physical domain, we have to apply Laplace inversion formula to the equations (1.49)-(1.54) respectively. Here we adopt the method of Bellman et al. [45] for inversion and choose a time span given by seven values of time t_i, i =1 to 7 at which u_r, σ_{rr}, $\sigma_{\phi\phi}$, P and C are evaluated from the negative of logarithms of the roots of the shifted Legendre polynomial of degree 7. For the illustration we consider copper material with material constants. The physical data in SI units for which given as follows [24]:

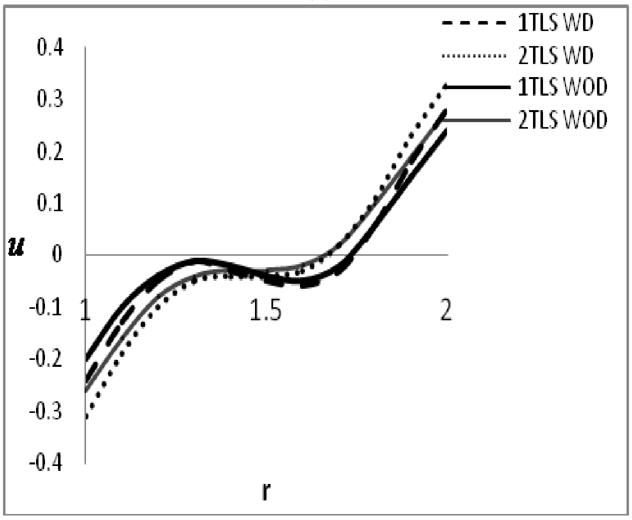
$\lambda = 7.76\times10^{10}\,N.m^{-2}$, $\mu = 3.86\times10^{10}\,N.m^{-2}$, $\rho = 8954\,kg.m^{-3}$, $K = 386\,W.m^{-1}K^{-1}$, $c_E = 383.1\,J.kg^{-1}.K^{-1}$, $T_0 = 293K$, $\alpha_c = 1.98\times10^{-4}\ m^3kg^{-1}$, $\alpha_t = 1.78\times10^{-5}K^{-1}$, $\varepsilon = 0.0168$, $c = 1.2\times10^4 m^2K^{-1}s^{-2}$, $d = 0.9\times10^6 m^5kg^{-1}s^{-1}$, $\alpha_1 = 5.43$, $\alpha_2 = 0.533$, $\alpha_3 = 36.24$.

Also we have taken $\phi_0 = 1$, $\tau_0 = 0.01$, $\tau^0 = 0.1$ and $R = 1$ for computational purposes.

Figures 1-6 representing the variation of displacement, radial stress, shear stress, conductive temperature, chemical potential and mass concentration along the radius of the sphere for both the one-temperature Lord-Shulman (1TLS) (for $\omega = 0$)and the two-temperature Lord-Shulman (2TLS) (for $\omega = 0.1$) theories. The computation were carried out for both large time (t=0.35) and small time (t=0.026). The variation of the field is observed when the step input of conductive temperatures with $\phi_1 = 1$ and $\phi_2 = 1$ applied on the inner boundary a=1 and outer boundary b=2 respectively. The step input of chemical potential with $P_1 = 1$ and $P_2 = 1$ are also applied on the boundaries of the shell. Our computations were also carried out in absence of diffusion, by using Equations (1.57), (1.59)-(1.61). In figure 1-6 dashed lines represent the case in the thermoelastic diffusive medium in the context of 1TLS theory ($\omega = 0$) (1TLS WD). Doted lines represent the distribution in thermoelastic diffusive medium in the context of 2TLS ($\omega = 0.1$) (2TLS WD) theory. The solid lines and thin lines correspond to the thermoelastic distributions (that is, in the absence of diffusion) under the 1TLS theory ($\omega = 0$) (1TLS WOD) and the 2TLS theory ($\omega = 0.1$) (2TLS WOD) respectively.



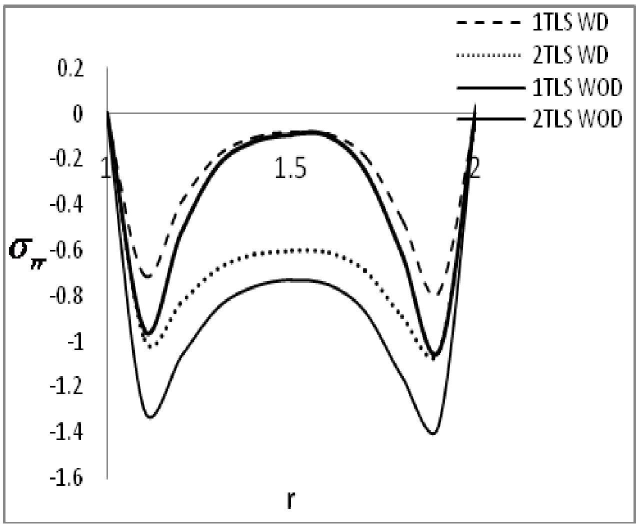
(a)



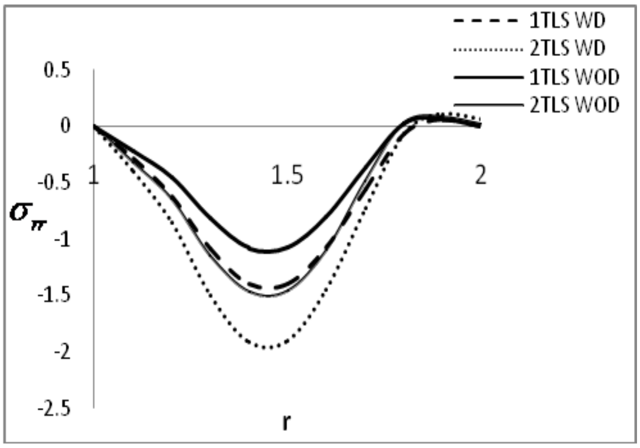
(b)

***Figure 1*: Distribution of u_r at t=0.026 (a) and t=0.35 (b).**

Figures 1(a) and 1(b), depicts the variation of the displacement component (u) against radial distance (r) inside the spherical shell for both small time (t =0.026) and large time (t =0.35), respectively. From figure 1(a) it is observed that, a mild effect of diffusion is in the interval $1 < r < 1.2$. Moreover, the displacement component is compressive in nature near both the boundaries at lower time. The graphs of displacement under with diffusion and without diffusion for 1TLS ($\omega = 0$) and 2TLS ($\omega = 0.1$) theories are almost merged. As may seen from the figure 1(b), it is observed that the radial displacement (u) takes negative values near the inner boundary ($1 < r < 1.7$) of the sphere for both the cases with diffusion (WD) and without diffusion (WOD) for both 1TLS model and 2TLS model. The absolute value of this field also increases with the increase of time in both the media.



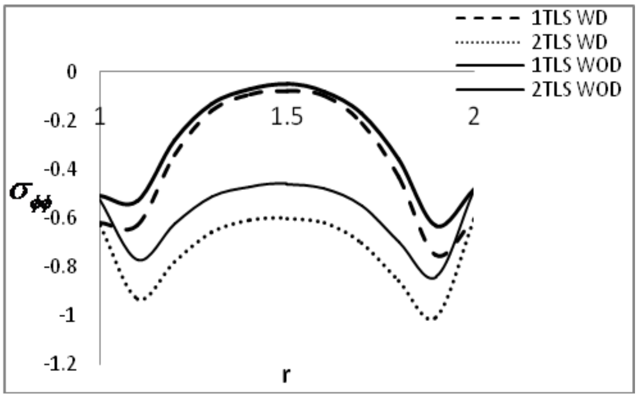
(a)



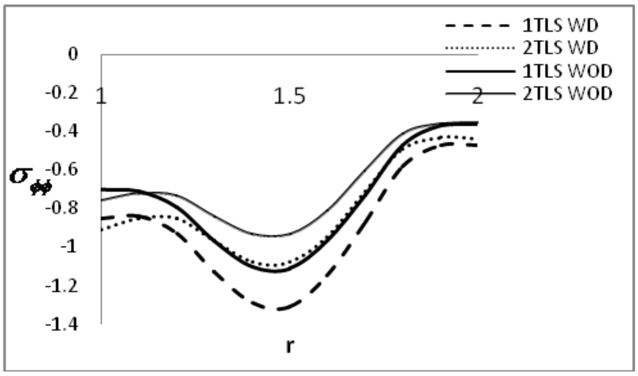
(b)

***Figure 2*: Distribution of σ_{rr} at t= 0.026 (a) and t=0.35 (b).**

Figures 2(a) and 2(b) are plotted to show the variation of radial stress (σ_{rr}) against radial distance (r) inside the spherical shell. At both boundaries the radial stress is noted to be zero, which also agrees with the theoretical boundary conditions. At time t =0.026, the radial stress σ_{rr} is compressive in nature throughout the medium ($1 < r < 2$), whereas with the advancement of time it becomes fully compressive at the middle zone for both theories. For both t =0.026 and t =0.35, the influence of diffusion is significant towards the middle of the shell under the 1TLS ($\omega = 0$) and the 2TLS ($\omega = 0.1$) theories.



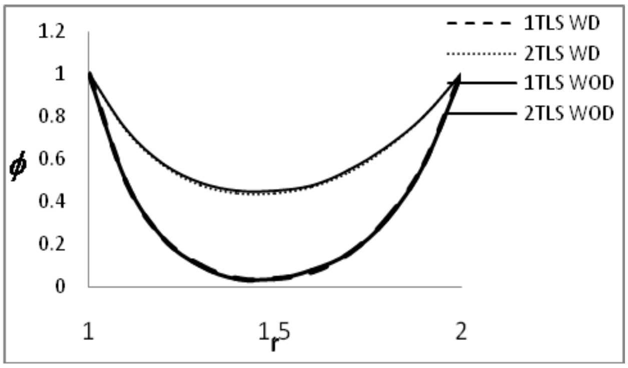
(a)

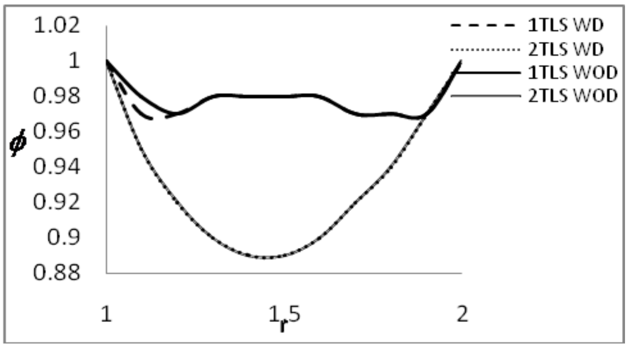


(b)

Figure 3: **Distribution of $\sigma_{\phi\phi}$ at t=0.026 (a) and t= 0.35 (b).**

Figures 3(a) and 3(b), the variation of shear stress ($\sigma_{\phi\phi}$) against radial distance (r) inside the spherical shell are observed. At both small and large times, it may seen from the figure, the hoop stress is fully compressive in nature in all cases. At small time the effect of diffusion is very prominent inside the shell, whereas, with the increase of the time it decreases under both 1TLS ($\omega = 0$) and 2TLS ($\omega = 0.1$) theories.

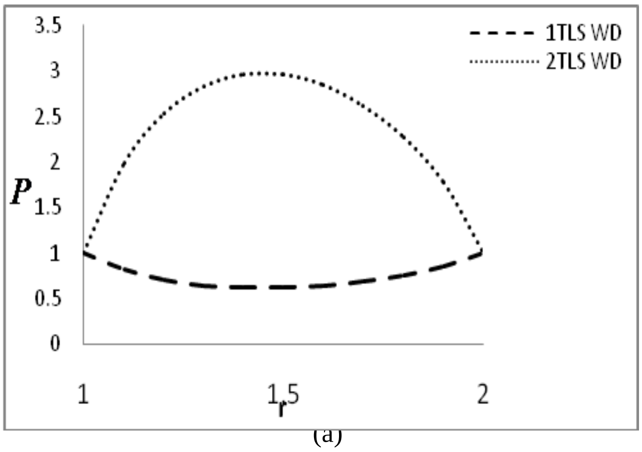




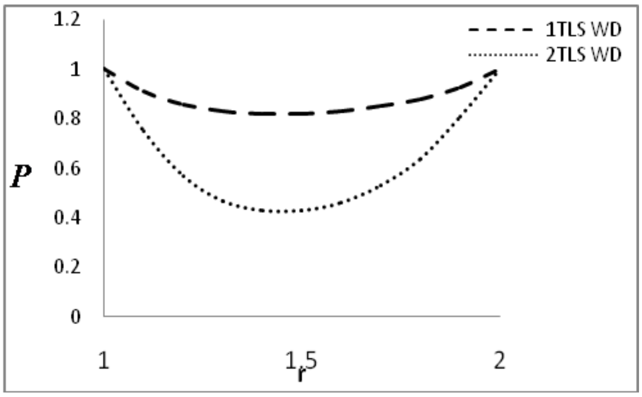
(b)

Figure 4: **Distribution of ϕ at t=0.026 (a) and t= 0.35 (b).**

Figures 4(a) and 4(b) display the variation of temperature ϕ against radial distance (r) inside the spherical shell. We can see that the magnitude of temperature field shows the maximum value at both the boundaries. At small time, with the increase of radial distance towards the middle it decreases and becomes minimum at the middle of the shell. For both larger and small time the difference of numerical value for both with diffusion and without diffusion under 1TLS ($\omega = 0$) and 2TLS ($\omega = 0.1$) theories are also noted.



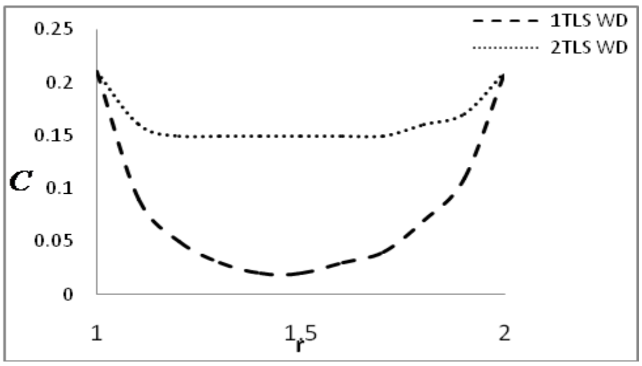
(a)



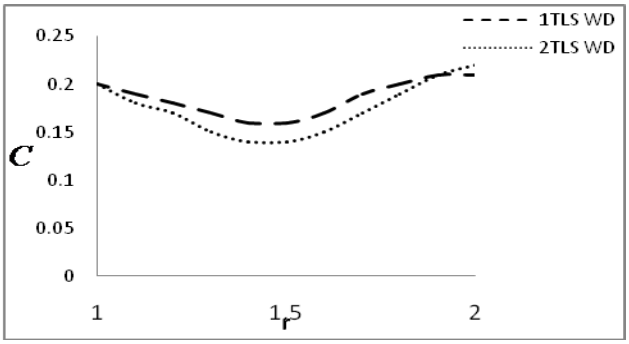
(b)

Figure 5: **Distribution of P at t=0.026 (a) and t= 0.35 (b).**

Figures 5(a) and 5(b) represent the variation of chemical potential (P) against radial distance (r) inside the spherical shell for the thermoelastic diffusive medium. At both boundaries the chemical potential is noted to be in agreement with the boundary conditions. The difference between the chemical potential are more prominent under the 1TLS ($\omega = 0$) theory as compared to the 2TLS ($\omega = 0.1$) theory.



(a)



(b)

Figure 6: **Distribution of C at t=0.026 (a) and t= 0.35 (b).** Figures 6(a) and 6(b) shows the variation of concentration (C) against radial distance (r) inside the spherical shell for the thermoelastic diffusive medium. Like temperature field the mass concentration shows the maximum value at both the boundaries and becoming minimum towards the middle. The 1TLS ($\omega = 0$) theory predicts a significantly different trend as compared to 2TLS ($\omega = 0.1$) theory.

VI. CONCLUSIONS:

The problem of investigating the thermoelastic displacements, stresses, conductive temperature, thermoelastic temperature produced in a homogenous isotropic spherical shell of the thermoelastic diffusive medium is stated in the context of 1TLS and 2TLS theories. We also compare our results with the corresponding results in thermoelastic medium (without diffusion). According the analysis above and the numerical results presented in figs.1-6, we can conclude the following important points:

(i) The presence of diffusion plays an important role in all the quantities. The influence of diffusion is more significant on displacement, radial stress and shearing stress, as compared to conductive temperature and thermoelastic temperature fields. At small time, the influence of diffusion on stresses near the boundary is more significant. With the increase of time the region of influence shifts towards the middle.

(ii) The significant difference is also noted in the physical quantities for one temperature and two temperature LS models. Two temperature theory is more realistic then the one temperature theory in the case of both generalized thermoelasticity with diffusion and without diffusion. This study is very important for microscale problems, because in these cases the material parameters are temperature dependent.

ACKNOWLEDGEMENTS

We are grateful to Prof. S.C. Bose of the Department of Applied Mathematics, University of Calcutta for his valuable suggestions and guidance in preparation of the paper.

FUNDING:

This research received no specific grant from any funding agency in the public, commercial or not-for-profit sectors.

REFERENCES:

[1] M. A. Biot,"*Thermoelasticity and irreversible thermodynamics*," J Appl Phys, vol.27, 1956, pp. 240-253.
[2] H.W. Load, Y. A Shulman, "*Generalized dynamical theory of thermoelasticity*," J Mech Phys Solid, vol. 15, 1967, pp. 299-309.
[3] R. Dhaliwal, H. Sherief, Quart. Applied Math, vol. 33, 1980, pp. 1-11.
[4] J. Ignaczak, "*A Strong Discontinuity Wave in Thermoelastic with Relaxation Times,"* J. Thermal Stresses, vol. 9, 1985, pp. 25-40.
[5] H. H. Sherief, "On uniqueness and stability in generalized thermoelasticity," Quarterly of Applied Mathematics, vol. 33, 1987, pp.773-778.
[6] M. Sherief, Anwar, Journal of Thermal Stresses, vol. 11, 1988, pp. 353-365
[7] W. Nowacki, "*Dynamical problems of thermoelastic diffusion in solids,"* I.Bull Acad Pol Sci Tech, vol. 22, 1974, pp. 55-64.
[8] W. Nowacki, "*Dynamical problems of thermoelastic diffusion in solids II,"* Bull Acad Pol Sci Tech, vol. 22, 1974, pp.129-135.
[9] W. Nowacki, "*Dynamical problems of thermoelastic diffusion in solids III,"* Bull Acad Pol Sci Tech, vol. 22, 1974, pp. 257-266.
[10] W. Nowacki, *"Dynamical problems of thermoelastic diffusion in elastic solids,"* Bull Acad Pol Sci Tech, vol. 15, 1974, pp. 105-128.
[11] J. Gawinecki, P. Kacprzyk, and P. Bar-Yoseph, "*Intial boundary value problem for some coupled nonlinear parabolic system of partial differential equations appearing in thermoelastic diffusion in solid body*," J.Math Appl, vol. 19, 2000, pp. 121-130.
[12] J. Gawinecki, P. Kacprzyk, *"A. Global solution of the Cauchy problem in the nonlinear thermoelastic diffusin in solid body,"* (PAMM) Proc Appl Math Mech, vol. 1, 2002, pp. 446-447.
[13] P. J. Chen, M. E Gurtin, and Z. anzew, Journal of Mathematical Physics, vol. 19, 1968, pp. 614-622.
[14] P. J. Chen, M. E Gurtin, and W.O. Williams, *"A note on non-simple heat conduction,"* Zeitschrift für angewandte Mathematik und Physik, vol. 19, 1968, pp. 969-985
[15] P. J. Chen, M. E Gurtin, and W.O. Williams, Zeitschrift für angewandte Mathematik und Physik, vol. 20, 1969, pp. 107-121.
[16] W. E. Warren, P. J, Chen, Acta Mechanica, vol. 16, 1973, pp. 83-98
[17] D.Lesan, Journal of Applied Mathematics and Physics, vol. 21, 1970, pp. 583-601.
[18] R. Quintanilla, Acta Mechanica, vol. 168, 2004, pp. 61-73.
[19] A. E. Green, P. M. Naghdi, "*A re-examination of the basic postulates of thermomechanics*," Proc Roy Soc London Ser A, vol. 432, 1991, pp. 171-194.
[20] A. E. Green, P. M. Naghdi,"*On undamped heat waves in an elastic solid*,"J Therm Stresses, vol. 15 1992, pp. 253-264.
[21] A. E. Green, P. M. Naghd,"*Thermoelasticity without energy dissipation*," J Elasticity, vol. 31, 1993, pp. 189-209.
[22] H. H. Sherief, F. A. Hamaza, and H. A. Salah, "*The theory of generalized thermoelastic diffusion,"* Int J Eng Sci, vol. 42, 2004, pp. 591-608.
[23] R. Kumar, T. Kansal, *"Propagation of lamd waves in transversely isotropic thermoelastic diffusion plate*," Int J Solid Struct, vol. 45, 2008, pp. 5890-5913.
[24] H. H. Sherief, and H. A. Salah, *"A half space problem in the theory of generalized thermoelastic diffusion,"* Int J Solids Struct, vol. 42, 2005, pp. 4484-4493.
[25] H. H. Sherief, N. M. El-Maghraby, "*A thick plate problem in the theory of generalized thermoelastic diffusion,"* Int J Thermophys, vol. 30, 2009, vol. 2044-2057.
[26] B. Singh, "*Reflection of P, and SV waves from free surface of an elastic solid with generalized thermodiffusion,"* J Earth Syst Sci, vol. 114, 2005, pp. 159-168.

[27] B. Singh, "*Reflection of SV waves from free surface of an elastic solid with generalized thermoelastic diffusion,"* J Sound Vib, vol. 291, 2006, pp. 764-778.
[28] M. Aouadi, "*A generalized thermoelastic diffusion problem for aan infinite long solid cylinder,"* Int J Math Math Sci, vol. 2006, 2006, pp. 1-15.
[29] M. Aouadi, *"A problem for an infinite elastic body with a spherical cavity in the theory of generalized thermoelastic diffusion,"* Int j Solid Struct, vol. 44, 2007, pp-5711-5722.
[30] M. Aouadi, *"Uniqueness and reciprocity theorem in the theory of generalized thermoelastic diffusion,"* J Therm Stresses, vol. 30, 2007, pp. 665-678.
[31] M. Aouadi, *"Spatial stability for quasi-state problem in thermoelastic diffusion theory*," Acta Appl Math, vol. 106, 2009, pp. 307-323.
[32] M. Aouadi, *"Polynomial and exponential stability for one-dimensional problem in thermoelastic diffusion theory,"* Appl Anal, vol. 89, 2010, pp. 935-948.
[33] M. Aouadi, *"Qualitative result in the theory of thermoelastic diffusion mixtures,"* J Therm Stresses, vol. 33, 2010, pp. 595-615.
[34] J. N. Sharma, *"Generalized thermoelastic diffusive waves in heat conducting materials*," J Sound Vib vol. 301, 2007, pp. 979-993.
[35] Y. D. Sharma, P. K. Sharma, *"On propagation of elasto-thermodiffusive surface waves in heat-conducting materials,"* J Sound Vib, vol. 315, 2008, pp. 927-938.
[36] J. N. Sharma, N. Thakur, and S. Singh, "*Propagation characteristics of elasto-thermodiffusive surface waves in semiconductor material half-space,"* Therm Stresses vol. 30 2007, pp. 357-380.
[37] J. N. Sharma, I. Sharma, and S. Chand, "*Elasto-thermodiffusive surface waves in a semiconductor half space underlying with varying temperature*," J Therm Stresses, vol. 31, 2008, pp. 956-975.
[38] R. Kumar, S. Kothari, and S Mukhopadhyay, *"Some theorems on generalized thermoelastic diffusion,"* Acta Mech, vol. 217, 2011, pp. 287-296.
[39] A. Kar, and M. Kanoria, "*Generalized thermo-visco-elastic problem of a spherical shell with three phase-lag effect,"* Appl Math Model, vol. 33, 2009, pp. 3287-3298.
[40] A. Kar, and M. Kanoria, *"Thermoelastic interaction with energy dissipation in an unbounded body with a spherical hole,"* Interational Journal of Solids Struct., vol. 44, 2007a, pp. 2961-2971.
[41] A. Kar, and M. Kanoria, "*Thermoelastic interaction with energy dissipation in a transversely isotropic thin circular disc*," Europian Journal of Mechanics A/Solids, vol. 26 2007b, pp. 969-981.
[42] H. M Youssef, and A. H. Al-Harby, "*State-space approach of two temperature generalized thermoelasticity of infinite body with a spherical cavity subjected to different type thermal loading*." Arch. Appl. Mech, Vol. 77, 2007, pp. 675-687.
[43] H. M Youssef, E. A. Al-Lehaibi, *"State-space approach of two temperature generalized thermoelasticity of one dimention problem,"* Int. J. Solids Struct, vol. 44, 2007, pp. 1550-1562.
[44] R. Kumar, R. Prasad, and S. Mukhopadhyay, *"Variational and reciprocal principal of two temperature generalized thermoelasticity,"* Journal of Thermal Stresses, vol. 33, 2010, pp. 161-171.
[45] R. Bellman, R. E. Kalaba, and J. A. Lockette, "*Numerical inversion of Laplace transform,"* New York: Amercan Elsevier Publishing Company, 1966.
[46] S. Kothari, S. Mukhopadhyay, "*A study of influence of diffusion inside a spherical shell under thermoelastic diffusion with relaxation times,"* Math and Mech of Solids, 2012, pp. 1-16.

Mridula Kanoria, Ph.D, University of Calcutta, Kolkata, West Bengal, INDIA, Department of Applied Mathematics, University of Calcutta, 92 A. P. C Road, Kolkata-700009, West Bengal, India, University Of Calcutta, +919433775538

Debarghya Bhattacharya, M.sc., University Of Calcutta, +918961328903