Intuitionistic Fuzzy Contra Open Mappings in Intuitionistic Fuzzy Topological Space

A. MANIMARAN, K. ARUN PRAKASH

***Abstract*— In this paper, we introduce intuitionistic fuzzy contra open mappings. We study some of their properties and obtain their characterizations. Relationship between this new class and other classes of functions are established in intuitionistic fuzzy topological spaces.**

***Index Terms*— Intuitionistic fuzzy topological spaces, Intuitionistic fuzzy open sets and Intuitionistic fuzzy closed sets, Intuitionistic fuzzy contra open mappings.**

***AMS Classification*: 54A40, 54D20.**

I. INTRODUCTION

After the introduction of fuzzy sets by Zadeh [6], there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [1] is one among them. Using the notion of intuitionistic fuzzy sets, Coker [2] introduced the notion of intuitionistic fuzzy topological spaces. In this paper, we introduce the concept of intuitionistic fuzzy contra open mapping. We have also studied some of the properties of intuitionistic fuzzy contra open mapping and obtain their characterizations.

II. PRELIMINARIES

Before entering to our work, we recall the following notations, definitions and results of intuitionistic fuzzy sets. Throughout this paper, (X,τ), (Y,σ) and (Z,η) are always means intuitionistic fuzzy topological spaces in which no separation axioms are assumed unless otherwise mentioned.

Definition 2.1: [1] Let X be a nonempty set. An intuitionistic fuzzy set (IFS) A in X is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the mappings $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of the membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A respectively, $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$.

Obviously, every fuzzy set A on a non-empty set X is an *IFS* having the form

$$A = \{\langle x, \mu_A(x), 1-\mu_A(x)\rangle : x \in X\}.$$

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Definition 2.2:[1] Let X be a non-empty set and let the *IFS*'s A and B in the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$. Let $\{A_j : j \in J\}$ be an arbitrary family of *IFS*'s in (X,τ). Then,

i) if and only if [μ, and ı];

ii) $\bar{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle : :$;

iii) $\cap A_j = \{\langle x, \wedge \mu_{A_j}(x), \vee \nu_{A_j}(x)\rangle : x \in X$;

iv) $\cup A_j = \{\langle x, \vee \mu_{A_j}(x), \wedge \nu_{A_j}(x)\rangle : x \in X$;

v) $\underset{\sim}{1} = \{<x,1,0>; x \in X\}$ and $\underset{\sim}{0} = \{<x,0,1>; x \in X\}$;

vi) $\bar{\bar{A}} = A$, $\bar{\underset{\sim}{0}} = \underset{\sim}{1}$ and $\bar{\underset{\sim}{1}} = \underset{\sim}{0}$

Definition 2.3: [2] Let X and Y are two non-empty sets and $f:(X,\tau)\to(Y,\sigma)$ be a function. If $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$ is an *IFS* in Y, then the pre-image of B under f is denoted and defined by

$$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x))\rangle : x \in X\}$$

Since μ_B, ν_B are fuzzy sets, we explain that $f^{-1}(\mu_B)(x) = \mu_B(f(x))$.

Definition 2.4: An intuitionistic fuzzy topology [2] (*IFT*, for short) on a non-empty set X is a family τ of *IFS*'s in X satisfying the following axioms:

i) $\underset{\sim}{0}, \underset{\sim}{1} \in \tau$.

ii) $A_1 \cap A_2 \in \tau$ for some $A_1, A_2 \in \tau$.

iii) $\bigcup A_j \in \tau$ for any $\{A_j : j \in J\} \in \tau$.

In this case, the ordered pair (X,τ) is called an intuitionistic fuzzy topological space (*IFTS*, for short) and each *IFS* in τ is known as an intuitionistic fuzzy open set (*IFOS*, for short) in X. The complement of an intuitionistic fuzzy open set is called an intuitionistic fuzzy closed set (*IFCS*, for short).

Definition 2.5: [2] Let (X,τ) be an *IFTS* and let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ be an *IFS* in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure of A is defined by

$$\text{int}(A) = \bigcup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \}$$
$$\text{cl}(A) = \bigcap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}.$$

Remark 2.6:[2] For any *IFS* A in (X,τ) ,we have $\mathrm{cl}(\overline{A}) = \overline{\mathrm{int}(A)}$, $\mathrm{int}(\overline{A}) = \overline{\mathrm{cl}(A)}$.

Definition 2.7:[4] Let A be an IFS in an IFTS (X, τ). Then the intuitionistic fuzzy semi - interior and intuitionistic fuzzy semi - closure of A are defined by

$$\mathrm{sint}(A) = \cup \{G \mid G \text{ is an IFSOS in X and } G \subseteq A\},$$
$$\mathrm{scl}(A) = \cap \{K \mid K \text{ is an IFSCS in X and } A \subseteq K\}$$

respectively.

Obviously scl(A) is the smallest IFSCS which contains A and sint(A) is the largest IFSOS which is contained in A.

Definition 2.8: Let A be an IFS in an IFTS (X, τ). Then A is called an intuitionistic fuzzy semiopen set (IFSOS) [3] of X if $A \subseteq \mathrm{cl}(\mathrm{int}(A))$.

The complement of intuitionistic fuzzy semiopen set is called the intuitionistic fuzzy semiclosed set.

Definition 2.9: An *IFS* A in an *IFTS* (X,τ) is called an intuitionistic fuzzy regular open set [3] (*IFROS*) if int (cl(A)) = A. The complement of intuitionistic fuzzy regular open set is called intuitionistic fuzzy regular closed (*IFRCS*, for short). The family of all *IFROS* (*IFRCS*) of (X,τ) is denoted by *IFROS* (X) (*IFRCS* (X)).

Definition 2.11: [5] An intuitionistic fuzzy point (*IFP* for short), written $p_{(\alpha,\beta)}$ is defined to be an *IFS* of X given by

$$p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha,\beta) & \text{if } x = p \\ (0,1) & \text{otherwise} \end{cases}$$

Definition 2.12:[5] An IFP $p_{(\alpha,\beta)}$ in X is said to be quasi-coincident with an IFS $A = \langle x, \mu_A, \nu_A \rangle$, denoted by $p_{(\alpha,\beta)}\ q\ A$, if and only if $\alpha > \nu_A(p)$ or $\beta > \mu_A(p)$.

Definition 2.13: [3] Let A be an IFS in an IFTS (X, τ). Then A is called an intuitionistic fuzzy preclosed set (IFPCS) if $\mathrm{int}(\mathrm{cl}(A)) \subseteq A$.

III. INTUITIONISTIC FUZZY CONTRA OPEN MAPPING

Definition 3.1: A mapping $f:(X,\tau) \to (Y,\sigma)$ is said to be an intuitionistic fuzzy contra open mapping if $f(A)$ is an IFCS in Y for every IFOS A in X.

Example 3.2: : Let $X = \{a, b\}$, $Y = \{u, v\}$, $\tau = \{0_\sim, A, 1_\sim\}$ and $\sigma = \{0_\sim, C, 1_\sim\}$. Let $A = \left\{\left\langle x, \left(\frac{a}{0.5}, \frac{b}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.5}\right)\right\rangle, x \in X\right\}$ and $B = \left\{\left\langle y, \left(\frac{a}{0.4}, \frac{b}{0.5}\right), \left(\frac{a}{0.5}, \frac{b}{0.4}\right)\right\rangle, y \in Y\right\}$. Then, $\tau = \{0_\sim, A, 1_\sim\}$ and $\sigma = \{0_\sim, C, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Clearly, $f(A) = \bar{B}$, which is an IFCS in Y. Therefore, f is an intuitionistic fuzzy contra open mapping.

Theorem 3.3: Let $f:(X,\tau) \to (Y,\sigma)$ be a surjective mapping. Then the following statements are equivalent:

i) f is an intuitionistic fuzzy contra open mapping;

ii) $f(A)$ is an IFOS in Y for every IFCS A in X.

Proof: (i) $\Rightarrow$ (ii) Let A be an IFCS in X. Then A^c is an IFOS in X . By hypothesis $f(A^c)$ is an IFCS in Y . Since $f(A^c) = (f(A))^c$, $f(A)$ is an IFOS in Y.

(ii) $\Rightarrow$ (i) Let A be an IFOS in X. Then A^c is an IFCS in X. By Hypothesis $f(A^c) = (f(A))^c$ is an IFOS in Y. Hence $f(A)$ is an IFCS in Y. Thus f is an intuitionistic fuzzy contra open mapping.

Theorem 3.4: For a bijective mapping $f:(X,\tau) \to (Y,\sigma)$ the following statements are equivalent:

i) f is an intuitionistic fuzzy contra open mapping;

ii) for every IFCS A in X, $f(A)$ is an IFOS in Y;

iii) for every IFCS A in X and for any IFP $p_{(\alpha,\beta)} \in Y$, if $f^{-1}(p_{(\alpha,\beta)})\ q\ A$, then $p_{(\alpha,\beta)}\ q\ int(f(A))$;

iv) For any IFCS A in X and for any $p_{(\alpha,\beta)} \in Y$, if $f^{-1}(p_{(\alpha,\beta)})\ q\ A$, then there exists and IFOS B such that $p_{(\alpha,\beta)}\ q\ B$ and $f^{-1}(B) \subseteq A$.

Proof: The proof of (i) $\Rightarrow$ (ii) and (ii) $\Rightarrow$ (i) follows from Theorem 3.3.

(ii) $\Rightarrow$ (iii) Let $A \subseteq X$ be an *IFCS* and let $p_{(\alpha,\beta)} \in Y$, Assume that $f^{-1}(p_{(\alpha,\beta)})\ q\ A$. Then $p_{(\alpha,\beta)}\ q\ f(A)$. By hypothesis $f(A)$ is an IFOS in Y. then $f(A) = int(f(A))$. Hence $p_{(\alpha,\beta)}\ q\ int(f(A))$.

(iii) $\Rightarrow$ (ii) Let A be an IFCS in X and $p_{(\alpha,\beta)} \in Y$, Assume that $f^{-1}(p_{(\alpha,\beta)})\ q\ A$. Then $p_{(\alpha,\beta)}\ q\ f(A)$. By hypothesis, $p_{(\alpha,\beta)}\ q\ int(f(A))$. Hence $f(A) = int(f(A))$ which implies $f(A)$ is an IFOS in Y.

(iii) $\Rightarrow$ (iv) Let A be an IFCS in X and $p_{(\alpha,\beta)} \in Y$, Assume that $f^{-1}(p_{(\alpha,\beta)})\ q\ A$. Then $p_{(\alpha,\beta)}\ q\ f(A)$. By hypothesis, $p_{(\alpha,\beta)}\ q\ int(f(A))$. Thus $f(A)$ is an IFOS in Y . Put $f(A) = B$, then $p_{(\alpha,\beta)}\ q\ B$ and $f^{-1}(B) = f^{-1}(f(A)) \subseteq A$.

(iv) $\Rightarrow$ (iii) Let A be an IFCS and $p_{(\alpha,\beta)} \in Y$ such that $f^{-1}(p_{(\alpha,\beta)})\ q\ A$, By hypothesis there exists an IFOS B in Y such that $p_{(\alpha,\beta)}\ q\ B$ and $f^{-1}(B) \subseteq A$. Let $B = f(A)$. Then $p_{(\alpha,\beta)}\ q\ f(A)$ and since B is an IFOS, $f(A)$ is an IFOS which implies $p_{(\alpha,\beta)}\ q\ int(f(A))$.

Theorem 3.5: Let $f:(X,\tau) \to (Y,\sigma)$ be a bijective mapping and IFPC(Y) = IFC(Y). Suppose that one of the following properties hold:

(i) $f(cl(B)) \subseteq int(cl($ for each IFS in ;

(ii) $cl(int(f(B))) \subseteq f(ir$ for each IFS in ;

(iii) $f^{-1}\left(cl\left(int(A)\right)\right) \subseteq int(f^{-1}$ for each IFS in ;

(iv) $f^{-1}\left(cl(A)\right) \subseteq int(f$ for each IFOS in ;

then, f is an intuitionistic fuzzy contra open mapping.

Proof: (i)$\Rightarrow$(ii) can be proved by taking the complement in (i)

(ii)$\Rightarrow$(iii) Let A be an IFS in Y. Put $B = f^{-1}(A)$ in X, this implies $A \subseteq f(B)$ in Y. Now $cl\left(int(A)\right) \subseteq cl\left(int\left(f(B)\right)\right) \subseteq f\left(int(B)\right)$ by (ii). Therefore, $f^{-1}\left(cl\left(int(A)\right)\right) \subseteq f^{-1}\left(f\left(int(B)\right)\right) = int(B) = int\left(f^{-1}(A)\right)$.

(iii) $\Rightarrow$(iv) Let A be an IFOS in Y, then $A = int(A)$. By the hypothesis, $f^{-1}\left(cl\left(int(A)\right)\right) \subseteq int\left(f^{-1}(A)\right)$. Therefore, $f^{-1}\left(cl(A)\right) \subseteq int\left(f^{-1}(A)\right)$.

Suppose that (iv) holds. Let A be an IFOS in X. then $int\left(f(A)\right)$ is an IFOS in Y. Hence by hypothesis, $f^{-1}\left(cl\left(int\left(f(A)\right)\right)\right) \subseteq int\left(f^{-1}\left(int\left(f(A)\right)\right)\right) \subseteq int\left(f^{-1}\left(f(A)\right)\right) \subseteq int(A) \subseteq A$. Therefore, $cl\left(int\left(f(A)\right)\right) \subseteq f(A)$. Now, $f(A)$ is an IFPCS in Y. By hypothesis $f(A)$ is an IFCS in Y. Hence, f is an intuitionistic fuzzy contra open mapping.

Theorem 3.6: Let $f:(X,\tau) \to (Y,\sigma)$ be a surjective mapping. Suppose that one of the following properties hold:

(i) $f^{-1}\left(cl(A)\right) \subseteq int(f$ for each IFS in ;

(ii) $\left(cl\left(f(B)\right)\right) \subseteq f($ for each IFS in ;

(iii) $f\left(cl(B)\right) \subseteq i$ for each IFS in ;

Then f is an intuitionistic fuzzy contra open mapping.

Proof: (i) $\Rightarrow$(ii) Let us consider B be an IFS in X. Then $f(B)$ is an IFS in Y. By hypothesis $f^{-1}\left(cl\left(f(B)\right)\right) \subseteq int\left(f^{-1}\left(f(B)\right)\right) \subseteq int(B)$. Now, $cl\left(f(B)\right) \subseteq f\left(int(B)\right)$.

(ii)$\Rightarrow$(iii) can be proved by taking the complement in (ii).

Suppose that (iii) holds. Let A be an IFCS in X. then $cl(A) = A$ and $f(A)$ is an IFS in Y. Now, $f(A) = f\left(cl(A)\right) \subseteq int\left(f(A)\right) \subseteq f(A)$, by hypothesis. This implies $f(A)$ is an IFOS in Y. By Theorem 3.3, f is an intuitionistic fuzzy contra open mapping.

Theorem 3.7: Let $f:(X,\tau) \to (Y,\sigma)$ be a bijective mapping. Then f is an intuitionistic fuzzy contra open mapping if $cl\left(f^{-1}(A)\right) \subseteq f^{-1}\left(int(A)\right)$ for every IFS A in Y.

Proof: Let A be an *IFCS* in X. Then $cl(A) = A$ and $f(A)$ is an IFS in Y. By hypothesis $cl\left(f^{-1}\left(f(A)\right)\right) \subseteq f^{-1}\left(int\left(f(A)\right)\right)$. Since f is one to one $f^{-1}\left(f(A)\right) = A$. Therefore, $A = cl(A) = cl\left(f^{-1}\left(f(A)\right)\right) \subseteq f^{-1}\left(int\left(f(A)\right)\right)$. Now, $f(A) \subseteq f\left(f^{-1}\left(int\left(f(A)\right)\right)\right) = int\left(f(A)\right) \subseteq f(A)$. Hence, $f(A)$ is an IFOS in Y. By Theorem 3.5, f is an intuitionistic fuzzy contra open mapping.

Theorem 3.8: Let $f:(X,\tau) \to (Y,\sigma)$ be a surjective intuitionistic fuzzy contra open mapping, then the following conditions hold:

(i) $cl\left(f(B)\right) \subseteq f\left(int\left(cl(B\right.\right.$ for every IFOS in ;

(ii) $f\left(cl\left(int(B)\right)\right) \subseteq int\left(f(B\right.$ for every IFCS in ;

Proof: (i) Let B be an IFOS in X. Clearly, $int(B) = B$. By hypothesis $f(B)$ is an IFCS in Y. This implies that $cl\left(f(B)\right) = f(B) \subseteq f\left(int(B)\right) \subseteq f\left(int\left(cl(B)\right)\right)$.

(ii) can be proved by taking the complement in (i).

Theorem 3.9: Let $f:(X,\tau) \to (Y,\sigma)$ be a one-to-one mapping, then the following statements are equivalent:

i) f is an intuitionistic fuzzy contra open mapping;

ii) for each IFP $p_{(\alpha,\beta)} \in Y$ and for each IFCS B containing $f^{-1}\left(p_{(\alpha,\beta)}\right)$,there exists an IFOS A contained in Y and $p_{(\alpha,\beta)} \in A$ such that $A \subseteq f(B)$;

iii) for each IFP $p_{(\alpha,\beta)} \in Y$ and for each IFCS B containing $f^{-1}\left(p_{(\alpha,\beta)}\right)$,there exists an IFOS A contained in Y and $p_{(\alpha,\beta)} \in A$ such that $f^{-1}(A) \subseteq B$.

Proof: (i) $\Rightarrow$(ii) Let B be an IFCS in X and $p_{(\alpha,\beta)} \in Y$ such that $f^{-1}\left(p_{(\alpha,\beta)}\right) \in B$. Then, $p_{(\alpha,\beta)} \in f(B)$. By the hypothesis, $f(B)$ is an IFOS in Y. Let $A = f(B) = int\left(f(B)\right) \subseteq f(B)$.

(ii) $\Rightarrow$(i) Let B be an IFCS in X and $p_{(\alpha,\beta)} \in Y$ such that $f^{-1}\left(p_{(\alpha,\beta)}\right) \in B$. Then $p_{(\alpha,\beta)} \in f(B)$. By the hypothesis, $f(B)$ is an IFOS in Y. Let $A \subseteq f(B)$. This implies $f^{-1}(A) \subseteq f^{-1}\left(f(A)\right) \subseteq B$.

(iii) $\Rightarrow$(i) Let B be an IFCS in X and $p_{(\alpha,\beta)} \in Y$. Let $f^{-1}\left(p_{(\alpha,\beta)}\right) \in B$. Then by the hypothesis, there exists an IFOS A in Y such that $p_{(\alpha,\beta)} \in A$ and $f^{-1}(A) \subseteq B$. This implies $p_{(\alpha,\beta)} \in A \subseteq f\left(f^{-1}(A)\right) \subseteq B$. That is $p_{(\alpha,\beta)} \in f(B)$. Since A is an IFOS, $A = int(A) \subseteq int\left(f(B)\right)$. Therefore, $p_{(\alpha,\beta)} \in A \subseteq int\left(f(B)\right)$. But $f(B) = \bigcup_{p_{(\alpha,\beta)} \in f(B)} p_{(\alpha,\beta)} \subseteq int\left(f(B)\right) \subseteq f(B)$. Hence $f(B)$ is an IFOS in Y. hence f is an intuitionistic fuzzy contra open mapping.

Theorem 3.10: A mapping $f:(X,\tau) \to (Y,\sigma)$ is an intuitionistic fuzzy contra open mapping if $f\left(scl(B)\right) \subseteq int\left(f(B)\right)$ for every IFS B in X.

Proof: Let B be an *IFCS* in X. Then $cl(B) = B$. Since every IFCS is an IFSCS, $scl(B) = B$. By hypothesis $f(B) = f(scl(B)) \subseteq int(f(B))$. This implies that $f(B)$ is an IFOS in Y. Hence f is an intuitionistic fuzzy contra open mapping.

Theorem 3.11: If IFSO(Y) = IFO(Y), then a mapping $f:(X,\tau) \to (Y,\sigma)$ is an intuitionistic fuzzy contra open mapping if and only if $f(scl(B)) \subseteq sint\left(f(cl(B))\right)$ for every IFS B in X.

Proof:
Necessity: Let B be an *IFOS* in X. Then $cl(B)$ is an IFCS in X. By hypothesis, $f(cl(B))$ is an IFOS in Y. Since every IFOS is an IFSOS, $f(cl(B))$ is an IFSOS in Y. Therefore, $f(scl(B)) \subseteq f(cl(B)) = sint\left(f(cl(B))\right)$.

Sufficiency: Let B be an *IFCS* in X. Then $cl(B) = B$. By hypothesis $f(scl(B)) \subseteq sint\left(f(cl(B))\right) = sint(f(B))$. Then $f(B) \subseteq f(scl(B)) \subseteq sint(f(B)) \subseteq f(B)$. Therefore, $f(B)$ is an IFSOS in Y. By hypothesis, $f(B)$ is an IFOS in Y. Hence, f is an intuitionistic fuzzy contra open mapping.

Theorem 3.12: A mapping $f:(X,\tau) \to (Y,\sigma)$ is an intuitionistic fuzzy contra open mapping if and only if $f(scl(B)) \subseteq int\left(f(cl(B))\right)$for every IFS B in X.

Proof:
Necessity: Let B be an IFS in X. Then $cl(B)$ is an IFCS in X. By hypothesis, $f(cl(B))$ is an IFOS in Y. Then, $f(scl(B)) \subseteq f(cl(B)) = int\left(f(cl(B))\right)$.

Sufficiency: Let B be an IFCS in X. Then $cl(B) = B$. By hypothesis, $f(scl(B)) \subseteq int\left(f(cl(B))\right) = int\, f(B)$. Now, $f(B) \subseteq f(scl(B)) \subseteq int(f(B)) \subseteq f(B)$. Thus, f is an intuitionistic fuzzy contra open mapping.

Theorem 3.13: A mapping $f:(X,\tau) \to (Y,\sigma)$ is an intuitionistic fuzzy contra open mapping, then $f(cl(B)) \subseteq sint\left(f(cl(B))\right)$ for every IFS B in X.

Proof: Let B be an IFOS in X. Then $cl(B)$ is an IFCS in X. By hypothesis, $f(cl(B))$ is an IFOS in Y. Then, $f(cl(B)) = int\left(f(cl(B))\right) \subseteq sint\left(f(cl(B))\right)$..

Theorem 3.14: An intuitionistic fuzzy open mapping is an intuitionistic fuzzy contra open mapping if IFOS (Y) = IFCS(Y).

Proof: Let A be an IFOS in X. By hypothesis $f(A)$ is an IFOS and hence $f(A)$ is an IFCS in Y. Hence f is an intuitionistic fuzzy contra open mapping.

Definition 3.15: A mapping $f:(X,\tau) \to (Y,\sigma)$ is said to be an intuitionistic fuzzy almost contra open mapping, if the image of each IFROS in X is an IFCS in Y.

Theorem 3.16: Every intuitionistic fuzzy contra open mapping is an intuitionistic fuzzy almost contra open mapping.

Proof: Let $f:(X,\tau) \to (Y,\sigma)$ be an intuitionistic fuzzy contra open mapping and A be an IFROS in X. Since every IFROS is an IFOS, A is an IFOS in X. By hypothesis $f(A)$ is an IFCS in Y. Hence f is an intuitionistic fuzzy almost contra open mapping.

The converse of the above theorem is not true in general as seen from the following example.

Example 3.17: Let $X = \{a,b\}$, $Y = \{u,v\}$. Let $A = \left\{\langle x, \left(\frac{a}{0.5},\frac{b}{0.4}\right), \left(\frac{a}{0.5},\frac{b}{0.6}\right)\rangle, x \in X\right\}$, $B = \left\{\langle x, \left(\frac{a}{0.8},\frac{b}{0.7}\right), \left(\frac{a}{0.2},\frac{b}{0.3}\right)\rangle, x \in X\right\}$ and $C = \left\{\langle y, \left(\frac{a}{0.5},\frac{b}{0.6}\right), \left(\frac{a}{0.5},\frac{b}{0.4}\right)\rangle, y \in Y\right\}$. Then $\tau = \{0_\sim, A, B, 1_\sim\}$ and $\sigma = \{0_\sim, C, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Clearly A is an IFROS in X and $f(A)$ is an IFCS in Y. Therefore, f is an intuitionistic fuzzy almost contra open mapping. Since B is an IFOS in X, but $f(B)$ is not an IFCS in Y, f is not an intuitionistic fuzzy contra open mapping.

Theorem 3.18: Let $f:X \to Y$ and $g:Y \to Z$ be any two mappings, then $g \circ f:X \to Z$ and the following conditions hold:

i)If f is an intuitionistic fuzzy open mapping and g is an intuitionistic fuzzy contra open mapping, then $g \circ f:X \to Z$ is an intuitionistic fuzzy contra open mapping.

ii) If f is an intuitionistic fuzzy almost contra open mapping and g is an intuitionistic fuzzy contra open mapping, then $g \circ f$ is an intuitionistic fuzzy almost open mapping.

iii) If f and g are intuitionistic fuzzy contra open mappings, then $g \circ f$ is an intuitionistic fuzzy open mapping.

Proof: (i) Let A be an IFOS in X. By the hypothesis $f(A)$ is an IFOS in Y. Since g is an intuitionistic fuzzy contra open mapping $g(f(A)) = (g \circ f)(A)$ is an IFCS in Z. Hence, $g \circ f$ is an intuitionistic fuzzy contra open mapping.
(ii) Let B be an IFROS in X. By the hypothesis $f(B)$ is an IFCS in Y. Since g is an intuitionistic fuzzy contra open mapping $g(f(B)) = (g \circ f)(B)$ is an IFOS in Z. Hence, f is an intuitionistic fuzzy almost open mapping.
(iii) Let B be an IFOS in X. By the hypothesis $f(B)$ is an IFCS in Y. Since g is an intuitionistic fuzzy contra open mapping, then by Theorem 3.3, $g(f(B)) = (g \circ f)(B)$ is an IFOS in Z. Hence, f is an intuitionistic fuzzy open mapping.

Definition 3.19: A mapping $f:X \to Y$ is said to be an intuitionistic fuzzy contra irresolute open mapping, if $f(A)$ is an IFSCS in Y for every IFSOS A in X.

Example 3.20: Let $X = \{a,b\}$, $Y = \{u,v\}$. Let $A = \left\{\langle x, \left(\frac{a}{0.3},\frac{b}{0.4}\right), \left(\frac{a}{0.6},\frac{b}{0.6}\right)\rangle\right\}$ and $B = \left\{\langle x, \left(\frac{a}{0.2},\frac{b}{0.3}\right), \left(\frac{a}{0.7},\frac{b}{0.6}\right)\rangle\right\}$. Then, $\tau = \{0_\sim, A, 1_\sim\}$ and $\sigma = \{0_\sim, B, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$.

$IFSOS(X) = \{0_\sim, 1_\sim, G_{a,b}^{(l_1,m_1),(l_2,m_2)}$; $l_1, m_1 \in [0.3, 0.6]$, $l_2, m_2 \in [0.4, 0.6]$ $l_i + m_i \leq 1$, $i = 1,2\}$

$IFSCS(Y) = \{0_\sim, 1_\sim, H_{u,v}^{(a_1,b_1),(a_2,b_2)}$; $a_1, b_1 \in [0.2, 0.7]$, $a_2, b_2 \in [0.3, 0.6]$ $a_i + b_i \leq 1$, $i = 1,2\}$

Clearly, every IFSOS in X is an IFSCS in Y. Therefore, f is an intuitionistic fuzzy contra irresolute open mapping.

Theorem 3.21: Let $f: X \to Y$ be a surjective mapping, then the following statements are equivalent:

i) f is an intuitionistic fuzzy contra irresolute open mapping
ii) $f(A)$ is an IFSOS in Y for every IFSCS A in X.

Proof: $(i) \Rightarrow (ii)$ Let A be an IFSCS in X. Then, A^c is an IFSOS and by the hypothesis $f(A^c) = (f(A))^c$ is an IFSCS in Y. Hence, $f(A)$ is an IFSOS in Y.
$(ii) \Rightarrow (i)$ Let A be an IFSOS in X. By the hypothesis $(f(A))^c = f(A^c)$ is an IFSOS in Y. Then, $f(A)$ is an IFSCS in Y. Hence, f is an intuitionistic fuzzy contra irresolute open mapping.

Theorem 3.22: Every intuitionistic fuzzy contra irresolute open mapping is an intuitionistic fuzzy contra open mapping, if IFSC(Y) = IFC(Y)

Proof: Let $f: X \to Y$ be an intuitionistic fuzzy contra irresolute open mapping and A be an *IFOS* in *X*. Since every IFOS is an IFSOS, A is an IFSOS in X. By hypothesis $f(A)$ is an IFSCS in Y. Since IFSCS(Y) = IFCS(Y), $f(A)$ is an IFCS in Y. Hence f is an intuitionistic fuzzy contra open mapping.

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