Analysis of Model Predictive Control for Identified Fluid Catalytic Cracking Unit

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Abstract—This paper analyzes the Model Predictive Control technique for estimated Fluid Catalytic Cracking Unit (FCCU). It presents different estimated models of FCCU such as; impulse response model, estimated ARX model, state space model and output error model. These estimated models are then used as a model in Model Predictive Control (MPC) design. The main advantages of MPC is that it can handle hard input and output constraints and it can be used for Multi Input Multi Output processes (MIMO) without increasing the complexity in control design. MATLAB/Simulink is used to estimate the different models of FCCU and simulate the results of the controller. The simulation results shows that state space model estimated using N4SID logarithm provides better result for both identification and control.

Index Terms—FCCU, MPC, N4SID, Output Error Model, System Identification.

I. INTRODUCTION

System identification is a process to build a mathematical model of dynamical systems from observed input and output data. These methods are widely used in industry for over a decade. It is being used in process control, aerospace, automotive, disk drives and embedded systems to create system models for any systems. The advantage of system identification is that it is quick and applicable to almost all systems [Billings, 2013].

On another hand Model Predictive Control (MPC) is an advanced optimized control method that has been widely used in process industries over the last two decades. It is a form of control in which the model of the process being controlled is required. The controller uses the model of the process and the output measurements to calculate the current control actions and predict the future behavior of the processes. The control action is calculated by minimizing the cost function at each sampling instant.

This paper analyzes the estimated models of Fluid Catalytic Cracking Unit (FCCU) such as; impulse response model, estimated ARX model, state space model and output error model. These estimated models are then used as a model in Model Predictive Control (MPC) design. The response of the controller for these models are shown in this paper.

II. SYSTEM IDENTIFICATION

The procedure to identify the dynamical model for any systems are as follows. First we have to select an input signal and apply it to the plant to collect an output data. Input signal can be impulse signal, step signal, Pseudo Random Binary Noise (PRBS), Generalized Binary Noise (GBN), multiple sinusoids, etc. After getting the input and output data, the model structure of the plant is specified. There are three common types of models in system identification: white-box identification model, black-box identification model and grey-box identification model. White-box identification model structure is based on the first principles such as Newton’s law. In Black-box identification; the model structure is completely known and the model parameters are estimated from the measured data. In grey-box identification the model structure is partially known from the first principles and the rest is developed from the measured data. In black-box identification, the model structure and its parameters are completely unknown and they are estimated using observed input and output data. After deciding the model structure, the system identification algorithms is used to estimate the mathematical models of dynamical systems. After the mathematical models of dynamical systems are developed, the result is validated. If the estimated model is not good enough then other estimation methods can be tried.

Input signals play very important role in system identification because it is the only way to check the behavior of the process and collect the output data. In general we cannot introduce any random input to the process that is being estimated. We have to select an input signal that will carry enough knowledge about the system and affect smoothly all the operating frequencies. This can be achieved by using many different excitation signals, such as impulse signals, step signals, Pseudo Random Binary Sequences (PRBS), Generalized Binary Noises (GBN) and multiple sinusoids. The choice among these input signals depends on the type of identification technique used and the priori knowledge of the system under the test.

In practice, for any controlled process the effect of the noises on the system should be minimized. The Input signal is the only freedom that the user have to determine the signal-to-noise ratio. Thus the amplitude of the input signal should be large enough in order to improve the signal-to-noise ratio but it cannot be too large that the output run away from the equilibrium point [Nelles, 2011].

Pseudo Random Binary Sequence (RBS) is a two stage deterministic signal with a periodic sequence of length N that switches between two levels, e.g. +a and -a. To generate these
sequences, there are two possibilities: first possibility is to use a quadratic residue code method suggested by Godfrey [Godfrey, 1993] and the second possibility is to use feedback shift register [Godfrey, 1993], [Eykhoff, 1974].

PRBS signals have been used for nonparametric model identification, such as: frequency response estimation and correlation analysis. In process control white noise signal is harmful to the actuator because it over emphasizes the high frequency band at the cost of the low and the middle frequency band. In process control low pass character signals are preferred. Such signals can be obtained by increasing the clock period of the signal or by filtering the PRBS signals. The PRBS signal is preferred for system identification, because it excites all frequencies equally and imitate white noise in deterministic signal [Nelles, 2011]. In this paper PRBS signals are applied to FCCU to obtain an input-output data.

A. Impulse Response Model

Impulse response model refers to the models estimated using impulse response signal as an input. It is defined as

\[ \delta(t)=0 \text{ for all } t \neq 0 \]

\[ \int_{-\infty}^{\infty} \delta(t) \, dt = 1 \]  

From (1) and (2), it is clear that an ideal impulse function is a function that is zero everywhere but at the origin, where it is infinitely high. If a unit impulse function \( \delta(t) \) is applied as an input to a linear time invariant system \( \text{LTI} \) and the impulse response is denoted by \( g(t) \), then a time delay input signal \( \delta(t-\tau) \) will give a time delay output signal represented by \( g(t-\tau) \). If the input signal is represented as

\[ \int_{-\infty}^{\infty} \delta(t-\tau)u(t) \, dt = u(\tau) \]

then the output impulse response will be

\[ \int_{-\infty}^{\infty} g(t-\tau)u(\tau) \, dt = y(t) \]  

If \( g(t) \) is known, then for an input signal \( u(t) \), the corresponding output signal can be computed.

B. ARX Model

ARX model relates the current output \( y \) to a finite number of past outputs \( y(t-k) \) and inputs \( u(t-k) \). It is represented in linear difference equation as

\[ y(t) + a_1y(t-1) + \cdots + a_ny(t-n_a) = b_1u(t-1) + \cdots + b_nu(t-n_b) + e(t) \]  

The advantage of ARX model is that it is the most efficient polynomial estimation methods available. It is considered to be the simplest way to represent the dynamic processes driven by an input in the presence of error and disturbances.

ARX model can be estimated using least square method. Rewriting (5) in regression form gives

\[ \hat{y}(t, \theta) = \varnothing(t)^T \bar{\theta} \]

where

\[ \varnothing(t)^T = [u(t-1), u(t-2), \ldots, u(t-n_b)] \]

\[ \bar{\theta} = [b_1, b_2, \ldots, b_{n_b}] \]

hence the model parameter \( \theta \) can be estimated using the least square estimation given as

\[ \bar{\theta} = (\varnothing^T \varnothing)^{-1} \varnothing^T y \]

C. State Space Model

The state space representation of dynamical system given as

\[ x(t+1) = Ax(t) + Bu(t) + Ke(t) \]

\[ y(t) = Cx(t) + Du(t) + e(t) \]

where \( x(t) \) is the state vector, \( y(t) \) is the system output, \( u(t) \) is the system input, \( e(t) \) is the stochastic error, \( A, B, C, D \) and \( K \) are the system matrices.

Estimating the parameters in state space model is considered to be simple because the states or the order is the only parameter which needed to be estimated.

Subspace identification is a method used to estimate the state space matrices \( A, B, C, D \) from an input and output data. It was proposed by van Overschee and de Moor. Further development to the method was done by Larimore in 1990 in which he proposed canonical variate analysis (CVA) [Larimore, 1990]. In 1994, van Overschee and de Moor proposed new numerical algorithm N4SID which identifies the mixed deterministic-stochastic systems.

In N4SID, the observability and controllability of the system is not needed to be known in advance since the state space matrices are not calculated in their canonical form but it is calculated as the full state space matrices so there is no problem of identifiability [Overschee and Moor, 1994]. Let consider a state space model of combined deterministic stochastic system given as

\[ x_{k+1} = Ax_k + Bu_k + w_k \]

\[ y_k = Cx_k + Du_k + v_k \]

where \( A, B, C, D \) are the state space matrices, \( v \) is the state noise with covariance matrices \( E[w_kw_k^T] = \) and \( E[w_kv_k^T] = \) is the output measurement noise with covariance matrices \( E[w_kw_k^T] = \) and \( E[w_kv_k^T] = \)

If the system is observable then Kalman filter can be designed for the system to estimate the state variables according to

\[ \hat{x}_{k+1} = \tilde{A}x_k + Bu_k + \tilde{K}(y_k - \tilde{C}x_k - De_k) \]

where \( \tilde{A} = \left[ \begin{array}{c} A_k \end{array} \right] \), \( \tilde{B} = \left[ \begin{array}{c} B_k \end{array} \right] \), \( \tilde{C} = \left[ \begin{array}{c} C_k \end{array} \right] \), \( \tilde{D} = \left[ \begin{array}{c} D_k \end{array} \right] \)

and \( \tilde{K} = \left[ \begin{array}{c} K_k \end{array} \right] \)

From (17) and (18), an extended state space can be formulated as

\[ y_k = \bar{G}[x_k + \tilde{G}x_{k-1}(k) + D_ku_k(k) + e_k(k)] \]

where \( \bar{G} \) is the finite horizon and

\[ \bar{G} = \left[ \begin{array}{c} \tilde{C} \\ \vdots \\ \tilde{C}A_k^{s-1} \\ \tilde{C}A_k^{s-2}B_k \\ \vdots \\ \tilde{C}A_k^{s-2}B_k \\ \tilde{C}A_k^{s-3}B_k \\ \vdots \\ \tilde{C}B_k \end{array} \right] \]
By iterating (17) and (18), the following is obtained
\[ y_f(k) = \begin{bmatrix} y_k \\ y_{k+1} \\ \vdots \\ y_{k+f-1} \end{bmatrix} \\
\[ z_{f-1}(k) = \begin{bmatrix} z_{k+1} \\ \vdots \\ z_{k+f-2} \end{bmatrix} \]

By substituting (25) into (20) gives
\[ y_f(k) = \Gamma_p z_p(k) + \Gamma P x_{k-p} + \delta_f z_{f-1}(k) + D u(k) + \varepsilon_f(k) \]

In subspace identification methods following assumptions are made: the eigenvalues of \( A \) lies inside the unit circle, \( A \) is observable, \( B, C, D \) is controllable and \( w \) is a stationary, zero mean white noise with covariance \( \mathbb{E}[w w^T] \)

Let consider an input vector \( x(k) \) and output vector \( y(k) \), the linear regression can be expressed as
\[ y(k) = 8x(k) + v(k) \]
which can be written in matrix form as
\[ \begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(N) \end{bmatrix} = \Theta \begin{bmatrix} x(1) \\ x(2) \\ \vdots \\ x(N) \end{bmatrix} + V \]
Where \( V \) is a noise vector.
By minimizing the following function
\[ J = ||Y - VX||_F^2 \]
Where \( ||.||_F \) is the Frobenius norm, the least square solution is given as
\[ \hat{\Theta} = YX^T (XX^T)^{-1} \]

The model prediction is then given as
\[ \hat{Y} = 6 \hat{X} = YX^T (XX^T)^{-1}X = Y \Pi \hat{X} \]
The least square residual is given as
\[ \hat{Y} = Y - \hat{Y} = Y(1 - \Pi \hat{X}) = Y(1 - \Pi) \]

Based form (10) and (11), an extended state space model can be formulated as
\[ Y_f = \Gamma P x_k + H_f u + G_f e \]
where \( f \) is the future horizon and
\[ H_f = \begin{bmatrix} C & CA & \ldots & CA^{f-2} & CA^{f-1} & 0 \\ 0 & \ldots & \ldots & \ldots & \ldots & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ C \delta & D & \ldots & \ldots & \ldots & \ldots \\ CB & D & \ldots & \ldots & \ldots & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ CA^{f-2}B & CA^{f-3}B & \ldots & D & \ldots & \ldots \\ I & \ldots & \ldots & \ldots & D & \ldots \end{bmatrix} \]

The Kalman state is estimated from past input and output data based on equation 25 as
\[ x_k = L_p z_p(k) + A_p x_{k-p} \]
where
\[ x_{k-p} = \begin{bmatrix} x_k-p \\ x_k-p+1 \\ \vdots \\ x_k+p-N-1 \end{bmatrix} \]

From equations 41 and 35
\[ Y_f = \Gamma_p z_p + H_f u + G_f e \]
\[ = H_p z_p + H_f u + G_f e \]

where \( H_p = \Gamma_p \Gamma_p \)
Under open-loop conditions, \( E_f \) is uncorrelated to \( U_f \) so
\[ \frac{1}{N} E_f E_f^T \rightarrow 0 \text{ as } N \rightarrow \infty \]

Furthermore, \( E_f \) is uncorrelated to \( Z_p \) from the Kalman filter theory. Therefore
\[ \frac{1}{N} E_f Z_p^T \rightarrow 0 \text{ as } N \rightarrow \infty \]

In N4SID we have to eliminate first eliminate \( U_f \) by post-multiplying \( \frac{1}{U_f} \) on (42).
\[ Y_f \Pi \frac{1}{U_f} = H_p z_p \Pi \frac{1}{U_f} + H_f u \Pi \frac{1}{U_f} + G_f e \Pi \frac{1}{U_f} \]

Then the noise term is removed by multiplying \( Z_p^T \) from the result of equation (44).
\[ Y_f \Pi \frac{1}{U_f} Z_p^T = H_p z_p \Pi \frac{1}{U_f} Z_p^T + G_f e \Pi \frac{1}{U_f} Z_p^T \]

and
\[ H_p = Y_f \Pi \frac{1}{U_f} Z_p^T \Pi \frac{1}{U_f} Z_p^T \]

N4SID performs Singular Value Decomposition (SVD) on \( H_p z_p \Pi \frac{1}{U_f} Z_p^T \)

where \( S_n \) contains the n largest singular value and choose \( f = U_n S_n^{1/2} \)

D. Output Error Model
The output error model is defined, as follows
\[ \xi(t) + f_1 \xi(t-1) + \cdots + f_n \xi(t-n) = b_1 u(t-1) + \cdots + b_m u(t-m) \]
where \( \xi(t) \) is an undisturbed output, \( y(t) \) is the output at \( t \), \( f_1, \ldots, f_n \) and \( b_1, \ldots, b_m \) are the unknown parameter that needed to be estimated.Rewriting (49) in compact form gives
\[ \hat{\xi}(t, \Theta) = \frac{B(q)}{F(q)} u(t) + e(t) \]

where
\[ F(q) = \sum_{k=0}^{n_b} f_k q^{-k} = 1 + f_1 q^{-1} + \cdots + f_n q^{-n_b} \]
\[ B(q) = \sum_{k=1}^{n_a} b_k q^{-k} = b_1 q^{-1} + \cdots + b_{n_a} q^{-n_a} \]

The output error model is estimated using regularization method. The parameters are estimated by minimizing the
Analysis of Model Predictive Control for Identified Fluid Catalytic Cracking Unit

mean square error given by a sum of systematic error (bias) and random error (variance).

\[ \text{MSE} = |\text{Bias}|^2 + VCE \]  

(53)

III. MODEL PREDICTIVE CONTROL

Model Predictive Control (MPC) is considered to be an advanced optimized control method that has been widely used in process industries over the last two decades. The strategy that Model Predictive Controllers (MPC) uses to calculate the control actions characterized as : At the kth sampling instant, the values of the control action, \( u \), for the next \('M'\) sampling instants. \( \{u(k), u(k+1), ..., u(k+M-1)\} \) are calculated. They are calculated by minimizing the difference between the predicted outputs and the reference trajectories over the next \('P'\) sampling instants while satisfying the constraints. In MPC, the control horizon \('M'\) and the predicted horizon \('P'\) are the tuning parameters. After calculating the control moves for \( M \) sampling instants. The controller will implement the first control move \( u(k) \). At the next sampling instant, \( k+1 \), the control moves are recalculated for the next \( M \) sampling instants, \([k+1 to k+M]\), and the first control move \( u(k+1) \) is implemented. These steps are repeated at each sampling instant.

\[ J(k) = \sum_{i=0}^{M} \| y(k+i|k) - r(k+i) \|^2_{\xi(k)} + \sum_{i=0}^{P} \| \Delta u(k+i|k) \|^2_{\delta(k)} \]  

(54)

where represents the model predicted output, is the reference setpoint, \( \Delta \) is the change in manipulated input from one sample time to the next, \( \xi \) is a weight for the changes in the manipulated input, \( \delta \) is a weight for the changes in the predicted output, and the subscripts indicate the sample time. \( P \) is a prediction horizon and \( M \) is a control horizon.

The controller minimizes the objective function given in (54) to obtain the control moves. The objective function is minimized at each sampling instant using quadratic equation solvers.

There are many different quadratic solvers available such as; interior point, active set, augmented Lagrangian, conjugate gradient, KIWIK algorithm, etc. The details of these algorithms are not considered in this paper. In general the quadratic problem can be formulated as

\[ \min_{u} \left( r^Tu + \frac{1}{2}u^THu \right) \]  

Subject to \( Ax \leq b \)

where \( u \) is the optimal solution which gives the minimum, \( H \) is the positive definite Hessian matrix, \( A \) is a matrix of linear constraint coefficients, and \( b \) and \( f \) are the vectors. The \( H \) and \( A \) matrices are constants matrices which are calculated during the initialization of the controller and \( b \) and \( f \) vectors are calculated at the beginning of each sampling instant.

The quadratic solver tries to find the minimum of the function given in (55) which satisfies the constraints.

In MPC calculation, it is assumed that all the states of the process are measurable but that’s not true. In most of the application it is impossible to measure all the states and the estimation of the states are required. Observer, Kalman filter and Extended Kalman filter can be used for the state estimation.

IV. FLUID CATALYTIC CRACKING UNIT

The fluid catalytic cracking unit (FCCU) is complex interactive process in in petroleum refining industries. It takes chains of hydrocarbons and breaks them into smaller ones which allows refineries to utilize their crude oil resources more efficiently. It uses an extremely hot catalyst to crack the hydrocarbons into smaller ones. FCCU processes present challenging control problem because it has very complex kinetics of both cracking and coke burning reactions and it has strong interaction between the reactor and regenerator. A schematic of FCCU is shown in fig. 2.

The FCCU contains two main parts: the reactor and the regenerator. In reactor, the reaction between the mixture of hydrocarbon vapors and catalyst takes place. This reaction breaks the long molecules of hydrocarbons into smaller one which leaves from the top of the reactor. Steam is supplied to remove hydrocarbons from the catalyst. The cracking reaction...
produces carbon materials and un-cracked organic materials known as coke that reduces the catalyst activity. The catalyst is taken into regenerator where it is regenerated by burning off the deposited coke with air. The regenerated catalyst is then taken to the reactor to repeat the cycle. The combustion of the coke in the regenerator produces a heat absorbed by the regenerated catalyst. This absorbed heat provides the energy for the vaporization and cracking reactions that take place in the catalyst riser.

The FCCU considered here is developed from [Skogsted, 1993]. It has two manipulated variable and two controlled variables. The manipulated variables are: which is the flow rate of regenerated catalyst and which is the flow rate of air to regenerator. The controlled variable are: which is the riser outlet temperature and which is the regenerator temperature.

V. RESULTS AND DISCUSSIONS

PRBS signals applied to the model of FCCU developed in Simulink. The developed model is based on the work of Skogested, 1993. This model has two manipulated variables and two controlled variables. The input-output data is then used to estimate the different models of FCCU.

The estimated impulse model is shown in fig. 3 and fig. 4. From those figures, it is clear that the impulse model was unable to estimate the correct model. Impulse model produced a fit of 4.214% for $T_{RO}$ and -555.1% fit for $T_{RG}$.

ARX model is shown in fig. 5 and fig. 6. This model produced a fit of 86.8% for $T_{RO}$ and 85.5% fit for $T_{RG}$.

State Space Model is shown in fig. 7 and fig. 8. This model produced a fit of 87.83% for $T_{RO}$ and 84.68% fit for $T_{RG}$.
Analysis of Model Predictive Control for Identified Fluid Catalytic Cracking Unit

Output error model is shown in fig. 9 and fig. 10. This model produced a fit of 84.65% for $T_{r\theta}$ and 80.8% fit for $T_{r\theta}$.

Since the estimated impulse model was unable to produce a correct model, it was not used in MPC design. In MPC ARX model, state space Model and output error model are used. The results of MPC design is shown in fig. 11 and fig. 12 and fig. 13. The results show that the state space model estimated using N4SID produce better response than ARX model and ARX model produces better response than output error model.
VI. CONCLUSION

In this paper, system identification techniques are used to estimate the dynamical models of FCCU. Impulse model, ARX model, state space model and output error model are estimated and compared. The result of the simulation shows that an Impulse model was unable to give a correct fit, however; ARX model, state space model and output error model produced a good fit. The estimated models then used in MPC design to control the riser outlet temperature and regenerator temperature of FCCU unit. MPC design is simulated in MATLAB. The result of the simulation shows that the estimated state space model using N4SID algorithm gives better result than ARX model and ARX model gives better result than output error model.

REFERENCES