Two Methods of Obtaining a Minimal Upper Estimate for the Error Probability of the Restoring Formal Neuron

A. I. Prangishvili, O. M. Namicheishvili, M. A. Gogiashvili

***Abstract*— It is shown that a minimal upper estimate for the error probability of the formal neuron, when the latter is used as a restoring (decision) element, can be obtained by the Laplace transform of the convolution of functions as well as by means of the generating function of the factorial moment of the sum of independent random variables. It is proved that in both cases the obtained minimal upper estimates are absolutely identical.**

***Index Terms*—generating function, probability of signal restoration error, restoring neuron, upper estimate.**

I. Introduction

Let us consider the formal neuron, to the inputs of which different versions $X_1, X_2, \dots, X_n, X_{n+1}$ of one and the same random binary signal X arrive via the binary channels $B_1, B_2, \dots, B_n, B_{n+1}$ with different error probabilities $q_i \left(i=\overline{1,n+1}\right)$, and the neuron must restore the correct input signal X or, in other words, make a decision Y using the versions $X_1, X_2, \dots, X_n, X_{n+1}$. When the binary signal X arrives at the inputs of the restoring element via the channels of equal reliability, the decision-making, in which some value prevails among the signal versions, i.e. the decision-making by the majority principle, was for the first time described by J. von Neumann [1], and later V. I. Varshavski [2] generalized this principle to redundant analog systems.

In the case of input channels with different reliabilities, adaptation of the formal neuron is needed in order to restore the correct signal. Adaptation is interpreted as the control process of weights $a_i \left(i=\overline{1,n+1}\right)$ of the neuron inputs, which makes these weights match the current probabilities $q_i \left(i=\overline{1,n+1}\right)$ of the input channels. The purpose of this control is to make inputs of high reliability to exert more influence on decision-making (i.e. on the restoration of the correct signal) as compared with inputs of low reliability. Restoration is carried out by vote-weighting by the relation

$$Y = \operatorname{sgn}\left(\sum_{i=1}^{n+1} a_i X_i\right) = \operatorname{sgn} Z\,, \tag{1}$$

where

$$Z = \sum_{i=1}^{n+1} a_i X_i\ . \tag{2}$$

Both the input signal X and its versions $X_i (i=\overline{1,n})$ are considered as binary random variables coded by the logical values $(+1)$ and (-1). It is formally assumed that the threshold Θ of the restoring neuron is introduced into consideration by means of the identity $\Theta \equiv a_{n+1}$, where $(-\infty < a_{n+1} < \infty)$ and the signal $X_{n+1} \equiv -1$. The main point of this formalism is that the signal $X_{n+1} \equiv -1$ is dumped from some imaginary binary input B_{n+1} for any value of the input signal X, whereas the value q_{n+1} is the a priori probability of occurrence of the signal $X = +1$ or, which is the same, the error probability of the channel B_{n+1}. Quite a vast literature [3]-[7] is dedicated to threshold logic which takes into consideration the varying reliability of channels, but in this paper we express our viewpoint in the spirit of the ideas of W. Pierce [8].

Let us further assume that

$$\operatorname{sgn} Z = \begin{cases} -1 \ \text{ if } Z < 0 \\ +1 \ \text{ if } Z \geq 0 \end{cases}. \tag{3}$$

When $Z = 0$, the solution Y at the output of the restoring formal neuron has the form +1 according to (3). The probability that the restored value Y of the signal X is not correct is expressed by the formula

$$Q = \operatorname{Prob}\{Y \neq X\} = \operatorname{Prob}\{\eta < 0\}. \tag{4}$$

Here $\eta = XZ$ is a discrete random variable with probability distribution density $f(v)$. This variable is the sum of independent discrete variables $\eta_i = a_i X X_i$, and the function $f_i(v_i)$ describes the probability distribution density of individual summands η_i. For the realizations of random variables η and η_i we introduce the symbols v and v_i, respectively.

It is easy to observe that the variable η_i takes the values $+a_i$ and $-a_i$ with probabilities $1-q_i$ and q_i, respectively. Therefore, if we use the Dirac delta function $\delta(t)$, then the probability density $f_i(v_i)$ can be represented as follows

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A. I. Prangishvili, Faculty of Informatics and Control Systems, Georgian Technical University, 0171 Tbilisi, Georgia, +995 591191700, (e-mail: a_prangi@gtu.ge).

O. M. Namicheishvili, Faculty of Informatics and Control Systems, Georgian Technical University, 0171 Tbilisi, Georgia, +995 593573139, (e-mail: o.namicheishvili@gtu.ge).

M. A. Gogiashvili, School (Faculty) of Informatics, Mathematics and Natural Sciences, St. Andrew the First-Called Georgian University of the Patriarchate of Georgia, 0162 Tbilisi, Georgia, +995 599305303, (e-mail: maia.gogiashvili@yahoo.com).

$$\left.\begin{array}{l} f_i(v_i) = (1-q_i)\delta(v_i - a_i) + q_i\delta(v_i + a_i) \\ v_i = +a_i, -a_i \\ i = \overline{1, n+1} \end{array}\right\}. \qquad (5)$$

Such formalism is completely justified and frequently used due to the following two properties of the delta-function

$$\left.\begin{array}{l} \delta(t) \geq 0, \forall t \in \mathbb{R} \\ \int_{-\infty}^{+\infty} \delta(t)dt = 1 \end{array}\right\}.$$

However $f_i(v_i)$ can also be represented as

$$\left.\begin{array}{l} f_i(v_i) = q_i^{(a_i - v_i)/2a_i} \cdot (1-q_i)^{(a_i + v_i)/2a_i} \\ v_i = +a_i, -a_i \\ i = \overline{1, n+1} \end{array}\right\}. \qquad (6)$$

The random variable η is the sum of independent discrete random variables η_i. Its distribution density $f(v)$ can be defined in the form of convolution of probability distribution densities of summands $f_i(v_i)$:

$$f(v) = \mathop{*}_{i=1}^{n+1} f_i(v_i), \qquad (7)$$

where $*$ (superposition of the addition and multiplication signs) is the convolution symbol.

It is obvious that in view of formula (7) the error probability at the decision element output can be written in two equivalent forms

$$\begin{aligned} Q &= \text{Prob}(v<0) = \int_{-\infty}^{0} f(v)dv = \int_{-\infty}^{0} \mathop{*}_{i=1}^{n+1} f_i(v_i)dv = \\ &= \int_{-\infty}^{0} \mathop{*}_{i=1}^{n+1} \left[(1-q_i)\delta(v_i - a_i) + q_i\delta(v_i + a_i)\right]dv \end{aligned} \qquad (8)$$

and

$$\begin{aligned} Q &= \sum_{v<0} f(v) = \sum_{v<0} \mathop{*}_{i=1}^{n+1} f_i(v_i) = \\ &= \sum_{v<0} \mathop{*}_{i=1}^{n+1} \left[q_i^{(a_i - v_i)/2a_i} \cdot (1-q_i)^{(a_i + v_i)/2a_i}\right], \end{aligned} \qquad (9)$$

where the probability distribution density $f_i(v_i)$ is defined by (5) in the first case and by (6) in the second case. Integration or summation in both cases is carried out continuously or discretely over all negative values of the variable v. Formulas (8) and (9) give an exact value of the error probability of restoration of a binary signal by the formal neuron.

Note that for practical calculations, formula (9) can be written in a more convenient form. Indeed, the complete number of discrete values of the variable v is 2^{n+1} since

$$v = \widetilde{a}_1 + \widetilde{a}_2 + \cdots + \widetilde{a}_n + \widetilde{a}_{n+1},$$

where $\widetilde{a}_i$ is equal either to $+(a_i)$ or to $-(a_i)$, whereas the proper sign of the weight a_i is meant to be within the round brackets.

By formula (9), to each discrete value of the sum v there corresponds the term Q_j $(j = \overline{1, 2^{n+1}})$ which is the product of $(n+1)$ co-factors of the form q_k or $(1-q_k)$.

In particular

$$\left.\begin{array}{l} Q_j \equiv f(v) = \mathop{*}_{i=1}^{n+1} f_i(v_i) \equiv \widetilde{q}_1 \cdot \widetilde{q}_2 \cdot \cdots \widetilde{q}_n \cdot \widetilde{q}_{n+1} \\ j = \overline{1, 2^{n+1}} \end{array}\right\},$$

where

$$\widetilde{q}_k = \begin{cases} q_k & \text{if} \quad v_k = -(a_k) \\ 1-q_k & \text{if} \quad v_k = +(a_k) \end{cases}$$

for any $k \left(k = \overline{1, n+1}\right)$.

Thus formula (9) can also be written in the form

$$Q = \sum_{v<0} Q_j = \sum_{v<0} \widetilde{q}_1 \cdot \widetilde{q}_2 \cdot \cdots \cdot \widetilde{q}_n \cdot \widetilde{q}_{n+1}, \qquad (10)$$

which is more adequate for cognitive perception and practical realization.

II. FINDING A MINIMAL UPPER ESTIMATE BY THE FIRST METHOD

From the expression

$$Q = \text{Prob}(\eta < 0) = \int_{-\infty}^{0} f(v)dv$$

it follows that for a real positive number s $(s > 0)$

$$Q \leq \int_{-\infty}^{0} e^{-sv} f(v)dv \leq \int_{-\infty}^{\infty} e^{-sv} f(v)dv.$$

But the left-hand part of this inequality is the Laplace transform of the function $f(v)$

$$\mathfrak{L}[f(v)] = \int_{-\infty}^{\infty} e^{-sv} f(v)dv,$$

where $\mathfrak{L}$ is the Laplace transform operator.

Therefore

$$Q \leq \mathfrak{L}[f(v)]. \qquad (11)$$

The random value η with realizations v is the sum of independent random variables η_i having realizations v_i. In that case, as is known, the Laplace transform for the convolution $f(v)$ of functions $f_i(v_i)$ is equal to the product of Laplace transforms of convoluted functions:

$$\mathfrak{L}[f(v)] = \prod_{i=1}^{n+1} \mathfrak{L}[f_i(v_i)].$$

The latter implies that

$$Q \leq \prod_{i=1}^{n+1} \mathfrak{L}[f_i(v_i)]. \qquad (12)$$

By expression (5) for functions $f_i(v_i)$ and the Laplace transform definition, we obtain

$$\begin{aligned} \mathfrak{L}[f_i(v_i)] &= \int_{-\infty}^{\infty} e^{-sv_i} f_i(v_i)dv_i = \\ &= \int_{-\infty}^{\infty} e^{-sv_i} \left[(1-q_i)\delta(v_i - a_i) + q_i\delta(v_i + a_i)\right]dv_i \end{aligned}.$$

Using this expression in formula (12) we have

$$Q \leq \prod_{i=1}^{n+1} \int_{-\infty}^{\infty} e^{-sv_i} \left[(1-q_i)\delta(v_i - a_i) + q_i\delta(v_i + a_i)\right]dv_i. \qquad (13)$$

Here we should make use of one more property of the Dirac delta function

$$\int_{-\infty}^{\infty} g(t)\delta(t - t_0)dt = g(t_0).$$

With this property taken into account, from formula (13) we obtain

$$Q \le \prod_{i=1}^{n+1} \left[(1-q_i)e^{-a_i s} + q_i e^{+a_i s} \right]. \tag{14}$$

Here s, as mentioned above, is an arbitrary real positive number. Before we continue simplifying the right-hand part of inequality (14), we have to define a value of s for which expression (14) gives *a minimal upper estimate*.

Passing to the natural logarithm of inequality (14) we come to the expression

$$\ln Q \le \sum_{i=1}^{n+1} \ln \left[(1-q_i)e^{-a_i s} + q_i e^{+a_i s} \right].$$

Let us define here partial derivatives with respect to arguments a_i by using the elementary fact that

$$y' = \frac{dy}{dx} = f'(x) \cdot e^{f(x)}$$

if $y = e^{f(x)}$, and also the fact that $\frac{d}{dx} \ln f(x) = f'(x) \cdot \frac{1}{f(x)}$.

Hence we obtain

$$\frac{\partial \ln Q}{\partial a_i} \le \sum_{i=1}^{n+1} \frac{1}{[(1-q_i)e^{-a_i s} + q_i e^{+a_i s}]} \cdot [s q_i e^{+a_i s} - s(1-q_i)e^{-a_i s}].$$

For the right-hand part of this inequality to be equal to zero, it suffices that the following condition be fulfilled

$$s q_i e^{+a_i s} - s(1-q_i)e^{-a_i s} = 0,$$

whence it follows that

$$e^{+2a_i s} = \frac{1-q_i}{q_i},$$

or, which is the same,

$$a_i s = \frac{1}{2} \ln \frac{1-q_i}{q_i}.$$

If the weights a_i of the neuron inputs are put into correspondence with error probabilities q_i of these inputs by the relations

$$a_i = \ln \frac{1-q_i}{q_i}, \tag{15}$$

then the sought value of s will be

$$s = \frac{1}{2}. \tag{16}$$

Using equality (16) in formula (14), we obtain a minimal upper estimate for the error probability Q of the restoring neuron. Indeed, for the right-hand part of expression (14) the following chain of identical transforms is valid:

$$q_i e^{\frac{a_i}{2}} + (1-q_i)e^{-\frac{a_i}{2}} = 2\sqrt{q_i(1-q_i)} \cdot \frac{q_i e^{\frac{a_i}{2}} + (1-q_i)e^{-\frac{a_i}{2}}}{2\sqrt{q_i(1-q_i)}} =$$

$$= 2\sqrt{q_i(1-q_i)} \cdot \frac{\sqrt{\frac{q_i}{1-q_i}} \cdot e^{\frac{a_i}{2}} + \sqrt{\frac{1-q_i}{q_i}} \cdot e^{-\frac{a_i}{2}}}{2}.$$

Let us take into account here that

$$\left. \begin{aligned} \sqrt{\frac{q_i}{1-q_i}} &= \exp\left[\ln\left(\sqrt{\frac{q_i}{1-q_i}} \right) \right] = \exp\left[-\frac{1}{2} \ln \frac{1-q_i}{q_i} \right] \\ \sqrt{\frac{1-q_i}{q_i}} &= \exp\left[\ln\left(\sqrt{\frac{1-q_i}{q_i}} \right) \right] = \exp\left[+\frac{1}{2} \ln \frac{1-q_i}{q_i} \right] \end{aligned} \right\}.$$

Besides, we denote

$$\frac{1}{2}\left(a_i - \ln \frac{1-q_i}{q_i} \right) = \lambda_i.$$

Then we have

$$q_i e^{\frac{a_i}{2}} + (1-q_i)e^{-\frac{a_i}{2}} = 2\sqrt{q_i(1-q_i)} \cdot \frac{e^{\lambda_i} + e^{-\lambda_i}}{2}.$$

The second co-factor in the right-hand part of this expression is the hypebolic cosine of the argument λ_i:

$$\frac{e^{\lambda_i} + e^{-\lambda_i}}{2} = \operatorname{ch} \lambda_i.$$

Therefore

$$q_i e^{\frac{a_i}{2}} + (1-q_i)e^{-\frac{a_i}{2}} = 2\sqrt{q_i(1-q_i)} \cdot \operatorname{ch} \lambda_i =$$

$$= 2\sqrt{q_i(1-q_i)} \cdot \operatorname{ch}\left(\frac{a_i - \ln \frac{1-q_i}{q_i}}{2} \right).$$

Finally, for estimate (14) we can write

$$Q \le \prod_{i=1}^{n+1} \left\{ 2\sqrt{q_i(1-q_i)} \cdot \operatorname{ch}\left(\frac{a_i - \ln \frac{1-q_i}{q_i}}{2} \right) \right\}.$$

For the error probability Q, the right-hand part of the above inequality is the upper estimate Q^+:

$$Q^+ = \prod_{i=1}^{n+1} \left[2\sqrt{q_i(1-q_i)} \cdot \operatorname{ch}\left(\frac{a_i - \ln \frac{1-q_i}{q_i}}{2} \right) \right].$$

The minimum $Q^+_{\min}$ of this upper estimate Q^+ is equal to

$$Q^+_{\min} = \prod_{i=1}^{n+1} \left[2\sqrt{q_i(1-q_i)} \right] = 2^{n+1} \cdot \prod_{i=1}^{n+1} \left[\sqrt{q_i(1-q_i)} \right]. \tag{17}$$

It is attained when for the zero argument the hyperbolic cosine attains the minimum equal to 1.

This estimate confirms in a certain sense the advantage of the choice of weights of the restoring neuron in compliance with the error probabilities of input signals according to the following relations

$$\left. \begin{aligned} a_i &= \ln \frac{1-q_i}{q_i} \\ i &= \overline{1, n+1} \end{aligned} \right\}.$$

III. OBTAINING A MINIMAL UPPER ESTIMATE BY THE SECOND METHOD

Simulteously, for the probability Q it is useful to obtain a minimal upper estimate in the closed analytic form by one more new approach.

As is known [9], the generating function $\gamma_v(S)$ of the factorial moment of the sum η of independent random variables η_i is equal to the product of generating functions $\gamma_{v_i}(S)$ of the factorial moments of individual summands, i.e.

$$\gamma_v(S) = \prod_{i=1}^{n+1} \gamma_{v_i}(S), \tag{18}$$

where

$$\gamma_v(S) = \mathrm{M}\left[S^{\eta}\right] = \sum_v S^v \cdot f(v), \tag{19}$$

$$\left.\begin{aligned} &\gamma_{v_i}(S) = \mathrm{M}\left[S^{\eta_i}\right] = \sum_{v_i} S^{v_i} \cdot f_i(v_i) \\ &i = \overline{1, n+1} \end{aligned}\right\}. \tag{20}$$

Here M is the mathematical expectation symbol and S is an arbitrary complex number for which series (19) and (20) exist on some segment of the real axis containing the point $S = 1$.

Since in relation (20), summation is carried out on the set of two possible values $+a_i$ and $-a_i$ of the variable v_i, using (6) we have

$$\left.\begin{aligned} &\gamma_{v_i}(S) = (1-q_i)S^{a_i} + q_i S^{-a_i} \\ &i = \overline{1, n+1} \end{aligned}\right\}. \tag{21}$$

The substitution of (21) into relation (18) gives

$$\gamma_v(S) = \prod_{i=1}^{n+1}\left[(1-q_i)S^{a_i} + q_i S^{-a_i}\right].$$

When $v < 0$, the value S^v satisfies the condition

$$S^v = \frac{1}{S^{(v(}} > 1,$$

if of course

$$0 < S < 1. \tag{22}$$

Let us assume that inequality (22) is fulfilled. Then the following relation is valid:

$$Q = \sum_{v<0} f(v) < \sum_{v<0} S^v \cdot f(v).$$

Since every summand $S^v \cdot f(v)$ is non-negative, we have the inequality

$$\sum_{v<0} S^v f(v) \le \sum_v S^v \cdot f(v).$$

Therefore

$$Q < \gamma_v(S). \tag{23}$$

The right-hand part of this expression can be taken as the upper estimate Q^+ of the error probability Q of the restoring neuron

$$Q^+ = \prod_{i=1}^{n+1}\left[(1-q_i)S^{a_i} + q_i S^{-a_i}\right].$$

The latter relation is easily rewritten in the equivalent form

$$Q^+ = \prod_{i=1}^{n+1} Q_i^+ = \prod_{i=1}^{n+1}\left[(1-q_i)w_i + \frac{q_i}{w_i}\right], \tag{24}$$

where

$$Q_i^+ = (1-q_i)w_i + \frac{q_i}{w_i}$$

and, along with this,

$$w_i = S^{a_i}, 0 < w_i < \infty, (i = \overline{1, n+1}). \tag{25}$$

Now we can find the minimum $Q_{\min}^+$ of expression (24) and the value w_{0i} of w_i will attach a minimum to the upper estimate of the error probability Q^+ of the restoring neuron. For this, we use the conditions

$$\left.\begin{aligned} &\frac{\partial Q^+}{\partial w_i} = 0 \\ &i = \overline{1, n+1} \end{aligned}\right\}.$$

Hence

$$w_{0i} = \sqrt{\frac{q_i}{1-q_i}}, \ (i = \overline{1, n+1}). \tag{26}$$

If (26) is substituted into expression (24), then by the second method for a minimal upper estimate of the error probability of the restoring neuron we obtain the relation

$$Q_{\min}^+ = 2^{n+1} \cdot \prod_{i=1}^{n+1}\left[\sqrt{q_i(1-q_i)}\right], \tag{27}$$

which coincides with result (17) obtained by the first method.

The weights $a_i (i = \overline{1, n+1})$ which match the error probabilities $q_i \left(i = \overline{1, n+1}\right)$ are defined from relations (26) with notation (25) taken into account:

$$\left.\begin{aligned} &a_i = \frac{1}{2\ln S} \cdot \ln\frac{q_i}{1-q_i} \\ &i = \overline{1, n+1} \end{aligned}\right\}.$$

Since the value S satisfies condition (22), we have $\ln S < 0$ and therefore

$$\left.\begin{aligned} &a_i = K \cdot \ln\frac{1-q_i}{q_i} \\ &i = \overline{1, n+1} \end{aligned}\right\}, \tag{28}$$

where

$$\left.\begin{aligned} &K = \frac{1}{2|\ln S|} \\ &0 < K < \infty \end{aligned}\right\}. \tag{29}$$

Thus, the weights $a_i (i = \overline{1, n+1})$, which are consistent with the error probabilities $q_i \left(i = \overline{1, n+1}\right)$ and attach a minimum to the upper estimate of the error probability of the restoring neuron, are defined to within the a general positive factor K.

IV. CONCLUSION

A minimal upper estimate of the error probability of the restoring formal neuron is defined by formula (17) or, which is the same, by formula (27). In both cases the result can be written in the form

$$Q_{\min}^+ = \exp\left(-\sum_{i=1}^{n+1} A(q_i)\right), \tag{30}$$

where

$$A(q_i) = \left|\ln\left[2\sqrt{q_i(1-q_i)}\right]\right|. \quad (31)$$

In view of relations (31) confirming the non-negativity of the values $A(q_i)$, formula (30) implies that an increase of the number n of inputs of the formal decision neuron bings about a monotone decrease of the minimal upper estimate of the error probability of restoration of the binary signal $Q^+_{\min}$ by the exponential law if, certainly, the error probabilities $q_i\left(i=\overline{1,n+1}\right)$ at these inputs are not equal to $\frac{1}{2}$ when the minimal upper estimate of the error probability $Q^+_{\min}$ is equal to 1.

This result is demonstrates an essenatial inner connection with Shannon's theorem [10]. According to this theorem, the number of messages of length n (duration τ) composed of individual symbols – both in the absence and in the presence of fixed and probabilistic constraints (in the latter case it is assumed that the source is ergodic) – grows by the asymptotically exponential law as n (or τ) increases. In particular we understand this connection as follows: as the number n of inputs of the restoring formal neuron increases, the initial information to be used in making the decision Y increases by the exponential law if there are a number of possible versions of the input signal, while the minimal upper estimate $Q^+_{\min}$ of the probability Q that the made decision is erroneous decreases by the same exponential law.

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Archil I. Prangishvili was born April 13, 1961, in Tbilisi (Georgia). During 1978-1983, he was the student of Faculty of Automatics and Computer Engineering at Georgian Polytechnic Institute. During 1983-1987, he was the Post graduated student at Georgian Polytechnic Institute. Currently, he is the Doctor of Technical Sciences, Full professor at Faculty of Informatics and Control Systems of Georgian Technical University, full member (academician) of the Georgian National Academy of Sciences, President of the Georgian Engineering Academy, Member of the International Engineering Academy and the International Academy of Informatization of the UN (United Nations), Rector of Georgian Technical University. Archil I. Prangishvili is the expert in the field of Computer Science, Applied Expert Systems, Artificial Intelligence and Control Systems, head of Editorial Board of the Journal "Automated Control Systems" and member of the Editorial Board of the Georgian Electronic Scientific Journal (GESJ): Computer Science and Telecommunications. Number of published works - more than 120. including 7 monographs and 5 textbooks.

Oleg M. Namicheishvili received the Master of Science (Radio physics) from Ivane Javakhishvili Tbilisi State University in 1965. During 1965-1968, he studied at the Graduate School of Tbilisi State University and he was a trainee at the Department of theory of probability in Moscow State University. In 1968 he received the degree of PhD (Engineering Cybernetics and Information Theory) and in 1989 he was doing D.Sc.(Tech.) in Georgian Technical University. Over a long period of time Oleg Namicheishvili was an internship at CNET (Centre National d' Études des Télécommunications, France, Lannion) - a French research laboratory in telecommunications. He was now working in Faculty of Informatics and Control Systems of Georgian Technical University as Full Professor and ALT (Accelerated Life Tests) Project Director. He has a teaching job and his research affected the reliability of information systems. He has published more than 150 research papers in various journals. He is a full member (academician) of the Georgian Engineering Academy and Deputy Editor-in-Chief of the Georgian Electronic Scientific Journal (GESJ): Computer Science and Telecommunications.

Maia A. Gogiashvili was born September 23, 1971 in Tbilisi (Georgia). In 1994 she graduated Ivane Javakhishvili Tbilisi State University, faculty of Physics. Since 1994 she is teacher of physics and mathematics at Grigol Robakidze University. From 2005 she is an expert at the National Examinations Center of the Ministry of Education and Science of Georgia. Currently she is a doctoral candidate of St. Andrew the First-Called Georgian University of the Patriarchate of Georgia at Faculty of Information Science, Mathematics and Natural Sciences. She has published more than 3 research papers in various scientific journals.