

# Prediction of Flow Resistance in Gravel Bed River

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**Abstract**— This paper provides a framework for predicting of Darcy Weisbach resistance and understanding modeling of flow resistance to Gravel Bed Rivers on global scale shows the only pioneering contribution in India perhaps is made by Garde<sup>1</sup> (2001). The literature survey had been updated till date. A modified flow resistance equation for modeling of gravel bed has been investigated after comparison of different sets of formulae as reported by investigators till date. Toniolo et al.<sup>2</sup> (2010) (published in American Society for Civil Engineers, JHE) calculated Mannings 'n' by traditional equation developed in 19<sup>th</sup> century for observed hydraulic parameters for gravel bed Anaktuvuk River in USA; which has been modified for this prototype gravel bed water body by investigation of a set of formulae on flow resistance found in literature till date. Finally a simple formula is formulated empirically to find the more accurate flow resistance in Gravel Bed Rivers.

**Index Terms**— Stereophotogrammetry, Digital elevation model, Semivariogram, Darcy Weisbach resistance coefficient, Probability Distribution Function, Correlation

## I. INTRODUCTION

Gravel-Bed Rivers are those rivers, which flow over very coarse material in the range of gravel cobbles and boulders. Mountain Rivers are characterized by turbulent flow, coarse bed material, steep channel slope and shallow depths. Available hydraulic methods and numerical models, developed for lower-gradient Rivers, are generally unverified and sometimes inadequate for studying rivers in mountainous areas. For example, Jarrett<sup>3</sup> (1991) stated that existing methods and models underestimate flow resistance. Rouse<sup>4</sup> (1965) classified flow resistance into four components:

- (1) Surface on skin friction,
- (2) Form resistance or drag,
- (3) Wave resistance from free surface distortion, and
- (4) Resistance associated with local acceleration or flow unsteadiness.

By using the Darcy Weisbach resistance coefficient  $f$ , the resistance equations were expressed as the following dimensionless symbolic function:

$$f = F(R, k, \eta, N, Fr, U) \quad (1)$$

in which  $R$ =Reynolds number;  $k$ =relative roughness, usually expressed as  $k_s/R$ , where  $k_s$  is the equivalent wall surface roughness in m and  $R$  is hydraulic radius of the flow in m;  $\eta$ =cross-sectional geometric shape;  $N$ =non-uniformity of the channel in both profile and plan;  $Fr$ =Froude number;  $U$ =degree of flow unsteadiness; and  $F$  represents a function.

However, the objective of the present paper is to examine the flow resistance in gravel Bed Rivers by the various precursors in this field and make predictions for fostering the better understanding.

## II. DATA AND INVESTIGATORS

Williams<sup>5</sup> (1978) conducted studies on the definition of bankful discharge of 23 small and medium size streams in the USA. Leopold and Wolman<sup>6</sup> (1957) collected the data from USA Rivers only 15 runs of data have been used in the present studies. The United State Bureau of reclamation investigated in San Luis Valley of USA for 14 canals of Gravel Bed.

The data reported by Kellerhals<sup>7</sup> (1967) was from the various rivers south central British Columbia, Quesnel, Caribo, Taseko, Chilko and Thomson. In total 20 number of observation been included in the present study. Bray<sup>8</sup> (1979) presented basic data from 67 natural gravel bed rivers reaches Alberta Canada. The data of Samide<sup>9</sup> (1971) were collected at irregularly shaped cross sections on the Saskatchewan river at Nordegg bridge and Bragg creek of Elbow river. Basically these data are taken from Brownlie<sup>10</sup> (1981). The 17 data records on the Oak Creek were obtained by Milhaus<sup>11</sup> (1973) have been included in the present study. The data collected by other various investigators such as Griffiths<sup>12</sup> (1981), Thorne and Zevenbergen<sup>13</sup> (1985), Michalik<sup>14</sup> (1989), Colosimo *et al.*<sup>15</sup> (1998), Gladky<sup>16</sup> (1979), Church and Rood<sup>17</sup> (1983), Hey and Thorne<sup>18</sup> (1986), Bathrust<sup>19</sup> (1978), and, Thompson and Campbell<sup>20</sup> (1979) also have been included in the present study. The number of data, range of bed slope and sediment size of included data is mentioned in the *Table 1* mentioned below. This table summarizes the work done by the various scientists in Gravel Bed Rivers across the world.

## III. LITERATURE REVIEW

Gravel bed rivers study poses some complexities that should be mentioned here. The first related to size distribution of bed material as bed material size varies from cobbles to fine sand. The difficulty is experienced in choosing the characteristic size of the bed material because of wide variation in the bed material size. Simultaneously difficulties are caused by the formation of the pavement and its destruction that affect the size distribution of the transport material which also causes a sudden increase in the sediment transport rate. Other difficulties are to calculate the hydraulic geometry, sediment transport and

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Table I : Summary of the Data

S.No.	Investigators	Country	Number of data	Range of slope S	Range of sediment size $d_{50}$ (m)
1.	Williams	U.S.A.	23	0.00085-0.0416	0.021-0.19
2.	Leopold and Wolman	U.S.A.	15	0.0002-0.036	0.034-0.50
3.	U.S.B.R.	San Luis Valley canals	14	0.008-0.00965	0.021-0.082
4.	Kellerhals	Canada	20	0.00017-0.0065	0.007-0.265
5.	Bray	Alberta, Canada	67	0.00022-0.015	0.027-0.145
6.	Samide	Canada	54	0.00154-0.00745	0.014-0.076
7.	Milhaus	Oak creek, U.S.A.	17	0.0079-0.0125	0.008-0.027
8.	Griffiths	New Zealand	52	0.00119-0.00714	0.012-0.051
9.	Thorne and Zevenbergen	Boulder Creek, U.S.A.	12	0.00143-0.0198	0.130-0.162
10.	Michalik	Poland for two rivers	6	0.0002-0.0004	0.031-0.274
11.	Colosimo <i>et al.</i>	South Italy	43	0.0032-0.019	0.05-0.12
12.	Gladky	Poland, Raba R.	30	0.00074-0.00087	0.013
13.	Church and Rood		50	0.002-0.081	0.011-0.268
14.	Hey and Thorne	U.K.	62	0.00119-0.01522	0.014-0.176
15.	Griffiths	New Zealand	46	0.0009-0.011	0.013-0.301
16.	Bathrust	U.K.	9	0.0081-0.0116	0.212
17.	Thompson and Campbell	New Zealand	26	0.0037-0.052	0.03-0.47

the resistance. The resistance plays significant role in all the cited complexities.

Smith<sup>21</sup> (1978) has identified five unit bar features such as:

- longitudinal or spool bars,
- Transverse bars,
- Point bars,
- Diagonal bars and
- Alternate bar

These are affecting the resistance in the flow.

The resistance relationship is the relationship between average velocity  $U$  (m/s) in the channel, water depth  $D$  or hydraulic radius  $R$  in m, longitudinal channel slope  $S$  and the coefficient dependent on the resistance offered by the channel boundary.

Equating the relationship for  $U$ , the following relationship obtained between  $n$ ,  $C$  and  $f$ .

$$\left( \frac{U}{\sqrt{gRS}} \right) = \left( \frac{R^{\frac{1}{6}}}{n\sqrt{g}} \right) = \frac{C}{\sqrt{g}} = \sqrt{\frac{8}{f}} \quad (2)$$

Where  $U$  is Average Velocity of flow in m/s,  $g$  is acceleration due to gravity in  $m/s^2$ ,  $R$  is hydraulic radius in m,  $S$  is longitudinal bed slope of the River,  $n$  is Manning's coefficient,  $C$  is Chezy's coefficient and  $f$  is Darcy Weisbach resistance coefficient. Colosimo *et al.* (1988) recognized that

when Froude number  $\left( \frac{U}{\sqrt{gRS}} \right)$  is greater than 1.65 the

flow in the open channel becomes unstable and hence resistance should be related to relative roughness.

Jie Qin and Sai Leung Ng<sup>22</sup> (2012) investigated through the semivariogram method, based on the second order structure function, can be used to investigate spatial correlation and fractal characteristics of rough surface. The following main conclusions arise from the results.

1. In conjunction with a global vertical scale parameter, such as the standard deviation, the steepness parameter can be used to describe roughness that is dominated.
2. Mixed-fractal behaviour is an important phenomenon for gravel surfaces. At the grain scale, a new parameter—the coarseness parameter—was proposed to account for the influence of roughness in conjunction with the steepness parameter and standard deviation. Both the steepness and coarseness parameters were found to have significant influence on the effective roughness.
3. Simulated elevation fields in conjunction with the  $k - \epsilon$  turbulence model can be used to study the characteristics of random roughness. Special attention should be paid to the boundary description approach when applying the  $k - \epsilon$  model for random rough surfaces. Where  $k$  is turbulent kinetic energy and  $\epsilon$  is energy dissipation.

Bertin and Friedrich<sup>23</sup> (2014) studied to evaluate the hydraulic roughness for gravel beds available measurement techniques. The use of an acoustic bed profiler, a hand-held laser scanner and stereophotogrammetry allowed acquisition of Digital Elevation Models (DEM). Analysis techniques, such as determination of the vertical roughness heights, Probability Distribution Function (PDF) and generalized second-order structure functions, are used to study the random field of bed elevations, as represented by the DEMs. The results show that all three measurement techniques used for this study are capable of recording DEMs with sampling distances small enough to examine an evolving gravel bed at the grain scale in laboratory conditions, using a sediment mixture with  $D_{50} = 7$  mm. Statistical analysis resulted in describing vertical as well as horizontal roughness characteristics. Although, visually, differences in the DEMs obtained with different measurement techniques are observed, the results of the chosen statistical analysis do not disclose the visual differences to the same extent. Improving DEM quality is not only critical for a realistic representation of a surface, but also for interpretation of surface scaling properties, such as obtained with structure function analysis. The accuracy of the DEM obtained with stereophotogrammetry in the first part of this study is inferior to the DEMs obtained with the other techniques, which did not agree with what is theoretically achievable with a stereophotogrammetric system. The authors thus presented changes made in the setup, resulting in the highest gravel-bed DEM resolution obtained with stereophotogrammetry, 30.8 million points per square meter.

Table I represents various investigators who have collected the data of longitudinal bed slope and sediment size ranges. It can be seen that the slope varies from 0.00017 to 0.081 where as the sediment size  $d_{50}$  of the bed material for the gravel Bed Rivers varies from 0.008 meter to 0.50 meter.

Whereas calculated 'n' by Manning's equation for different set of measured data of the gravel bed River at Alaska, USA) has been found in the range of 0.036 to 0.056 (on an average being 0.047) as given in Toniolo *et al.* (2010), a set of friction resistance formulae shows that friction 'f' in the range of 0.066 to 0.075 (semi log formula by Garde), 0.072 to 0.089 (power law by Garde), 0.071 to 0.081 (by Rickenmann and Recking<sup>24</sup> (2011) closely matches amongst each other with overall average value by 0.075 as uniform friction coefficient (Darcy Weisbach) along the river. This value of 'f' could be predicted for deterministic 1D/2D mathematical modeling of hydraulics of Gravel Bed Rivers. Afzalimehr and Anctil<sup>25</sup> (1998) presented semi logarithmic formula which shows higher value of friction in the range of 0.100 to 0.140 for this prototype river and this new formula containing many other parameters are not appears to be applicable. Power law formula proposed by Rickenmann and Recking and Garde have been found appears to be suitable in this prototype river.

### I. PRESENT WORK

Based on the analysis of 706 measured data compiled by Garde (2001) following equation retrieved which have square standard error of the order of 0.095 and the correlation coefficient is 1.00 for the data of Garde, Limerinos<sup>26</sup>, Griffiths, Rickenmann, Keulegan<sup>27</sup> and Bray.

$$\sqrt{\frac{1}{f}} = \frac{2.5\left(\frac{h}{d}\right)^{2.1}}{\left[1 + \left(\frac{h}{d}\right)^2\right]} \quad (3)$$

Where  $f$  is Darcy Weisbach resistance coefficient,  $h$  is depth of flow in m and  $d$  is sediment size in m.

$$\text{Square Standard Error} = \frac{\sum (X_o - X_c)^2}{N - 2} \quad (4)$$

Where  $X_o$  are the observed values,  $X_c$  are the computed values and  $N$  are number of data.

Table II: Square Standard Error

Afzalimehr	Garde	Charlton <i>et al.</i>	Bray	Griffiths
0.001	0.001	0.001	0.00 1	0.001

The square standard error with various investigators is given in table II. The 706 published data compiled by Garde for (as shown in Table I) gravel bed river was used to analyze the resistance by different formulae as shown in the following Table III. It can be seen from the Fig.1 that the prediction made by Garde either power or semi-log are predicting the friction of the same order and seems to be ideal. If it is fact then the value of  $f$  predicted by Dieter Rickenmann and Alain Recking (2011) equation is much better and bounded by Garde's equation. Therefore, the predictions made by other investigators are of the higher side.

In the same way the regression analysis were carried out by using equation of Dieter Rickenmann and Alain Recking (2011), and Garde (2001) for the data of Toniolo *et al.* (2010) . It can be seen the frictional coefficient calculated by present equation is simple and easy to calculate.

### IV. CORRELATION

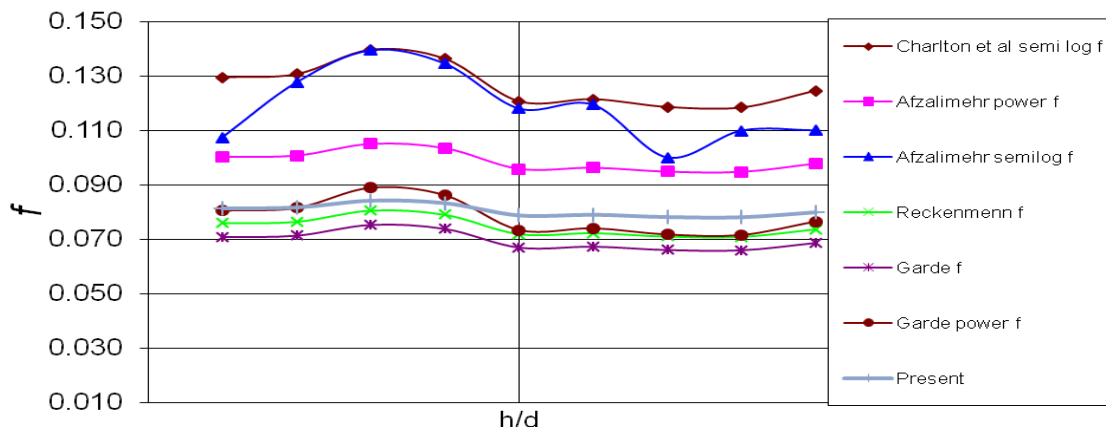
To verify the accuracy of the present work and how this equation correlates with the equations of the other scientists of the world namely Afzalimehr, Garde, Keulegan, Limerinos, Bray and Griffiths; the correlation Coefficient between present work. The result of the correlation is shown in figure 2. From the analysis, it can be seen that the correlation coefficient with Garde, Keulegan, Limerinos, Bray and Griffiths is 0.973 whereas the correlation coefficient with Afzalimehr is 0.785. The correlation coefficient was calculated by the following equation;

$$\text{Correlation}(x, y) = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \sum (y - \bar{y})^2}} \quad 5$$

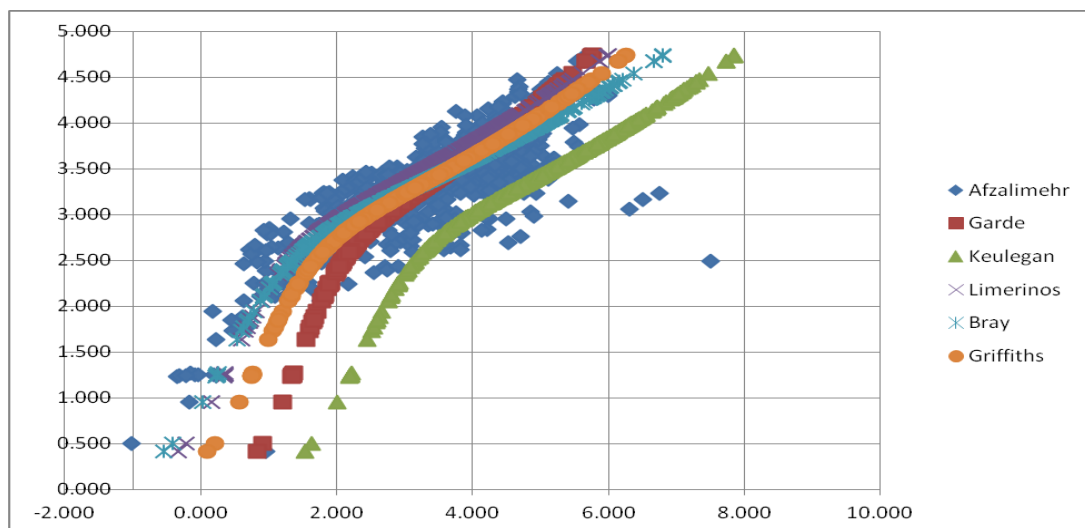
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**Table III : Formulae by Different Investigators**

Investigat ors	Power Formulation	Semi Logarithms
Charlton <i>et al.</i> (1978)	$\sqrt{1/f} = 1.27(h/d)^{0.23}$	$\sqrt{1/f} = 1.94 \log(h + d_{84}/d_{84}) + 0.78$
Bray (1979)	$\sqrt{1/f} = 1.36(h/d)^{0.281}$	$\sqrt{1/f} = 2.36 \log(h/d) + 0.248$
Griffiths (1981)	$\sqrt{1/f} = 1.33(R/d)^{0.287}$	$\sqrt{1/f} = 1.98 \log(R/d) + 0.76$
Garde (2001)	$1/\sqrt{f} = 1.229(h/d)^{0.302}$	$1/\sqrt{f} = 1.586 \log(h/d) + 1.355$
Hossein Afzalimehr (1998)	$\sqrt{1/f} = 1.91(h/d)^{0.148}$	$\sqrt{\frac{1}{f}} = 2.03 \log\left(\frac{\psi h}{d}\right) + 2.96F - 0.18 \frac{\tau_s}{\tau_c} - 0.83$
	$\sqrt{1/f} = 1.91(\psi h/d)^{0.144}$	
	$\sqrt{1/f} = 2.25(h/d)^{0.17} F^{0.33}$	
Dieter Rickenman and Alain Recking (2011)	$\sqrt{\frac{8}{f}} = 4.416 \left(\frac{d}{D_{84}}\right)^{1.904} \left[1 + \left(\frac{d}{1.283 D_{84}}\right)^{1.618}\right]^{-1.083}$	



**Fig.1. Comparison of Flow Resistance in Gravel Bed Rivers by Various Investigators**



**Fig. 2: Correlation of Present Work with other Investigators**

## V. VALIDITY

Although the present equation is an empirical equation for a large data for the sediment varies 7mm to 470mm, whereas the longitudinal bed slope varies from 0.00017 to 0.081 i.e. mild slopes to steep slope. The data used to formulate the present equation is from different parts of the world covers good size coverage and bed slope from 1:12 to 1:5890~1:6km. Therefore, this equation seems to be in better form than the others. Although the flow resistance in gravel bed rivers is site specific but is attempted to generalize the equation on the basis of quantum data.

## VI. CONCLUSION

The recent trend in modeling flow resistance of Gravel Bed Rivers continuous development of knowledge and latest technology should focus on the Gravel Bed Rivers which is in general found in Mountain area. A set of formulae for about 100 rivers around the world (706 measured points) as compiled by Garde and many other hydraulic parameter data on gravel bed rivers reported in literature till date are compiled for developing a new sets of power formula on flow resistance with many parameters for shallow and deep depth river. The present study shows the better prediction of the resistance in gravel Bed Rivers as the equation seems to be simple and covers wide range of the gravel size with different slopes.

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