

DEVELOPMENT OF TRADITIONAL OPTIMIZATION APPROACH FOR OPTIMAL SYNTHESIS OF PATH GENERATOR LINKAGE

Sandeep M. Deshmukh, M.K. Sonpimple, M.S. Giripunje

Abstract— The study of four-bar linkage is to trace a desired path is an important part of obtaining the optimal geometry of a four-bar linkage which is used in design. When the number of the precision points exceeds a certain number, it is not possible to apply or it is difficult to apply analytical methods. Alternatively some intelligence optimization methods can be used based on the complexity of the problem. Path synthesis of four-bar linkages with multiple numbers of precision points has been illustrated as an optimization problem. The objective to be minimized is sum square error of all the points reached by the coupler point. The important constraints included are, 1) Grashof's condition to have a crank rocker linkage, 2) the transmission angle criterion and, 3) input link angle sequence. The variables included are the link lengths and angles. The first multivariate optimization technique we will examine is one of the simplest: gradient descent (also known as steepest descent). Gradient descent is an iterative method that is given an initial point, and follows the negative of the gradient in order to move the point toward a critical point, which is hopefully the desired local minimum. A computer programme based on this algorithm is implemented in MATLAB to obtain the optimum dimension of four-bar linkage.

Index Terms—optimal synthesis, four bar linkage, objective function, steepest descent optimization algorithm.

I. INTRODUCTION

Gradient descent is popular for very large-scale optimization problems because it is easy to implement, can handle "black box" functions, and each iteration is cheap. Its major disadvantage is that it can take a long time to converge, also a discrete descent method often used to solve large combinatorial problems. Many machine design problems require creation of a device with particular motion characteristics. Synthesis of mechanism refers to design a linkage for a prescribed motion or path or velocity of tracing joint or link there are types of synthesis technique available in literature. The following methods of synthesis are commonly found in literature: (i) Qualitative synthesis, which is a creation of potential solution in the absence of an algorithm that configures or predicts solution, (ii) type synthesis, which is a definition of proper type of

mechanism best suited to the problem and is a form of qualitative synthesis and (iii) dimensional synthesis, referring to the determination of lengths of links necessary to accomplish the desired motion. Type synthesis refers to the kind of mechanism selected; it might be a linkage, a geared system, belts and pulleys, or even a cam system. This beginning phase of the total design problem usually involves design factors such as manufacturing processes, materials, safety, space, and economics. The study of kinematics is usually only slightly involved in type synthesis. Number synthesis deals with the number of links and the number of joints or pairs that are required to obtain certain mobility. Number synthesis is the second step in the design. The third step in design namely determining the dimensions of the individual links is called dimensional synthesis.

II. COUPLER POINT COORDINATES

In the problem of four-bar linkage synthesis there is some number of precision points to be traced by the coupler point P. To trace the coupler point, the dimension of the links (a, b, c, d, L_x , L_y) is to be determined along with the input crank angle θ_2 , so that the average error between these specified precision points ($P_{x_{di}}, P_{y_{di}}$), (where $i=1, 2, \dots, N$ with N as number of precision points given) and the actual points to be traced by the coupler point P gets minimized. The objective or error function can be calculated when the actual traced points (P_x, P_y) is evaluated which is traced by the coupler point P with respect to the main coordinate from X, Y as shown in Fig.2.1.

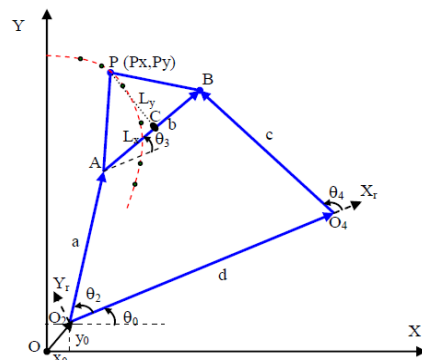


Fig.2.1 Four-bar linkage with ABP as coupler link

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First Author name, Mechanical Engineering, Priyadarshing college of engineering, Nagpur, India, +918237339932.

Second Author name, Mechanical Engineering, Priyadarshing college of engineering, Nagpur, India, +919423054182.

Third Author name, Mechanical Engineering, Priyadarshing college of engineering, Nagpur, India, +919763166605.

The position vector of the coupler point P reference frame X_r-Y_r can be expressed as a vector equation:

$$\vec{r}^P = \vec{a} + \vec{L}_x + \vec{L}_y \quad (2.1)$$

which can be represented in its components according to:

$$P_{x_r} = a \cos \theta_2 + L_x \cos \theta_3 + L_y (-\sin \theta_3) \quad (2.2)$$

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$$P_y = a \sin \theta_2 + L_x \sin \theta_3 + L_y \cos \theta_3 \quad (2.3)$$

Here, for calculation the coupler point coordinates (Px, Py), we have to first compute the coupler link angle θ_3 using the following vector loop equation for the four-bar linkage:

$$\vec{a} + \vec{b} - \vec{c} - \vec{d} = \mathbf{0} \quad (2.4)$$

This equation also can be expressed in its components with respect to relative coordinates:

$$a \cos \theta_2 + b \cos \theta_3 - c \cos \theta_4 - d \quad (2.5)$$

$$a \sin \theta_2 + b \sin \theta_3 - c \sin \theta_4 = 0 \quad (2.6)$$

We can compute the angle θ_3 for known values of θ_2 and eliminating θ_4 so, the result will be

$$K_1 \cos \theta_3 + K_4 \cos \theta_2 + K_5 = \cos(\theta_2 - \theta_3) \quad (2.7)$$

$$\text{where } K_1 = d/a, K_4 = d/b \text{ and } K_5 = \frac{c^2 - d^2 - a^2 - b^2}{2ab} \quad (2.8)$$

For this equation following two solutions are obtained:

$$\theta_3^1 = 2 \tan^{-1} \left(\frac{-E + \sqrt{E^2 - 4DF}}{2D} \right) \quad (2.9)$$

$$\theta_3^2 = 2 \tan^{-1} \left(\frac{-E - \sqrt{E^2 - 4DF}}{2D} \right) \quad (2.10)$$

whereas,

$$D = \cos \theta_2 - K_1 + K_4 \cos \theta_2 + K_5, E = -2 \sin \theta_2$$

$$\text{and } F = K_1 + (K_4 - 1) \cos \theta_2 + K_5 \quad (2.11)$$

These solutions may be (i) real and equal (ii) real and unequal and (iii) complex conjugates. If the discriminates $E^2 - 4DF$ is negative, then solution is complex conjugate, which simply means that the link lengths chosen are not capable of connection for the chosen value of the input angle θ_2 . This can occur either when the link lengths are completely incapable of connection in any position. Except this there are always two values of θ_3 corresponding to any one value of θ_2 . These are called, (i) crossed configuration (plus solution) and (ii) Open configuration of the linkage (minus solution) and also known as the two circuits of the linkage. The other methods such as Newton-Raphson solution technique can also be used to get approximate solution for θ_3 . The position of coupler P, with respect to world coordinate system XOY is finally defined by:

$$P_x = x_0 + P_x \cos \theta_0 - P_y \sin \theta_0 \quad (2.12)$$

$$P_y = y_0 + P_x \sin \theta_0 + P_y \cos \theta_0 \quad (2.13)$$

1) POSITION ERRORS AS OBJECTIVE FUNCTION:

The objective function is usually used to determine the optimal link lengths and the coupler link geometry. In path synthesis problems, this part is the sum squares which computes the position error of the distance between each calculated precision point (Px_i, Py_i) and the desired points (Pxd_i, Pyd_i) which are the target points indicated by the designer. This is written as:

$$f(X) = \sum_{i=1}^N [(Pxd_i - Px_i)^2 + (Pyd_i - Py_i)^2] \quad (2.14)$$

Whereas, X is set of variables to be obtained by minimizing this function. Some authors have also considered additional objective functions such as the deviation of minimum and maximum transmission angles μ_{\min} and μ_{\max} from 90° , for all the set of initial solutions considered.

2) THE CONSTRAINTS OF THE LINKAGE

The synthesis of the four-bar mechanism greatly depends upon the choice of the objective function and the equality or the inequality constraints which is imposed on the solution to get the optimal dimensions. Generally the objective function is minimized under certain conditions so that the solution is satisfied by a set of the given constraints. The bounds for variables considered in the analysis are treated as 20 one set of constraints, while the other constraints include: Grashof condition, input link order constraint and the transmission angle constraint.

2.1) Grashof criterion

For Grashof criterion, it is required that one of the links of mechanism, should revolve fully by 360° angle. There are three possible Grashof linkages for a four-bar crank chain: (a) Two crank-rocker mechanisms (adjacent link to shortest is fixed) (b) One double crank mechanism (shortest link is fixed) and (c) One double rocker mechanism (opposite to shortest link is fixed). Of all these, in the present task, only crank-rocker mechanism configuration is considered. Here, the input link of the four-bar mechanism to be crank. Grashof criterion states that the sum (L_s+L_l) of the shortest and the longest links must be lesser than the sum (L_a+L_b) of the rest two links. That is:

$$(L_s + L_l) \leq L_a + L_b$$

$$\text{(or)} \quad 2(L_s + L_l) \leq a + b + c + d$$

$$\text{(or)} \quad g1 = 2(L_s + L_l) / (a + b + c + d) - 1 \leq 0 \quad (2.15)$$

In the present work violation is defined as follows:

$$\text{Grashof's} = 1 \text{ if } g1 > 0$$

$$\text{Or } = 0 \text{ if } g1 \leq 0$$

2.2) Input link angle order Constraint:

Usually a large combination of the mechanisms exists that generates the coupler curves passing through the desired points, but those solutions may not satisfy the desired order. To ensure that the final solution honors the desired order, testing for any order violation is imposed. This is achieved by requiring that the direction of rotation of the crank as defined by the sign of its angular increments $\Delta \theta_2^i = (\theta_2^i - \theta_2^{i-1})$, between the two positions i and i-1, where $i=3,4,5,\dots,N$, have same direction as that between the 1st and the 2nd positions ($\theta_2^2 - \theta_2^1$). That checks the following:

Is $\text{sign}(\Delta\theta_2^i) = \text{sign}(\theta_2^1 - \theta_2^i)$ for all $i=3$ to N ? (2.16)

whereas, $\text{sign}(Z)=1$ if $Z \geq 0$

$$= -1 \text{ if } Z < 0 \quad (2.16a)$$

If this condition is not satisfied the solution is rejected.

2.3) Transmission Angle Constraint

For a crank-rocker mechanism generally the best results the designers recognize when the transmission angle is close to 90 degree as much as possible during entire rotation of the crank. Alternatively, the transmission angle during entire rotation of crank should lie between the minimum and maximum values. This can be written as one of the constraints as follows. First of all, the expressions for maximum and minimum transmission angles for crank-rocker linkage are defined.

$$\begin{aligned} \mu_{\max} &= 2 \cos^{-1} \left(\frac{b^2 - (d+a)^2 + c^2}{2bc} \right) \\ \mu_{\min} &= 2 \cos^{-1} \left(\frac{b^2 - (d-a)^2 + c^2}{2bc} \right) \end{aligned} \quad (2.17)$$

The actual value of transmission angle at any crank θ_2^i angle is given by:

$$\mu_{\max} = 2 \cos^{-1} \left(\frac{b^2 - a^2 - d^2 + c^2 + 2ad \cos \theta_2^i}{2bc} \right) \quad (2.18)$$

The condition to be satisfied is: $\mu_{\min} \leq \mu \leq \mu_{\max}$ (2.19)

The constraint given by equations (2.15), (2.16) and (2.19) are handled by penalty method. That is the non-dimensional constraint deviation is directly added to the objective function for minimization. For example, constraint eq.(2.19) if not satisfied, the penalty term is given as follows:

$$Trans = \sum_{i=0}^n (1 - Trans_{\min})(\mu - \mu_{\min})^2 + (1 - Trans_{\max})(\mu - \mu_{\max})^2$$

Whereas,

$$\begin{aligned} Trans_{\min} &= \text{sign}(b^2 + c^2 - (d-a)^2 - 2bc \cos \mu_{\min}) \\ Trans_{\max} &= \text{sign}(2bc \cos \mu_{\max} - b^2 + c^2 - (a+d)^2) \end{aligned}$$

Thus the solution seeks to obtain a feasible set of optimum values.

2.3) Variable Bounds

All variables considered in the design vector should be defined within pre specified minimum and maximum values. Often, this depends on the type of problem. For example, if we have 19 variables in a 10 point optimization problem, all the variables may have different values of minimum and maximum values. Generally, in non-conventional optimization techniques starting with set of initial vectors, this constraint is handled at the beginning itself, while defining the random variable values. That is we use the following simple generation rule:

$$X = X_{\min} + \text{rand}(X_{\max} - X_{\min})$$

Whereas, rand is a random number generator between 0 and 1.

III. OVERALL OPTIMIZATION PROBLEM

The objective function is the sum of the error function and the penalties assessed to violation the constraints as follows:

$$F(k) = f(X) + W1 \times \text{Grashof} + W2 \times \text{Tran},$$

whereas,

W1 is the weighting factor of the Grashof's criteria and W2 is the weighting factor of the Transmission angle constraints. These additional terms act as scaling factors to fix the order of magnitude of the different variables present in the problem or the objective function.

IV. GRADIENT-BASED METHODS

Gradient-based search method discussed in this section exploits the derivative information of the function and is usually faster search methods. Since these methods use gradient information and since the objective functions in many engineering optimization problems are not differentiable, they cannot be applied directly to engineering design problems. For the same reason, the Gradient-based method cannot be applied to problems where the objective function is discrete or discontinuous or the variable is discrete. On the contrary, in problems where the derivative information is easily available, gradient-based methods are very efficient. However, the concept of gradient in understanding the work principle of the optimization algorithm is so intricate that gradient based methods are most used in engineering design problems by calculating the derivatives numerically. In the following subsection, we describe algorithm of these kind.

- I) Cauchy's (Steepest Descent) method
- II) Newton's method
- III) Marquadt's method
- IV) Conjugate gradient method
- V) Variable-metric method (DFP method)

Optimization algorithms have been discussed to solve multivariable functions. Thereafter, four direct search algorithms have been discussed followed by five different gradient-based search methods.

Among the Gradient-based methods, the Steepest Descent method is implemented in a present work. Steepest Descent method works on the principle of generating new search direction iterative and performing a unidirectional search along each direction.

A. STEEPEST DESCENT METHOD

The method of steepest descent is the simplest of the gradient methods. Imagine that there's a function $F(x)$, which can be defined and differentiable within a given boundary, so the direction it decreases the fastest would be the negative gradient of $F(x)$. To find the local minimum of $F(x)$, The Method of The Steepest Descent is employed, where it uses a zigzag like path from an arbitrary point X_0 and gradually slide down the gradient, until it converges to the actual point of minimum.

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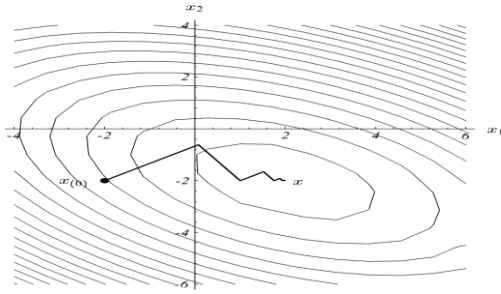


FIG. 4.1 The Method of Steepest Descent finds the local minimum through iterations, as the figure shows, it starts with an arbitrary point x_0 and taking small steps toward the direction of Gradient since it is the direction of fastest changes and stops at the minimum

It is not hard to see why the method of steepest descent is so popular among many mathematicians: it is very simple, easy to use, and each repetition is fast. But the biggest advantage of this method lies in the fact that it is guaranteed to find the minimum through numerous times of iterations as long as it exists. However, this method also has some big flaws: If it is used on a badly scaled system, it will end up going through an infinite number of iterations before locating the minimum, and since each of steps taken during iterations are extremely small, thus the convergence speed is pretty slow, and this process can literally take forever. Al-though a larger step size will increase the convergence speed, but it could also result in an estimate with large error.

B. The gradient is the steepest descent direction.

The first order Taylor approximation of $f(x)$ about $f(x_1)$ is,

$$f(x) = f(x_1) + \nabla f(x_1) \cdot (x - x_1) + O(\|x - x_1\|^2). \quad (4.1)$$

Consider moving from x_1 a small amount h in a unit direction u . We want to find the u that minimizes $f(x_1+hu)$. Using the Taylor expansion, we see that

$$f(x_1 + hu) - f(x_1) = h\nabla f(x_1)u + h^2O(1). \quad (4.2)$$

If we make the h^2 term insignificant by shrinking h , we see that in order to decrease $f(x_1+hu) - f(x_1)$ the fastest we must minimize $\nabla f(x_1) \cdot u$. The unit vector that minimizes $\nabla f(x_1) \cdot u$ is $u = -\nabla f(x_1) / \|\nabla f(x_1)\|$ as desired.

4.3) Algorithm

The algorithm is initialized with a guess x_1 , a maximum iteration count N_{max} , a gradient norm tolerance ϵ_g that is used to determine whether the algorithm has arrived at a critical point, and a step tolerance ϵ_x to determine whether significant progress is being made. It proceeds as follows.

- 1 For, $k = 1, 2, \dots, N_{max}$:
- 2 $x^{k+1} \leftarrow x_k - \alpha_k \nabla f(x^k)$
- 3 If $\|\nabla f(x^{k+1})\| < \epsilon_g$ then return "Converged on critical point"
- 4 If $\|x_k - x^{k+1}\| < \epsilon_x$ then return "Converged on an x value"
- 5 If $f(x^{k+1}) > f(x^k)$ then return "Diverging"
- 6 Return "Maximum number of iterations reached"

The variable α_k is known as the step size, and should be chosen to maintain a balance between convergence speed and avoiding divergence. Note that α_k may depend on the step k . Note that the steepest descent direction at each iteration is orthogonal to previous one. Therefore the method zigzags in the design space and is rather inefficient. The algorithm is guaranteed to converge, but it may take an infinite number of iterations. The rate of converge is linear. Usually, a substantial decrease is observed in the few iteration, but the method is very slow. Where,

x^k, x^{k+1} = Values of variable in k and $k+1$ iteration

$f(x)$ = Objective function to be minimized

$\nabla f(x)$ = Gradient of objective function

α_k = The size of the step in direction of travel

The Method of The Steepest Descent, also known as The Gradient Descent, is the simplest of the gradient methods. By using simple optimization algorithm, this popular method can find the local mini-mum of a function. Its concept is very easy to under-stand: we start by simply picking an arbitrary point x_0 that is within a function's range and take small steps towards the direction of greatest slope changes, which is the direction of the gradient, and eventually, after many iterations, we can find the minimum of the function.

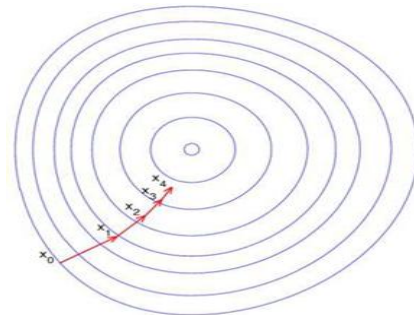


FIG. 4.2 The Method of Steepest Descent finds the local minimum through iterations, as the figure shows, it starts with an arbitrary point x_0 and taking small steps toward the direction of Gradient since it is the direction of fastest changes and stops at the minimum

It is popular because of its conceptual simplicity, easy to use, with fast iterations and it some badly scaled system, then its slow convergence will cause it to run numerous iterations process that will take forever before the minimum is located. There are many useful applications of the method of steepest descent; the most common would be using it for a complex integral in order to find the saddle points. This is a truly diverse function that no personal with math background should overlook.

V. PATH SYNTHESIS PROBLEM HAVING SIX POINTS AND FIFTEEN VARIABLES

1) Six Points Path Generation and 15 design variables.

The first case is a path synthesized problem with given six target points arranged in a vertical line without prescribed timing.

Design variables are:

$$X = [a, b, c, d, l_y, l_x, \theta_2^2, \theta_2^3, \theta_2^4, \theta_2^5, \theta_2^6, \theta_0, l_{y0}, l_{x0}]$$

Target points: = [(20, 20), (20, 25), (20, 30), (20, 35), (20, 40), (20, 45)]

Limits of the variable:

$$a, b, c, d, \in [5,60];$$

$$lyo, lxo, ly, lx, \in [-60,60];$$

$$\theta_2^1, \theta_2^2, \theta_2^3, \theta_2^4, \theta_2^5, \theta_2^6 \in [0, 2\pi];$$

The synthesized geometric parameters and the corresponding values of the precision points (Pxd, Pyd) and the traced points by the coupler point (Px,Py) and the difference between them are shown in table (5.1) and table (5.1) respectively . Although the constraint of the sequence of the input angles during the evolution is ignored in this case .The accuracy of the solution in case has been remarkably improved using the present method. Fig (5.1.A) shows the convergence of Steepest descent algorithm .Fig (5.1.B) shows the six target points and the coupler curve obtained using the Steepest descent search method

Table (5.1.1.) Synthesized results for case 1.

a	b	c	d	lx	ly	θ_2^1	θ_2^2	θ_2^3	θ_2^4
6.0	23.	46.	59.	-23.	19.	4.2	4.6	5.0	5.380
437	93	88	52	740	137	820	48	087	4
	27	74	80	6	8		0		

θ_2^5	θ_2^6	θ_0	lxo	lyo
5.8136	6.3502	1.2193	43.8344	22.5562

Table(5.1.1.) The actual points which is traced by the coupler link and the precision points.

SL NO	Px	Pxd	(Px-pxd)	(Px-pxd) ²	Py	Pyd	(Py-pyd)	(Py-pyd) ²
1	17.7	20.0	-2.2	5.12	20.86	20.0	0.86	0.745
	372	000	628	04	31	00	31	0
2	19.2	20.0	-0.7	0.53	25.04	25.0	0.04	0.002
	704	000	296	24	61	00	61	1
3	20.3	20.0	0.33	0.11	29.81	30.0	-0.1	0.035
	348	000	48	21	26	00	873	1
4	20.9	20.0	0.90	0.81	34.81	35.0	-0.1	0.035
	026	000	26	42	09	00	891	7
5	20.7	20.0	0.78	0.61	39.91	40.0	-0.0	0.008
	865	000	68	85	07	00	893	0
6	19.2	20.0	-0.7	0.52	43.86	45.0	-1.1	1.282
	760	000	240	42	74	00	326	8

Multiple Solution of Coupler curve

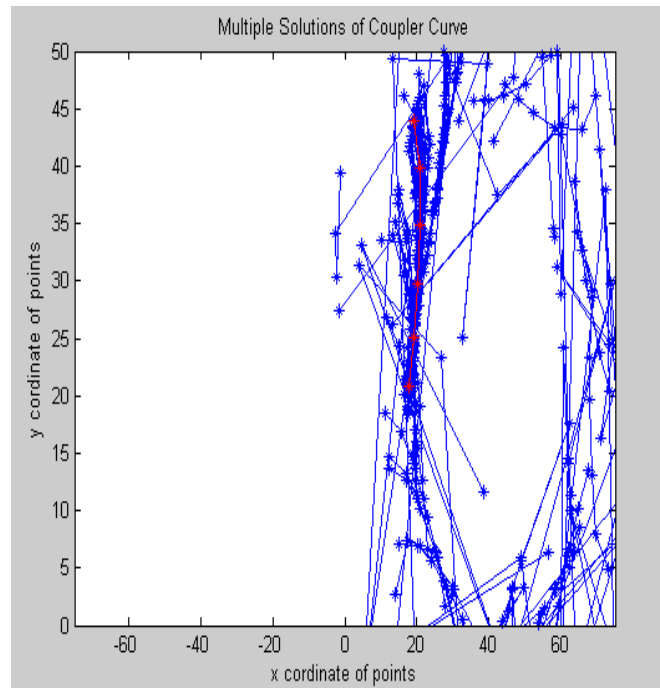


FIG (5.1.A) multiple solution of coupler curve

Coupler curve for 6points, 15 variables

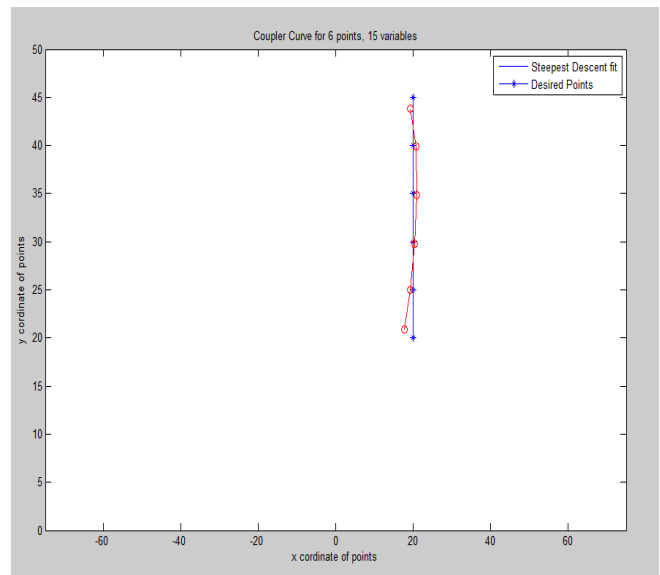


Fig (5.1.B) six target points and the coupler curve obtained

VI. CONCLUSION

In the present work we have consider a crank rocker mechanism of four-bar linkage. The objective function namely path error varies with respect to the number of precision points specified. It is found that in case the computational time for convergence of 10,000 cycles changes. In some examples even the constraint violation is maintained, the minimum value of the objective function is found to be close to the published results available in the literature by other methods. In each case the convergence of multiple iteration graph, coupler curves & tables of optimum dimensions and final coupler point coordinate were reported. In this method, we can find out the simplicity, an algorithm is

used to solve the optimization problems. The creative idea of dividing the path into smaller parts and finding the minimum error of each part from the points made the problem easier and help us to decrease the error. Especially when the number of precision points increases, convergence occurs earlier. This work presents an error function which could measure the goodness of matching between the predefined curve and generated curve, as a function of coupler motion. This error function defined on the basis of curvature of the desired and generated curve. As a result, the method is applied to four bar planar mechanism and showed that it needs lesser calculation time. Although the illustrative examples were four bar planar mechanism, the objective function is constructed so that the algorithm can be easily applied to other path generation mechanisms.

VII. FUTURE SCOPE

Even this Paper has concentrated on path synthesis part with some important constraints, some more constraints like mechanical advantage of the linkage, and flexibility effects can be also considered to get the accuracy. Also as in hybrid synthesis approach, the same linkage may be adopted both for path synthesis applications as well as motion synthesis applications. The objective function should be modified so as to get a different optimum link dimensions. Finally fabrication of the proto-type of this linkage may be done to know the difference between theoretically obtained coupler coordinates and actual values achieved. These approaches are very useful for the kinematician those who are working in the field of mechanism synthesis, robotics, assembly line, automation material handling and conveyors.

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