Design and Development of Experimental setup for Identification of SDoF with Harmonic Excitation

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Abstract— **The primary objective of vibration analysis is to study response behavior of dynamic systems and excitation forces associated with it. The technology of vibration testing has rapidly evolved since World War II and the technique has been successfully applied to a wide spectrum of products. Vibration testing is usually performed by applying a vibratory excitation to a test object and monitoring the structural integrity and performance of the intended function of the object.**

In this paper theoretical, experimental and numerical analysis of SDoF system is carried out to find out damping coefficient. The system identification is also carried out to identify the various parameters of the system. e.g. ω_n, m, k, ξ, etc. of a single degree of freedom system.

Index Terms— **Dynamic systems, Excitation, Monitoring, Structural Integrity, Vibration Testing.**

I. Introduction

Vibratory motion is repeated indefinitely and exchange of energy takes place. The structures designed to support the high speed engines and turbines are subjected to vibration. Due to faulty design and poor manufacture there is unbalance in the engines which causes excessive and unpleasant stresses in the rotating system because of vibration. The vibration causes rapid wear of machine parts such as bearings and gears. Unwanted vibrations may cause loosening of parts from the machine. Because of improper design or material distribution, the wheels of locomotive can leave the track due to excessive vibration which results in accident or heavy loss. Many buildings, structures and bridges fall because of vibration. If the frequency of excitation coincides with one of the natural frequencies of the system, a condition of resonance is reached, and dangerously large oscillations may occur which result in mechanical failure of the system. Hence vibration analysis is important in design field. The single-degree-of-freedom (SDOF) system is the most widely used and simplest model for vibration analysis. The SDOF system is a simple but worthy model because it quantifies many results of an isolation system [1].

A. Determination of Equivalent Viscous Damping from Frequency Response Curve [2]:

Damping is a phenomenon by which mechanical energy is dissipated, usually converted as a thermal energy in dynamic systems [3].The damping in a system can be obtained from free vibration decay curve where the free vibration test is not practical. The damping may be obtained from the frequency response curve of forced vibration test.

The frequency – response curve as obtained for a system excited with a constant force, as shown in Fig. 1.1

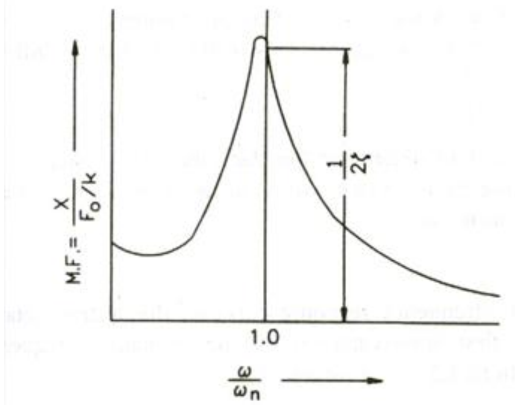

Fig. 1.1 Determination of Equivalent Viscous Damping from Frequency Response Curve

The magnification at resonance is given by ½ ξ. It is difficult, however to get the exact resonance point since the peak point occurs slightly away from the resonance. If the amplitude of vibration and the magnification can be found out at resonance then the damping factor is given by –

$$\xi = \frac{1}{2(M.F.)_{res}}$$

The fact that the phase difference between the exciting force and displacement is 90^0 at resonance is made to use in locating the resonant point. The phase difference (ϕ) between the force and the displacement is given by

$$\phi = \sin^{-1}\frac{X}{Y}$$

To determine the damping in a system if you have just frequency response curve of the system with constant excitation, damping in the system is determined by assuming the damping is of viscous nature.

As we know that,

$$\frac{X_P}{X_{st}} = \frac{1}{2\xi} \quad (1.1)$$

By drawing a horizontal line at X = 0.707 X_p, cutting the response curve at two points the corresponding value of abscissa being ω_1 and ω_2.

The magnification factor is given by equation,

Manuscript received Feb. 11, 2014.

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$$\frac{X}{X_{st}} = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

For point A and B then equation can be written as,

$$\frac{0.707X_p}{X_{st}} = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}} \qquad (1.2)$$

In above equation ω_p has been written in place of ω_n because it is taken that $\omega_n = \omega_p$ as a first approximation. Also the values of ω in the equation corresponding to ω_1 and ω_2.

From equation (1.1) an d (1.2),

$$\frac{0.707X_p}{2\xi} = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

Finally we have,

$$\frac{\omega_2-\omega_1}{2} = 2\xi \qquad (1.3)$$

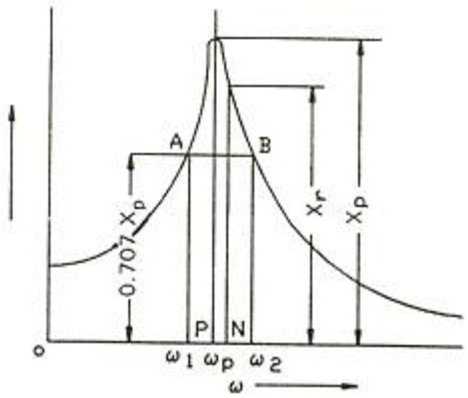


Fig 1.2 Determination of Equivalent Viscous Damping from Frequency Response Curve

Or $\frac{AB}{OP} = 2\xi$

$$\xi = \frac{1}{2}\frac{AB}{OP} \qquad (1.4)$$

Taking these measurement from the frequency response curve after making necessary construction, we get the first approximate value of ξ.

To get an accurate value of ξ, use ξ as obtained above to determine the nature frequency of the system from equation,

$$\left(\frac{\omega_p}{\omega_n}\right) = \sqrt{1-2\xi^2}$$

$$\omega_n = \left(\frac{\omega_p}{\sqrt{1-2\xi^2}}\right) \qquad (1.5)$$

Mark this point as N on the abscissa and draw an ordinate as shown in Fig.1.2 . The height of this ordinate upto the curve gives the resonance amplitude x_r.

Now repeat the process by first drawing a horizontal line at a height $X = 0.707\ X_r$ and then finding the frequencies ω_1 and ω_2 corresponding to the point of intersection. Then in a similar manner it can be shown that –

$$2\xi = \frac{\omega_2-\omega_1}{\omega_n} \qquad (1.6)$$

Hence an accurate value of ξ can be found by construction and measurements.

B. System Identification from Frequency Response:

System identification means to identify the various parameters of the system. e.g. ω_n, m, k, ξ, etc. of a system.

In order to do this, a known constant force F_o at a variable frequency is applied to the system and the frequency response curve similar to the Fig. 1.2.is obtained.

Different parameters can be obtained as follows,

I. Obtain ω_n and ξ as per equation,

$$\omega_n = \left(\frac{\omega_p}{\sqrt{1-2\xi^2}}\right), \text{ and}$$

$$2\xi = \frac{\omega_2-\omega_1}{\omega_n}$$

II. The magnification at resonance ($\omega = \omega_n$) can be obtained as,

$$\frac{X_P}{X_{st}} = \frac{1}{2\xi}$$

Where X_r is the resonant amplitude

$$\frac{X_r}{\left(F_0/K\right)} = \frac{1}{2\xi}$$

$$K = \frac{F_o}{2\xi X_r}$$

III. The natural frequency ω_n having been obtained at step I above and k at step II

$\omega_n = \sqrt{\frac{K}{m}}$ thus the mass is obtained as

$$m = \frac{K}{\omega_n^2}$$

IV. Similarly the damping coefficient c can be obtained as,

$$C = 2\xi\sqrt{Km}$$

II. DESIGN AND EXPERIMENTATION

A. *Conceptual Design*

After studying the problem thoroughly different possible solutions came up out of that most practical and cost effective experimental setup were developed [4] as shown in Fig.2.1. For excitation electric motor and the cam and follower mechanism is used. The cam is coupled with the motor and motor is connected to the dimmer-stat. Helical spring is used in order to include the freedom. There are various devices which are used to measure the vibration level in the system.

Fig.2.1 Experimental setup for Analysis of SDOF Vibration

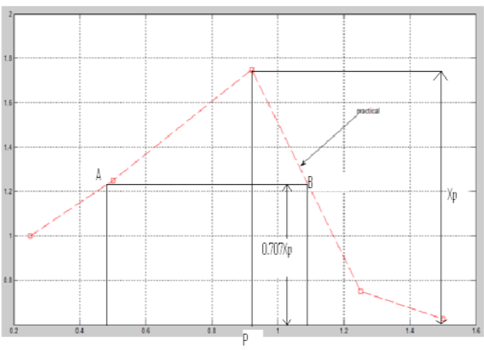
Following are the experimental results found out during experiment for analysis of SDoF vibrations.

Table 2.1.Magnification Factor and Frequency Ratio

X(mm)	U(mm)	ω(rad/sec)	ω_n(rad/sec)	X/U	ω/ω_n
4	4	18.485	73.94	1	0.25
5	4	36.97	73.94	1.25	0.50
7	4	68.29	73.94	1.75	0.92
3	4	92.425	73.94	0.75	1.25
2.5	4	110.91	73.94	0.625	1.5

The graph of magnification factor vs. frequency ratio is plotted as shown in Fig.2.2

X/U



ω/ω_n →

Fig 2.2 Graph between Magnification Factor vs. Frequency Ratio for Experimental Results

III. THEORETICAL ANALYSIS

For Theoretical Analysis consider the system excited by support as shown in Fig.3.1, According to support excitation theory,

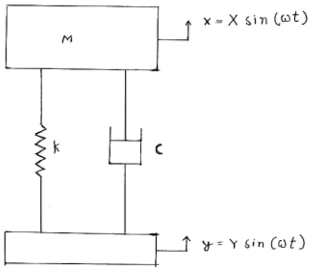


Fig. 3.1 Support Excitation

$$\frac{X}{U} = \frac{\sqrt{1+(2\xi\lambda)^2}}{\sqrt{(1-\lambda^2)^2+(2\xi\lambda)^2}} \tag{3.1}$$

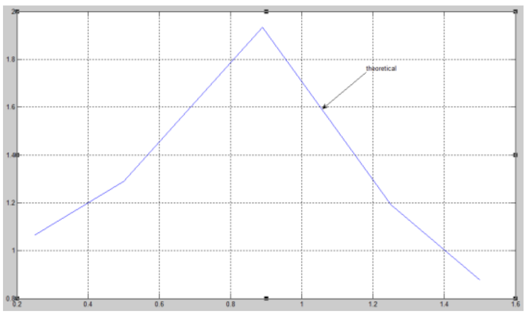
Where, $\lambda = \frac{\omega}{\omega_n}$

Values of magnification factor are established out for various frequency ratios.

$\lambda\ (\frac{\omega}{\omega_n})$	0.25	0.5	0.89	1.25	1.5
X/U	1.065	1.289	1.934	1.192	0.876

Graph is plotted as shown in Fig.3.2 for various values of magnification factor which are found out for various frequency ratios.

X/U



ω/ω_n →

Fig.3.2 **Graph between Magnification Factor vs. Frequency Ratio for Theoretical Results**.

IV. NUMERICAL ANALYSIS

For numerical analysis MATLAB software is used to validate of the experimental and theoretical results. MATLAB program is developed to find out various values of magnification factor for various frequency ratios. Equation 3.1 is modeled in MATLAB to obtain the following results.

$\lambda\ (\frac{\omega}{\omega_n})$	02	0.4	1	1.2	1.4
X/U	1.041	1.176	1.894	1.441	1.023

Graph is plotted as shown in Fig.3.3 for various values of magnification factor which are found out for various

Parameter	**ξ**	**K (N/m)**	**$\omega_p = \omega_n$ (rad/sec)**	**C (N-sec/m)**
Value	0.3108	4291	73.94	64.38

frequency ratios.

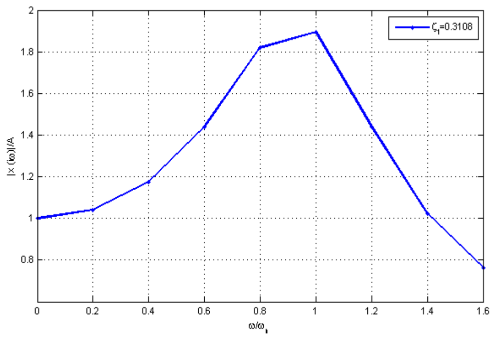


Fig 4.3 Graph between Magnification Factor vs. Frequency Ratio for Numerical Results

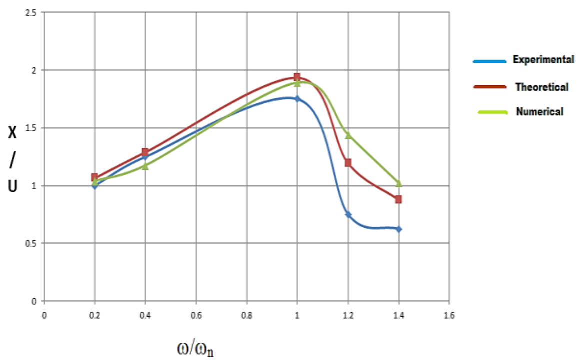


Fig 4.4 Magnification Factor vs. Frequency Ratio for Theoretical, Experimental and Numerical Results

Now, from theoretical results the various parameters of the system.e.g. ω_n, m, k, ξ, etc. of system were determined as follows

Parameter	ξ	K (N/m)	$\omega_p = \omega_n$ (rad/sec)	C(N-sec/m)
Value	0.3108	4291	73.94	70.52

If you are using *Word,* use either the Microsoft Equation Editor or the *MathType* add-on (http://www.mathtype.com) for equations in your paper (Insert | Object | Create New | Microsoft Equation *or* MathType Equation). "Float over text" should *not* be selected.

V. Conclusion

In this paper analysis of SDoF system is carried theoretically, experimentally and numerically to find out damping coefficient. System identification is also carried out to identify the various parameters of the system. e.g. ω_n, m, k, ξ, etc. of a SDoF system. It is found out that there is slight variation in experimental, theoretical and numerical values which are may be due to errors such as measuring or computational errors.

Acknowledgment

The preferred spelling of the word "acknowledgment" in American English is without an "e" after the "g." Use the singular heading even if you have many acknowledgments. Avoid expressions such as "One of us (S.B.A.) would like to thank ... ." Instead, write "F. A. Author thanks ... ." **Sponsor and financial support acknowledgments are placed in the unnumbered footnote on the first page**.

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