Combined DCT and Companding with LDPC for PAPR reduction in OFDM signals

S.Balaji, **G.Senthilkumar**

***Abstract*— Orthogonal frequency division multiplexing (OFDM) is an efficient method of data transmission. It is a base for high speed communication system. The main drawback of OFDM system is that suffers high peak to average power ratio(PAPR). Due to high PAPR leads inefficient use of high power amplifier and this could affect the efficiency of transmission. A number of approaches have been proposed to deal with PAPR problem. In this paper a PAPR reduction in OFDM system is based on low-density parity check codes (LDPC) with a standard discrete cosine transform (DCT) and companding technique. This proposed scheme involves applying of LDPC codes on the original signal, applying of DCT on the LDPC output and utilizes the companding technique for reducing the PAPR of the OFDM system further. The performance of the PAPR is evaluated using a computer simulation. The simulation result indicates that the proposed scheme may obtain about 1dB PAPR reduction compared with the conventional reduction systems.**

***Index Terms*— Orthogonal Frequency Division Multiplexing (OFDM), Peak-to-Average-Power Ratio(PAPR), Quadrature Phase Shift Keying(QPSK), Bit-Error-Rate(BER),Discrete Cosine Transform(DCT), Low Density Parity Check(LDPC).**

I. INTRODUCTION

OFDM (Orthogonal Frequency Division Multiplexing) is a promising technique that is able to provide high data rates over multipath fading channels. However, OFDM systems have the inherent problem of a high peak-to–average power ratio (PAPR), which causes poor power efficiency which leads performance degradation in the transmitted signal. To reduce the PAPR, many techniques have been proposed, such as clipping, coding, partial transmit sequence (PTS), selected mapping (SLM)[1-3] and non-linear companding transforms[4,5]. These schemes are primarily signal scrambling techniques, such as PTS and signal distortion techniques such as the clipping and companding techniques. Among those PAPR reduction methods, the simplest scheme to use in the clipping process. However, use of the clipping processing causes both in–band distortion and out-of-band distortion, and causes an increased bit-error-rate (BER) in the system.

As an alternative approach, the companding technique shows better performance than the clipping technique, because the inverse companding transform (expanding) can be applied at the receiver end to reduce the distortion of signal. A DCT may also reduce the PAPR of an OFDM signal, but does not increase the BER of system [6]. So, this proposed PAPR reducing technique based on LDPC and DCT with companding provide further reduction of PAPR of the OFDM system. The forward error correction code LDPC encoder is placed next to data source. The encoder data in the OFDM signal are modulated by an IFFT [Inverse Fast Fourier Transform] after being processed with the DCT and companding, which can reduce PAPR to some extent.

II. PAPR IN OFDM SYSTEMS

The PAPR is the relation between the maximum powers of a sample in a given OFDM transmit symbol divided by the average power of that OFDM symbol. PAPR occurs when in a multicarrier system the different sub-carriers are out of phase with each other. At each instant they are different with respect to each other at different phase values. When all the points achieve the maximum value simultaneously, this will cause the output envelope suddenly raises which causes a 'peak' in the output envelope, due to the presence of large no. of independently modulated sub–carriers is an OFDM system, the peak value of the system can be very high as compared to the average of the whole system. This ratio of the peak to average power value is termed as 'Peak-to-Average Power Ratio'. In OFDM systems, the whole systems bandwidth is divided into many orthogonal sub–channels with narrow bandwidth and the data symbol typically modulated by PSK (Phase Shift Keying) or QAM (Quadrature Amplitude Modulation) are transmitted independently on the subcarriers. An OFDM signal consists of N symbols $X = \{x_k, k=0,1,2,\ldots N-1\}$ and each symbol is modulated by one of the set of sub-carriers $[f_k, k=0,1,2,\ldots N-1]$, where N is the number of sub-carriers. The 'N' sub-carriers are chosen to be orthogonal, that is $f_k = k\Delta f$, where $\Delta f = 1/NT$ (Hz) and T is the original symbol period. Therefore, the complex envelope of the transmitted OFDM signal can be written as,

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi f_k t} \ ,0 \le t \le NT \qquad \text{--- (1)}$$

The PAPR of OFDM signals x(t) is defined as the ratio between the maximum instantaneous power and its average power during its OFDM symbol.

$$\text{PAPR}[x(t)] = \frac{max_{0\le t\le NT}|x(t)^2|}{P_{avg}} \qquad \text{----- (2)}$$

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Where, P_{avg} is the power of x(t) and it can be computed in the frequency domain. Because Inverse Fast Fourier Transform (IFFT) is a (scaled) unitary transformation.

$$P_{avg} = \frac{1}{NT}\int_0^{NT}|x(t)|^2 dt \quad ------ \quad (3)$$

In equation (3), the PAPR reduction of OFDM signals is mainly achieved by minimizing the maximum instantaneous signal power $\max_{0\leq t\leq NT}|x(t)|^2$.

For better approximation, the continuous time OFDM signal x(t) samples over sampled by a factor of L at frequency f_s = L/T, where L is the over sampling factor. This is extended from original signal x(t) by using the zero-padding scheme. i.e., by inserting (L-1) N zeros in the middle of the x(t). The over sample IFFT output with operation length NL can be expressed as

$$x(n) = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} X(k)e^{j\frac{2\pi nk}{LN}} ,0 \leq n \leq LN-1 \quad ----- \quad (4)$$

The PAPR computed from the 'L' times over sampled time domain OFDM signal sample can be defined as

$$\text{PAPR}\{x[n]\} = \frac{\max_{0\leq n\leq LN}|x(n)^2|}{E[|x(n)^2|]} \quad -- (5)$$

Where, E[.] – denotes the expression operator and it will be taken over all OFDM signals.

III. LDPC ENCODER

Low-density parity-check (LDPC) code is a linear error correcting code which is a method of transmitting a message over a noisy transmitting channel. This linear block code is represented as (n,k) code. When n is the code word length (in bits) and K is the no. of message bits i.e. (n-k) is the check bits. Also, these codes are defined as codes using a sparse parity–check matrix with the number of 1's per column (column weight Wc) and the number of 1's per row (row weights Wr), both of which are very small compared to the block length. Basically there are two different possibilities to represent LDPC codes. They can be described through graphical representation or matrix representation. The following priority check matrix with dimensions n x m for a (8,4) code. Further LDPC codes are classified into two groups, regular LDPC cods and irregular LDPC codes. Regular LDPC codes have a uniform column and row weight (i.e Wr = Wc) and irregular LDPC codes have a non-uniform column and row weight. For a matrix to be called low-density, The two conditions W_c << n & Wr<<m must be satisfied.

$$X = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

IV. DISCRETE COSINE TRANSFORM

Like other transforms such as the Hadamard transform, the DCT decorrelates the data sequence. To reduce the PAPR in an OFDM signal, a DCT is applied to reduce the autocorrelation of the input sequence before the IFFT operation is applied.[16].In this section, we review the DCT. The formal definition of a one-dimensional DCT of length N is given by the following formula.

$$X_c(k) = \alpha(k)\sum_{n=0}^{N-1} x(n)\cos\left[\frac{\pi(2n+1)k}{2N}\right]$$

for k=0,1,….N-1

Similarly, the inverse transformation is defined as

$$x(n) = \sum_{n=0}^{N-1}\alpha(k)X_c(k)\cos\left[\frac{\pi(2n+1)k}{2N}\right]$$

for n = 0,1,….N-1

For both equations (1) and (2) α(k) is defined as

$$\alpha(k) = \begin{cases} \frac{1}{\sqrt{N}}, for\ k = 0 \\ \frac{2}{\sqrt{N}}, for\ k \neq 0 \end{cases}$$

Expression (1) can be expressed in matrix form as:

$$X_c = C_N x$$

Where X_c and x are both vectors of dimension N x 1, and C_N, orthogonal matrix vectors. We can use this property of the DCT matrix and reduce the peak power of OFDM signals.

There is a close relation between the PAPR of an OFDM signal and the aperiodic autocorrelation function (ACF) of an input vector. Assume ρ(i) is the ACF of signal vector, then ρ(i) is expressed as

$$\rho(i) = \sum_{k=0}^{N-1-i} X_{k+1}X_k^* \text{ , for I = 0, 1,…N-1}$$

Where the superscript * denotes the complex conjugate. Then, the PAPR of the transformed OFDM signal is bounded by

$$PAPR \leq 1 + \frac{2}{N}\sum_{i=1}^{N-1}|\rho(i)|$$

Let $\lambda = \sum_{i=1}^{N-1}|\rho(i)|$, We found that an input vector passed by DCT transform before IFFT, the ρ(i) and thus PAPR could be reduced.

V. VCOMPANDING TECHIQUE

In this section, companding techniques [5] for the reduction of the PAPR in an OFDM signal is to be received specifically, in μ-Law companding, the PAPR is reduced at the expense of increasing the average power in order to overcome the problem of increase of average power and to have efficient PAPR reduction, a non-linear companding technique namely exponential companding has been developed in this companding transform, a compression is used at the transmit end after the IFFT process and is used expansion at the receiver end prior to the FFT (Fast Fourier Transform) process. For the discrete OFDM signal given below x(n), the companded signal S(n) can be given by

$$s(n) = c\{x(n)\} = \frac{vx(n)}{\ln(1+u)|x(n)|} \, ln\left(1 + \frac{u}{v}|x(n)|\right)$$

average amplitude of the signal and u is the companding parameter. Specifically, the companding transform should satisfy the following two conditions.

$$E(|s(n)|^2) \approx E(|x(n)|^2)$$
$$|s(n)| \geq |x(n)|, for |x(n) \leq v|$$
$$|s(n)| \leq |x(n)|, for |x(n) \geq v|$$

This transform reduces the PAPR of OFDM signal by amplifying the small signal and attenuating the period of high signal. On the receiver end, the receiver signal must be expanded by the inverse companding transform before it can be sent to the FFT processing unit. The expanded signal at the receiver is

$$y(n) = C^{-1}\{r(n)\} = \frac{vr(n)}{u|r(n)|}\left\{exp\left[\frac{|r(n)|\ln(1+u)}{v}\right] - 1\right\}$$

VI. PROPOSED SCHEME

To reduce the PAPR of an OFDM signal, the proposed techniques contain combined LDPC and DCT with Companding transform. The input data stream is first processed by LDPC, then with DCT. Next the signal processed by IFFT and companding transform.
The steps followed in the proposed scheme are described as below:
Step 1: Input sequence apply to LDPC encoder to generate parity - check Matrix X.

Step 2: Sequence X is transformed using the DCT matrix i.e. Y= DCT (X)

Step 3: Sequence Y is applied to IFFT .i.e $y = [y(1), y(2), y(3), \ldots . y(N)]^T$

Step 4: A companding transform is then applied to y. i.e. $s(n) = C\{y(n)\}$

Step 5: An inverse companding transform is applied to the received signal
r(n). i.e. $\hat{y}(n) = C^{-1}\{r(n)\}$

Step 6: A FFT transform is applied to the signal $\hat{y}$. i.e. $\hat{y} = FFT[\hat{y}(n)]$
where $\hat{y} = [\hat{y}(1), \hat{y}(2), \ldots . \hat{y}(N)]^T$

Step 7: An IDCT transforms applied to the signal $\hat{y}$ .i.e. $\hat{X} = IDCT[\hat{y}]$

Step 8: LDPC decoding is applied to $\hat{X}$

Step 9: The sequence $\hat{X}$ demapped to form output bit stream.

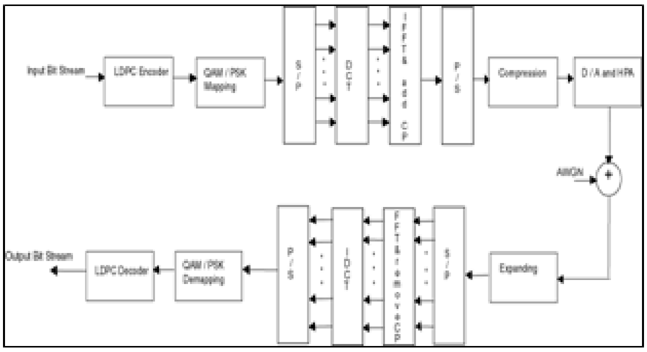

Fig.1.Block diagram of OFDM system with LDPC, DCT and Companding

VII. SIMULATION RESULT

In this section, the computer simulated output presented used to evaluate PAPR reduction capability relatively BER of the proposed scheme. The channel was modeled as additive white Gaussian noise (AWGN). In the simulation, an OFDM system with a sub-carrier of N = 128 and QAM modulation was considered. The performance of the PAPR reduction scheme using the complementary cumulative distribution function (CCDF) of the PAPR of the OFDM system was evaluated.

A. CCDF PERFORMANCE

The performance of PAPR can be evaluated using the cumulative distribution in OFDM signal. The cumulative distribution function (CDF) is one of the most regularly used parameters, which is used to measure the efficiency of PAPR technique. The CDF of the amplitude of a signal sample is given by

$$F(z) = 1 - \exp(z)$$

However, the complementary CDF is used instead of CDF, which helps us to measure the probability that the PAPR of a certain data block exceeds the given threshold. The CCDF of the PAPR of the data block is desired is our case to compare various reduction techniques. This is given by

$$P(PAPR > z) = 1 - P(PAPR > z) = 1 - (1 - \exp(-z))^N$$

Fig.2 shows the CCDF performance of the DCT scheme with various companding factor, u. With this method, the peak power at CCDF = 10^{-3} is reduced by 1dB to 2dB based on companding factor.

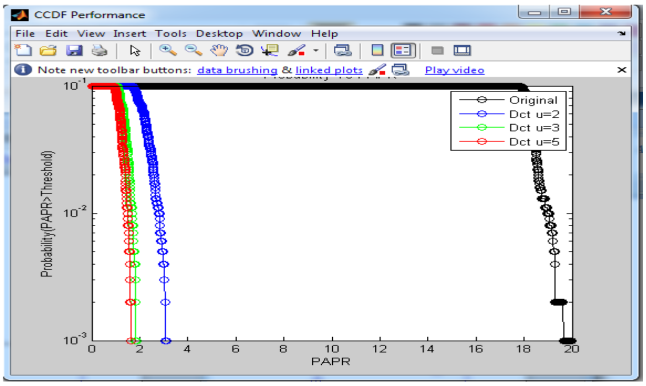

Fig.2 Comparison of CCDF among DCT and Companding

Fig.3 shows the CCDF performance of a companding algorithm for PAPR reduction with LDPC. the companding factor, u for the companding procedure of the second step were fixed to 2,3 and 5. With this companding method with LDPC, the power at CCDF = 10^{-3} is reduced to some extent.

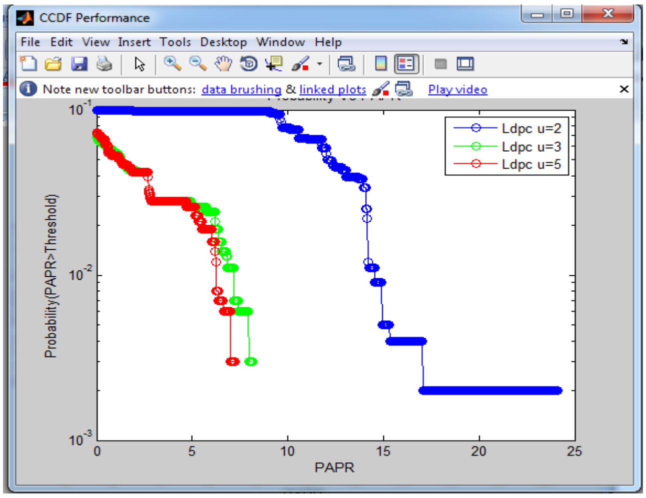

Fig 3. Comparisons of CCDF among LDPC and Companding

Fig. 4 shows the CCDF performance of a companding algorithm with LDPC and DCT. At CCDF = 10^{-3} , the DCT scheme with LDPC reduces the PAPR further and depends on companding factor also.

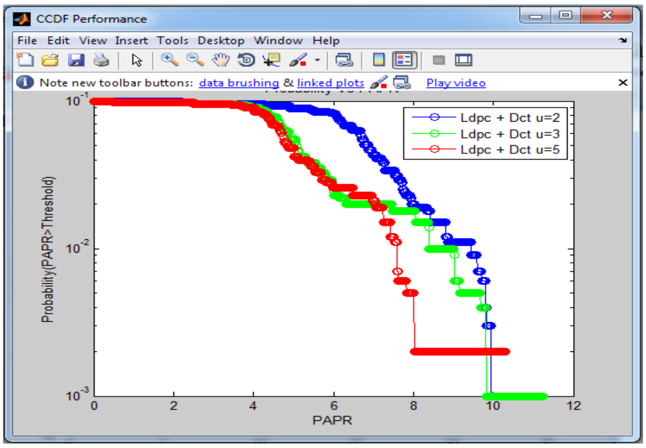

Figure 4.Comparisons of CCDF with LDPC,DCT and Companding

B. ALGORITHM COMPLEXITY

Compared to the ordinary companding algorithm, the computational complexity of the proposed scheme is increased because of LDPC and DCT is also used along with companding algorithm.

VIII. CONCLUSION

In this paper while taking both PAPR performance and BER performance into account, we proposed a LDPC, DCT and computing scheme for the reduction of the PAPR OF OFDM system. The PAPR reduction performance of the proposed scheme was evaluated using a computer simulation. The simulation result shows that the PAPR reduction is improved if LDPC is inducted when compared with those of DCT and companding techniques are done or both combined.

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